Chapter 2
Direct Variation

Mathematical Overview

Proportional relationships are introduced by having students conduct an Activity and an Investigation that relate stretch to weight using bungee jumping simulations. Students learn that an equation of the form $y = kx$, where k is the constant of proportionality, represents a direct variation function and that all graphs of direct variation functions are straight lines that pass through the origin. Students look for patterns in their data that can be described by the equation $y = kx$. They also use their equations to find unique output values for possible input values. The connection between k and the slope of the graph of their function is made. Students use the formula $\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$ to calculate the slope of a line that passes through any two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ on a coordinate grid. One other type of variation, inverse variation, of the form $y = \frac{k}{x}$, is presented in the Extension to this chapter.

Lesson Summaries

Lesson 2.1 ***Activity: Bungee Jumping***

In this Activity, student groups use an empty can, large paper clips, and a rubber band to simulate the testing of a bungee jump cord. Students first measure and record the weight of an object, put the object in the can, then measure and record the height of the bottom of the can. As students continue adding identical objects to the can and recording weights and heights in their tables, they create a set of data that they use to make a scatter plot. From their plot they identify a pattern and use that pattern to predict what might happen if they continued to add objects to the can. This data table and scatter plot will be used again in Lesson 2.2.

Lesson 2.2 ***Investigation: Proportional Relationships***

During the Activity in Lesson 2.1, students collected data and created tables of the data. In this lesson, students extend their tables to include a column of "stretch-to-weight ratios." They then use the ratios to find an equation that models the relationship between the weight on the rubber band and how much the band stretches. As students progress through this Investigation, they come to understand how the value of k, the constant of proportionality, is related to the points they plotted in their scatter plot and are able to write an equation relating stretch to weight. They also use their equation to predict the length of stretch for a weight they did not test.

Lesson 2.3 ***Direct Variation Functions***

In this lesson, students learn to recognize direct proportion relationships, and how to write equations that express direct variation. Students explore a context in which a regulating line is used to estimate the dimensions of similar rectangles. Students are given tables of data and asked to determine if one variable is directly proportional to the other variable. They also write equations that model direct variation functions and use their equations to make predictions.

Lesson 2.4 ***R.A.P.***

In this lesson, students **R**eview **A**nd **P**ractice solving problems that require the use of skills and concepts taught in previous math levels. The skills reviewed in this lesson are skills that are needed as a basis for solving problems throughout this course.

Lesson 2.5 ***Slope***

In this lesson, students learn that the slope (ratio of the rise to the run) of a line is a measure of its steepness. They also calculate the slope of a line and interpret it as a rate of change. A connection is made between the value of the constant of proportionality k and the slope of the graph of a function. Students conclude that the slope of a line is a constant and use a formula to calculate the slope of a line that passes through any two points on a coordinate grid. They also interpret the meaning of slope for real-world relationships.

Chapter 2 ***Extension: Inverse Variation***

In this Extension, students turn paper towel tubes into "telescopes." They use their "telescopes" to collect data on the diameter of the viewing circle and the length of the telescope. They describe the pattern they see in their data and learn that the variables vary inversely. This type of relationship is called an inverse variation and can be represented by an equation of the form $y = \frac{k}{x}$.

Lesson Guide

Lesson/Objectives	Materials	
Chapter 2 Opener: How Is Mathematics Related to Bungee Jumping? • recognize that safety in bungee jumping depends on a knowledge of mathematics.		
2.1 Activity: Bungee Jumping • collect experimental data. • make a scatter plot of experimental data.	*Per group:* • ruler • stand with hook, or other means of hanging the rubber band • long rubber bands* (7″ × $\frac{1}{8}$″) • metal can such as a soup can with two small holes punched in it • 2 jumbo paper clips • 6 identical objects such as "C" batteries, each weighing about 2 to 3 ounces • scale for measuring weight in ounces, accurate to 0.1 ounce * If long rubber bands are not available, a chain of three or four shorter $\left(3\frac{1}{2}'' \times \frac{1}{4}''\right)$ rubber bands can be used.	*Optional:* • nail and hammer • TRM table shell for Question 4. • TRM table shell for Exercise 5. • grid paper
2.2 Investigation: Proportional Relationships • identify proportional relationships. • identify independent and dependent variables. • find a constant of proportionality. • write an equation that expresses a proportional relationship.	*Per student:* • students' tables and scatter plots from the Activity in Lesson 2.1	*Optional:* • TRM table shell for Question 2 • grid paper
2.3 Direct Variation Functions • identify direct variation. • write direct variation equations.		*Optional:* • TRM table shell for Exercise 4. • grid paper
2.4 R.A.P. • solve problems that require previously learned concepts and skills.		*Optional:* • grid paper
2.5 Slope • find the slope of a line given rise and run. • find the slope of a line given two points on the line. • interpret the slope of a line as a rate of change.		*Optional:* • grid paper
Chapter 2 Extension: Inverse Variation • use inverse variation to model experimental data.	*Per group:* • 3 paper towel tubes • masking tape • meter stick • tape measure • scissors	*Optional:* • TRM table shell for Question 1. • grid paper

Pacing Guide

	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7	Day 8
Basic	p. 22, 2.1	2.1, 2.2	2.2, 2.3	2.3, 2.4	2.5	project	review	
Standard	p. 22, 2.1	2.1, 2.2	2.2, 2.3	2.3, 2.4	2.5	project	review	extension
Block	p. 22, 2.1	2.2, 2.3	2.3, 2.4	2.5, project	review, extension			

Supplement Support

See the Book Companion Website at www.highschool.bfwpub.com/ModelingwithMathematics and the Teacher's Resource Materials (TRM) for additional resources.

CHAPTER
2

Direct Variation

CONTENTS

CHAPTER 2

Direct Variation

CHAPTER 2 OPENER

5e Engage

Lesson Objective

- recognize that safety in bungee jumping depends on a knowledge of mathematics.

Vocabulary

none

Description

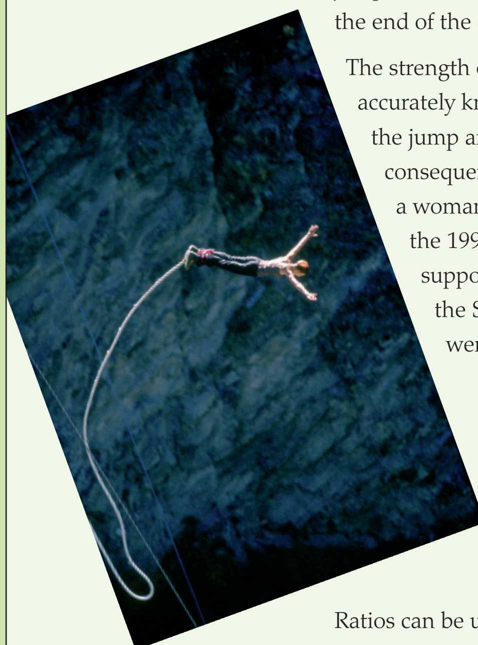
This reading helps students see that mathematics can be involved in the design and coordination of safe recreational activities.

TEACHING TIP

After students have read the Chapter Opener, ask the class if they are familiar with the sport of bungee jumping, and whether they know anyone who has done a bungee jump. Also have them discuss what kind of knowledge might have helped prevent the disaster mentioned in the reading.

How Is Mathematics Related to Bungee Jumping?

A bungee cord is an elastic cord that can be used to secure objects without tying knots. Specialized bungee cords are used in the sport of bungee jumping. One end of the cord is attached to a bridge or tower, and the other end is attached to the jumper. As the jumper falls, the cord stretches and slows the fall. At the bottom of the jump, the cord recoils and the jumper bounces up and down at the end of the cord.

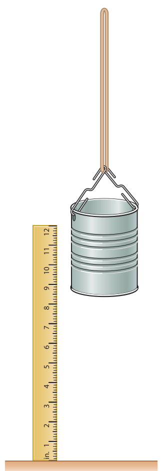


The strength of the cord used for a bungee jump must be accurately known. The cord must be adjusted for the height of the jump and for the weight of the jumper. Otherwise, the consequences can be disastrous. In one well-publicized case, a woman died practicing for a bungee jump exhibition for the 1997 Super Bowl halftime show. The bungee cord was supposed to stop her 100-foot fall just above the floor of the Superdome in New Orleans. At the time, officials were quoted in *The Boston Globe* as saying:

> Apparently, she made an earlier jump and didn't come as close as they wanted. They made some adjustments, and somebody made a miscalculation. I think it was human error.
>
> Bungee safety is a product of simple mathematics that factors height and weight in its calculations. It's so predictable.

Ratios can be used to model bungee jumping. Knowing how much the cord stretches for different jumper weights can help ensure that bungee jumps are safe.

ACTIVITY: Bungee Jumping



Constructing a mathematical model to describe a relationship between two variables often begins with collecting data and recording the values in a table. The data can then be graphed with a scatter plot.

A bungee cord is *elastic*, meaning that it can be stretched and then returned to its original length. You can use a large rubber band to simulate a bungee cord.

1. Start with an empty can, such as a soup can, that has two small holes punched opposite each other near the rim. Insert the end of a large opened paper clip in each of the holes. Insert a large rubber band through the free ends of the clips so that the can hangs from the rubber band.
2. Hang the rubber band so that the bottom of the can is at least 6 inches above a flat desk, a table, or the floor. Use a ruler to measure the height of the bottom of the can to the nearest $\frac{1}{8}$ inch.
3. Find the weight (in ounces) of one of the identical objects provided.
4. Make a table like the one shown here. Leave the last column blank for use in Lesson 2.2. Record the height of the bottom of the can above the flat surface.

Total Weight (ounces)	Height (inches)	Total Length of Stretch (inches)	
0		0	

5. Then place one object in the can. Record the weight of the object in the can and the new height of the bottom of the can.
6. Continue adding weight, one object at a time, until 6 objects are in the can. After each object is added, record the total weight of the objects in the can and the height of the bottom of the can.

Lesson 2.1 Activity Answers

1. Check students' set ups.
2. Sample answer: $8\frac{1}{2}$ inches
3. Sample answer: 2.3 ounces (size C battery)

4–7. See sample table to the right.

Total Weight (ounces)	Height (inches)	Total Length of Stretch (inches)
0	$8\frac{1}{2}$	0
2.3	8	$\frac{1}{2}$
4.6	$7\frac{5}{8}$	$\frac{7}{8}$
6.9	$7\frac{1}{4}$	$1\frac{1}{4}$
9.2	$6\frac{3}{4}$	$1\frac{3}{4}$
11.5	$6\frac{1}{4}$	$2\frac{1}{4}$
13.8	$5\frac{3}{4}$	$2\frac{3}{4}$

5e Engage

Lesson Objectives

- collect experimental data.
- make a scatter plot of experimental data.

Vocabulary

none

Materials List

Per group:

- ruler
- stand with hook, or other means of hanging the rubber band
- long rubber bands* ($7'' \times \frac{1}{8}''$)
- metal can such as a soup can with two small holes punched in it
- 2 jumbo paper clips
- 6 identical objects such as "C" batteries, each weighing about 2 to 3 ounces
- scale for measuring weight in ounces, accurate to 0.1 ounce

* If long rubber bands are not available, a chain of three or four shorter ($3\frac{1}{2}'' \times \frac{1}{4}''$) rubber bands can be used.

Description

Preparation:

Have students work in groups of 2 to 4. Make sure that students suspend their rubber band in such a way that they can use a ruler to measure the stretch of the rubber band. Stands and scales may be available from a science lab. Another possibility is hanging the rubber band from a doorknob. A nail and hammer can be used to punch two small holes on opposite sides near the top of the can, large enough for the paper clips to pass through.

Any identical weights may be used. Choose weights so that 6 of them will cause a stretch of a few inches.

During the Activity:

If students measure the height of the rubber band to $\frac{1}{8}$-inch precision, their scatter plots should look roughly linear. Perfectly linear data rarely occur, and students should be made aware of this fact.

Closing the Activity:

If necessary, prompt students to observe that their scatter plot has a nearly linear pattern. Even though a linear relationship is expected, some points in the scatter plot may deviate from a straight line. Point out to students that a straight line may be appropriate despite some minor fluctuations.

Remind students to keep their tables and graphs, which will be needed for Lesson 2.2.

TEACHING TIP

If necessary in **Question 7**, point out to students that they must subtract each measured height from their initial height measurement to find the total stretch of the rubber band.

TEACHING TIP

Question 8 Ask students how they know that the point (0, 0) belongs on their scatter plot.

TEACHING TIP

Question 8 Remind students that the scale on each axis of a graph must increase uniformly. Also check to make sure they have included units with the variable names.

COMMON ERROR

Exercise 1 Remind students about the need for uniform scales.

TEACHING TIP

Students may not be aware that graphs will only reveal true mathematical patterns in data if their scales are uniform. **Exercise 4** is intended to emphasize this fact. Graph A shows a misleading linear pattern because the scales are not uniformly increasing.

7. See sample table on page 23.

---

7. Use the data in your Height column to complete the Total Length of Stretch column.
8. Make a scatter plot of (*weight*, *stretch*) for your data. Let weight be the variable on the horizontal axis. Let stretch be the variable on the vertical axis. Label the axes with the variable names. Include their units. Also, label the axes with uniformly increasing scales for each variable.
9. Describe the pattern of points on your graph.
10. What do you think would happen if you continued to add more and more objects?

Save your data table and scatter plot for use in Lesson 2.2.

Practice for Lesson 2.1

For Exercises 1–3, use the following information.

Boiling water is poured over a teabag in a cup. The temperature is measured every 2 minutes. The results are shown in the table at the left.

Time (min)	Temperature (°F)
0	205
2	158
4	122
6	108
8	96

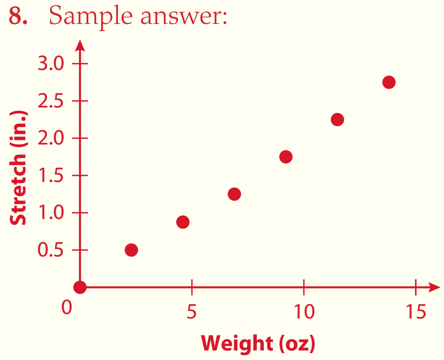
1. Make a scatter plot of the data in the table. Let time be the variable on the horizontal axis, and let temperature be the variable on the vertical axis. Plot points for (*time, temperature*). Label the axes with variable names, units, and uniformly increasing scales.
2. Describe the pattern in your graph.
3. What do you think would happen to the temperature after another 10 minutes?
4. The table below shows a comparison of several 2010 sporty cars. It lists acceleration as the time it takes to go from 0 to 60 miles per hour, as well as highway gas mileage.

Car	Time from 0 to 60 mph (seconds)	Gas Mileage (miles per gallon)
Mini Cooper S Clubman	8.1	34
Ford Mustang GT Coupe	5.2	24
Volkswagen Hatchback	7.6	31
Mitsubishi Eclipse Coupe	8.7	28
Chrysler Sebring Sedan	8.5	30
MazdaSpeed3	6.3	25

SOURCE: HTTP://WWW.AUTOS.COM/AUTOS/PASSENGER_CARS/SPORTY_CARS/ACCELERATION

---

8. Sample answer:



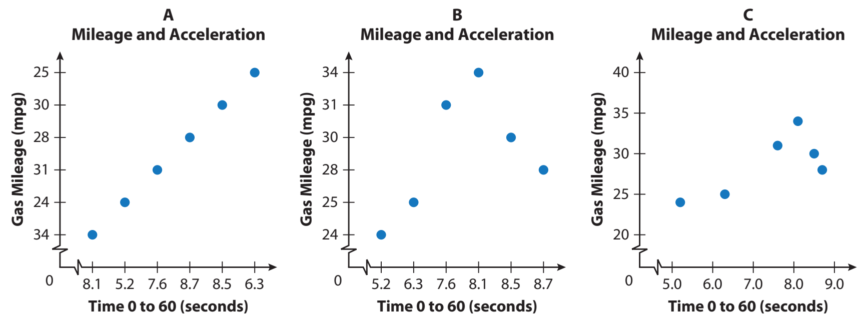
9. Sample answer: The points lie very close to a straight line that passes through the origin.

10. Sample answer: The rubber band would continue to stretch by about half an inch for each additional object, until it might break or the can might touch the floor.

Practice for Lesson 2.1 Answers

1–4 See answers on page 25.

Identify the graph that is the best model for the data. Explain why it is the best one and why each of the others does not accurately model the data.



For Exercises 5–8, use the following information.

A bungee cord has a length of 60 feet when it hangs with no weight on it. To test its strength, increasing amounts of weight are attached to its end. After each weight is added, the total length of the cord is measured. The results are shown in the table.

Total Weight (pounds)	Total Length (feet)	Length of Stretch (feet)
0	60	
64	76	
144	96	
176	104	
280	130	

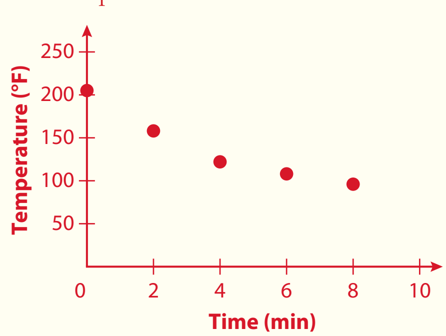
5. Complete the column labeled “Length of Stretch.”

6. Make a scatter plot of (*weight*, *stretch*). Label the axes with variable names, units, and uniformly increasing scales.

7. Describe the pattern of points on your graph.

8. What do you think would happen if more and more weight were added?

1. Sample answer:



2. Sample answer: The graph is slightly curved.

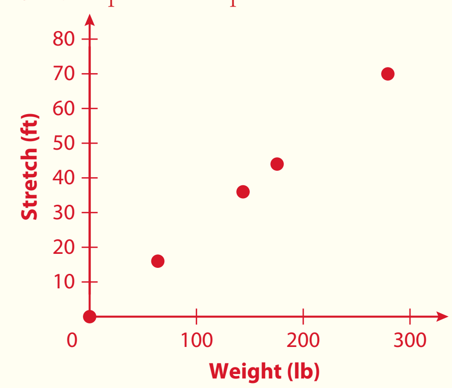
3. Sample answer: The temperature would continue to drop, but it would level off at room temperature.

4. Scatter plot C is correct. Both scales are uniform, so the plot correctly shows how gas mileage varies with acceleration. In A, points are just plotted in the order the data appear in the table, and the scales are jumbled. In B, the scales are both increasing but are not uniform, so the data are distorted.

5.

Length of Stretch (feet)
0
16
36
44
70

6. Sample scatter plot:



CONNECTION

The data for **Exercises 5–8** are an idealization. Real bungee cords may not have a perfectly linear response over their entire usable length.

TEACHING TIP

Exercise 5 Point out that the stretch of the cord must be found by subtracting 60 feet from each length in the table.

7. Sample answer: The points all lie on a straight line that passes through the origin.

8. Sample answer: The bungee cord would continue to stretch by about 1 foot for each 4 pounds of additional weight, until it might break.

LESSON 2.2

5e Explore

Lesson Objectives

- identify proportional relationships.
- identify independent and dependent variables.
- find a constant of proportionality.
- write an equation that expresses a proportional relationship.

Vocabulary

- constant of proportionality
- dependent variable
- independent variable
- proportional relationship

Materials List

- students' tables and scatter plots from the Activity in Lesson 2.1

Description

This lesson is designed as a small group investigation (2–4 students), using the same groups as in the Activity in Lesson 2.1. Students should work through **Questions 1–6** to extend their work from the previous lesson and discover that the ratios of stretch to weight for their rubber band are approximately constant. The ratios may not all be exactly equal, but they should be close.

Question 7 asks students to use their equation to make a prediction, which demonstrates the value of solving their first equation for S and expressing the relationship between S and W in the form of a function. Functions are discussed in Lesson 2.3.

Wrapping Up the Investigation:

Ask students to compare their analysis of the elasticity of their rubber band to what further information might be needed to prevent an accident like the one discussed in the Chapter Opener. For example, in addition to the stretching produced by hanging weights, the additional dynamic stretching due to the initial jump height would have to be considered.

Lesson 2.2 — INVESTIGATION: Proportional Relationships

An equation is one of the most useful kinds of mathematical models. It can allow you to predict the value of one quantity when you know the value of another. In this lesson, you will use ratios to find an equation that models the relationship between the weight on a rubber band and how much the band stretches.

Note

In some situations, neither variable is dependent on the other. In such cases, either variable can be placed on the horizontal axis. For example, if heights and weights of people are graphed, either height or weight could correctly go on the horizontal axis.

INDEPENDENT AND DEPENDENT VARIABLES

In the Activity in Lesson 2.1, you chose the values for the amount of weight. But you did not choose values for the length of the stretch. The stretch is called the **dependent variable** because its value depends on the value of the weight. Since values of weight can be freely chosen, weight is called the **independent variable**. The independent variable is almost always represented on the horizontal axis of a graph.

A PROPORTIONAL RELATIONSHIP

In the Activity in Lesson 2.1, the points on your scatter plot most likely fell in a nearly straight-line pattern. A straight line drawn through the origin may be a good visual model for the data. When a clear pattern exists on a graph of two variables, it is often possible to find an equation that relates the variables. Clues to an equation may sometimes be found by examining the ratios of the values of the variables.

1. Return to your data table from the Activity in Lesson 2.1. Label the fourth column "Stretch-to-Weight Ratio." Calculate the ratio of the total stretch to the total weight for each row of the table and record it. Write your ratios as decimals.

Total Weight (ounces)	Height (inches)	Total Length of Stretch (inches)	Stretch-to-Weight Ratio
0		0	-----

2. How do the ratios in your table compare to each other?
3. If the ratio of stretch S to weight W is always equal to the same constant value, there is a **proportional relationship** between S and W. The constant ratio is called the **constant of proportionality**. The letter k is often used to represent such a constant. So, $\frac{S}{W} = k$ is an equation relating the variables S and W.

TEACHING TIP

Question 1 Remind students that division by 0 is undefined, so the first cell in the fourth column of the table must be left blank.

TEACHING TIP

Questions 2 and 3 Remind students that real data rarely yield ratios that are exactly equal even when a relationship is proportional. Approximate equality of ratios, combined with a scatter plot that appears to follow a straight-line pattern, provide evidence of proportionality.

Lesson 2.2 Investigation Answers

1. Sample answer:

Total Weight (ounces)	Height (inches)	Total Length of Stretch (inches)	Stretch-to-Weight Ratio
0	$8\frac{1}{2}$	0	—
2.3	8	$\frac{1}{2}$	0.217
4.6	$7\frac{5}{8}$	$\frac{7}{8}$	0.190
6.9	$7\frac{1}{4}$	$1\frac{1}{4}$	0.181
9.2	$6\frac{3}{4}$	$1\frac{3}{4}$	0.190
11.5	$6\frac{1}{4}$	$2\frac{1}{4}$	0.196
13.8	$5\frac{3}{4}$	$2\frac{3}{4}$	0.199

For your data, k equals the average value of the ratios in the fourth column of your table. Find the constant of proportionality for your data.

4. Use your answer to Question 3 to write an equation relating S and W.
5. Solve your equation for S.
6. What are the units of the constant of proportionality k for your equations in Questions 4 and 5?
7. You can use your equation from Question 5 to predict the length of stretch for a weight that was not tested in your experiment. Choose a value for W. Then predict how much it would stretch your rubber band.

Practice for Lesson 2.2

1. How is the value of k in the Investigation related to the points in your scatter plot from the Activity in Lesson 2.1?

For Exercises 2–7, use the following information.

Most high-definition television sets have wider screens than other sets. Here is an example.

Screen dimensions for a range of models are given in the table.

Height (in.)	Width (in.)
18	32
$22\frac{1}{2}$	40
27	48
$31\frac{1}{2}$	56

2. For the dimensions of the first two TV models listed in the table, do the heights and widths form a proportion? Explain.
3. Is there a proportional relationship between width and height? Why or why not?

2. Sample answer: They are all equal to approximately 0.19.
3. Sample answer: 0.196
4. Sample answer: $\frac{S}{W} = 0.196$
5. Sample answer: $S = 0.196\,W$
6. inches per ounce, or in./oz
7. Sample answer: For $W = 3.0$ ounces, $S = (0.196)(3) = 0.588$ inches.

TEACHING TIP

Exercise 1 lays the groundwork for the study of slope in Lesson 2.5. Remind students that the coordinates of a point on a scatter plot represent the distances of the point from the axes.

TEACHING TIP

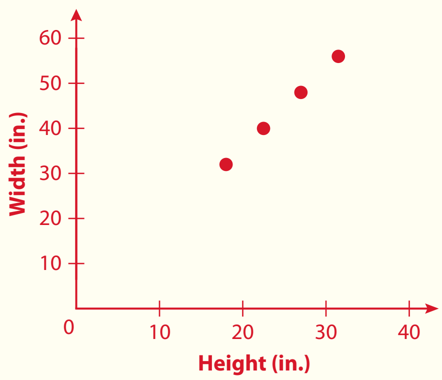
Make sure students see that proportional relationships are revealed by both constant ratios (**Exercises 2 and 11**) and linear patterns in graphs (**Exercises 6 and 9**).

Practice for Lesson 2.2 Answers

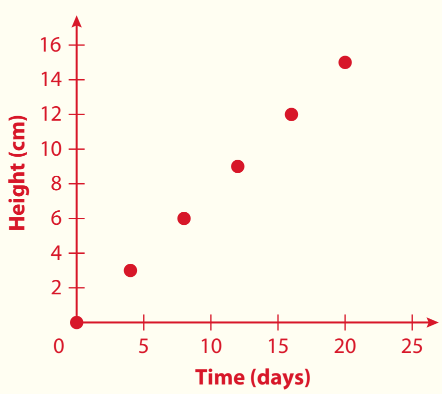
1. For each point, it approximately equals the ratio of the point's height to its distance to the right of the Stretch axis.
2. Sample answer: Yes, the ratios of height to width are equal: $\frac{18}{32} = \frac{22\frac{1}{2}}{40}$.
3. Yes, the ratio of width to height is always 16 to 9, or about 1.78. Or, the ratio of height to width is 9 to 16, or about 0.56.

LESSON 2.2

4. $w = \frac{16}{9}h$ or $h = \frac{9}{16}w$
5. They cannot be identified because neither variable depends on the other.
6. Sample answer:



7. Sample answer: No, there are no TV sets with dimensions of 0 inches and 0 inches.
8. The dependent variable is height, and the independent variable is time. The height of the plant depends on how long it has been growing.
9. Sample answer:



4. Write an equation to model the relationship between width w and height h.
5. If possible, identify the dependent variable and the independent variable. Explain.
6. Make a scatter plot for these data.
7. Does it make sense to include the point (0, 0) on your scatter plot? Why or why not?

For Exercises 8–13, use the following information.

For their science project, students plant a kudzu vine seed in order to investigate how quickly it grows. Every four days after the seed sprouts, they measure and record the height of their plant. Their results are shown in the table.

Time (days)	Height (cm)
0	0
4	3
8	6
12	9
16	12
20	15

8. If possible, identify the dependent variable and the independent variable. Explain.
9. Make a scatter plot for these data.

10. Does it make sense to include the point (0, 0) on your scatter plot? Why or why not?

11. Is there a proportional relationship between height and time? Why or why not?

12. Find the constant of proportionality, including units, for this situation.

13. Write an equation to model the relationship between height h and time t.

For Exercises 14–16, use the following information.

The amounts of a family's electric bills for four months are shown in the table.

Electricity Used (kilowatt-hours)	Amount of Bill
54	$14.56
120	$23.80
210	$36.40
285	$46.90

14. If possible, identify the dependent variable and the independent variable. Explain.

15. Make a scatter plot for these data.

16. Is there a proportional relationship between the variables? Why or why not?

10. Sample answer: Yes, because when the seed first sprouts, its height is 0 cm.

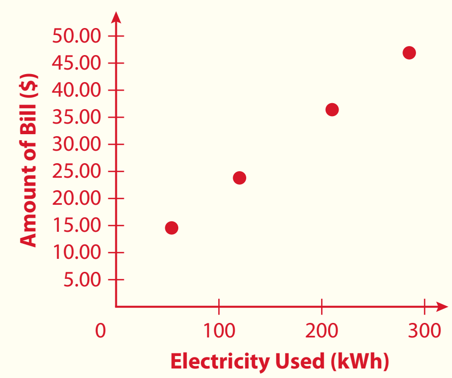
11. Yes, the ratio of height to time is constant.

12. $\frac{3 \text{ centimeters}}{4 \text{ days}} = 0.75$ centimeter per day

13. $h = \frac{3}{4}t$ or $h = 0.75t$

14. Electricity used is the independent variable. The bill amount is the dependent variable because it depends on how much electricity was used.

15. Sample answer:



16. No, the ratios of $\frac{\text{amount of bill}}{\text{electricity used}}$ are not constant.

LESSON 2.3

5e Explain

Lesson Objectives

- identify direct variation.
- write direct variation equations.

Vocabulary

- direct proportion
- direct variation
- direct variation function
- function
- input value
- origin
- output value
- variation
- varies directly

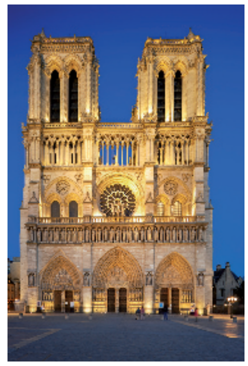
Description

In this lesson, students are introduced to the concept of a function. They identify direct variation functions that arise from proportional relationships and learn to recognize their graphs.

Lesson 2.3 Direct Variation Functions

Two variables have a proportional relationship if their ratio is constant. In this lesson, you will learn how to recognize direct proportion relationships, as well as how to write functions that express direct variation.

DIRECT PROPORTION



Architects and other designers often use *proportioning systems* to give structure to their work. A proportioning system can help ensure that the different parts of a design have shapes that fit together in a visually pleasing way.

The photograph at the left shows the Cathedral of Notre Dame in Paris, France. Many rectangular shapes can be seen in the design of the cathedral. A few of these are outlined in red in Figure 1.

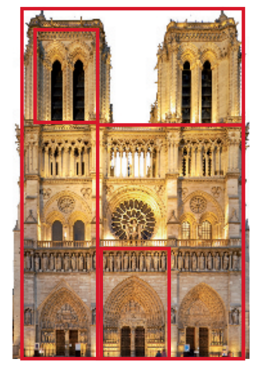


Figure 1

These rectangles have different sizes. But they all have the same shape. They are therefore similar to each other.

The entire front or *facade* of Notre Dame is about 304 feet high and 190 feet wide. The small rectangle in the upper-left tower is about 88 feet high and 55 feet wide. The height-to-width ratios of the building's front and the small rectangle form a proportion.

$$\frac{304}{190} = \frac{88}{55}$$

Both ratios are equal to $\frac{8}{5}$. The same is true for the other rectangles outlined in the figure. Since the ratio of height to width is constant, height and width are proportional to each other.

Recall

As used in mathematics, the word *line* is always assumed to mean a straight line.

Connection

The famous architect Le Corbusier stated:

> The regulating line brings in the tangible form of mathematics, which gives the reassuring perception of order. The choice of a regulating line fixes the fundamental geometry of the work.

In Figure 2, all of the rectangles shown in Figure 1 are drawn with their lower-left corners at the same position. Notice that their upper-right corners all lie on the same slanted line, which contains the diagonals of all of the rectangles.

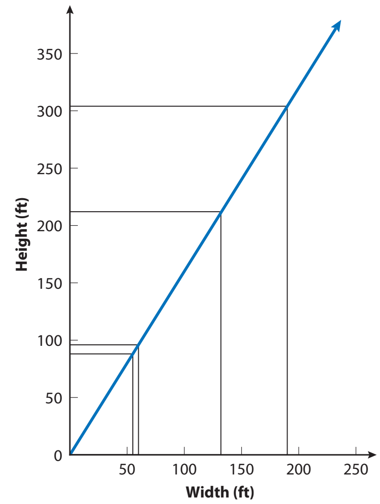


Figure 2

Designers call such a line a *regulating line*. Since all of the rectangles having their corners on this line are similar, such a line can be used to provide a uniform structure in a design.

EXAMPLE 1

The Notre Dame proportioning system can be used to find rectangles similar to the ones shown in Figure 1.

a. Use the regulating line in Figure 2 to *estimate* the height of a similar rectangle that has a width of 85 feet.

b. Use a proportion to *calculate* the height.

TEACHING TIP

As students examine the graph that shows the regulating line, have them look back at the rectangles in Figure 1. If necessary, point out where the diagonals of the rectangles are in the figure.

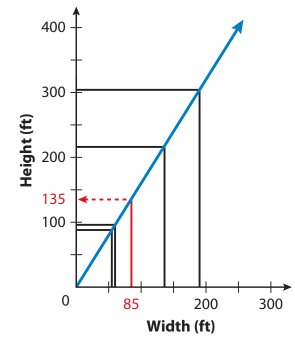
ADDITIONAL EXAMPLE 1

a. Use the regulating line for the Notre Dame proportioning system in Figure 2 to estimate the width of a similar rectangle with a height of 200 feet. about 120 ft

b. Use a proportion to calculate the width. 125 ft

Solution:

a. Estimate the height by drawing the rectangle so that its diagonal is on the regulating line.



The height of the similar rectangle appears to be about 135 feet.

b. Write a proportion for the problem.

$$\begin{array}{rcl} \text{height} \rightarrow & \dfrac{8}{5} = \dfrac{h}{85 \text{ feet}} & \leftarrow \text{height} \\ \text{width} \rightarrow & & \leftarrow \text{width} \end{array}$$

Solve for h.

Original equation	$\frac{8}{5} = \frac{h}{85}$
Find the cross products.	$5h = (8)(85)$
Simplify.	$5h = 680$
Divide each side by 5.	$\frac{5h}{5} = \frac{680}{5}$
Simplify.	$h = 136$

So, for a width of 85 feet, the height should be 136 feet.

Most of the relationships you have been exploring in this chapter are examples of **direct proportions**.

Note

For measured data, ratios of proportional quantities may sometimes be only approximately constant.

- One quantity is *directly proportional* to another if the ratios of the two quantities are constant.
- A graph of the quantities is a line that includes the point (0, 0).

There are several types of **variation** between two quantities. In general, the equation $y = kx$ expresses **direct variation**. The equation can be read "y is directly proportional to x" or "y **varies directly** with x."

TEACHING TIP

Point out to students the distinction between the use of the letters y and x that are used to represent variables and the letter k that is used to represent an unknown constant.

The stretch S of a rubber band is directly proportional to the weight W that is hung from it. This relationship can also be expressed as "S varies directly with W."

All direct variation graphs are straight lines that pass through the **origin** (0, 0). Another kind of variation is explored in this chapter's Extension.

FUNCTIONS

Recall that a mathematical model uses mathematics to describe relationships. In the stretch and weight relationship $S = kW$ from the bungee jump activity, for each value of W there is exactly one value of S. This kind of model is an example of a function.

> A **function** is a relationship between input and output in which each **input value** has exactly one **output value**.

A function provides a way of finding a unique output value for every possible input value.

The equation $S = kW$ expresses stretch as a function of weight. The input values are those of the independent variable W. For each value of the independent variable, the function returns an output value for the dependent variable S.

EXAMPLE 2

Nurses, doctors, and other types of caregivers often dispense medications. In many cases, the amount of medication depends on the weight of the person taking the medication. The table shows the dosage of a medication for various weights.

a. Verify that dosage is directly proportional to weight.

b. Write an equation that gives dosage d as a function of weight w.

c. Draw a graph of dosage versus weight.

d. Use your equation to determine the dosage for a person weighing 43 kilograms.

Weight (kg)	Dosage (mg)
10	250
20	500
30	750
40	1,000
50	1,250
60	1,500

Note

In graphs, the term *versus* (or the abbreviation *vs*) describes the position of the variables. The first variable (the dependent variable) is associated with the vertical axis and the second variable (the independent variable) with the horizontal axis.

TEACHING TIP

In case students ask, tell them that a function does not require that each *output* value have a unique *input* value.

CONNECTION

Example 2 refers to "weight in kilograms." A kilogram is actually a unit of mass, but some people (pharmacists, for example) in the U.S. often refer to weight in kilograms, grams, or milligrams.

ADDITIONAL EXAMPLE 2

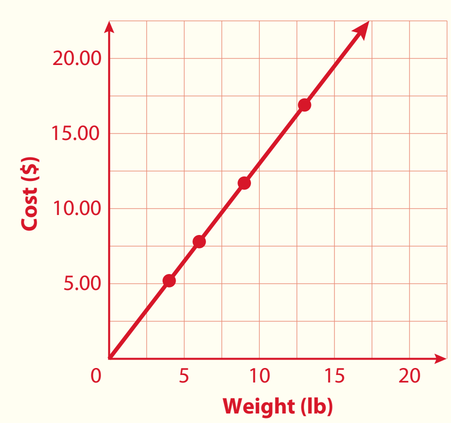
The table shows the cost of having various amounts of clothes washed at a drop-off laundry.

Weight of Clothes (lb)	Cost ($)
4	5.20
6	7.80
9	11.70
13	16.90

a. Verify that cost is directly proportional to weight. The ratios of cost to weight all equal 1.30.

b. Write an equation that models cost C as a function of weight w. $C = 1.30w$

c. Draw a graph of cost vs weight.



d. Use your equation to determine the cost for a load of 8 pounds of clothes. $10.40

TEACHING TIP

Example 2 You may want to point out that drawing a line through the points on the scatter plot makes sense because all intermediate weights and dosages are feasible. For some graphs, only certain discrete values make sense, so a line may not be appropriate even though lines are often drawn in such cases.

Solution:

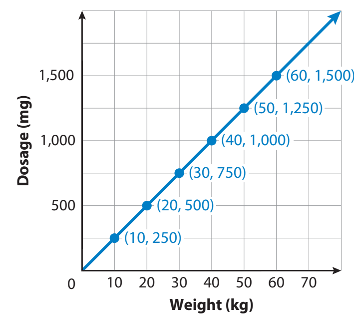
a. Find the ratios of dosage to weight.

Weight (kg)	Dosage (mg)	$\frac{\text{Dosage}}{\text{Weight}}$ (mg/kg)
10	250	$\frac{250}{10} = 25$
20	500	$\frac{500}{20} = 25$
30	750	$\frac{750}{30} = 25$
40	1,000	$\frac{1,000}{40} = 25$
50	1,250	$\frac{1,250}{50} = 25$
60	1,500	$\frac{1,500}{60} = 25$

The ratios of dosage d to weight w all equal 25 milligrams per kilogram (mg/kg). So, d is directly proportional to w.

b. Since $\frac{d}{w} = 25$, the equation $d = 25w$ is a function that models the relationship between the two variables.

c. Plot the data from the table. Then draw a line through the points.



d. To find the dosage for a person weighing 43 kg, use the equation $d = 25w$.

Original equation	$d = 25w$
Substitute.	$d = 25(43)$
Simplify.	$d = 1,075$

So, a dosage of 1,075 mg is needed.

The equations $S = kW$ and $d = kw$ both express direct variation relationships. And both produce unique output values for each input value. They therefore represent **direct variation functions**.

Practice for Lesson 2.3

1. The "triple-decker" apartment building shown here is another example of a proportioning system in architecture. Many of the rectangles have a length-to-width ratio of 7 to 4.
   a. Write an equation that models the length l of a rectangle as a function of width w.
   b. Use this proportioning system to find the length of a window that has a width of 30 inches.

For Exercises 2–5, tell whether the relationship is an example of a direct variation. If it is, explain how you know. Then write an equation relating the variables. If the relationship is not an example of direct variation, explain why.

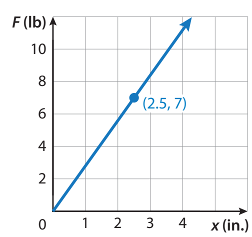
2. The table shows travel times for driving on a highway.

Time Elapsed (h)	Distance Traveled (mi)
0	0
3	180
6	360
9	540

3. The table lists some temperature conversions.

Celsius Scale (°C)	Fahrenheit Scale (°F)
5	41
10	50
15	59
20	68

4. It costs \$2.00 to begin a trip in a taxicab and an additional \$0.20 for each $\frac{1}{4}$ mile traveled. (*Hint*: Make a table of distances and costs.)
5. The graph shows the force F needed to stretch a spring by x inches. This is an example of *Hooke's Law* for an elastic spring.



Practice for Lesson 2.3 Answers

1a. $l = \frac{7}{4}w$

1b. 52.5 inches

2. This is a direct variation, since the ratio of distance traveled d to time t is always 60. An equation is $d = 60t$.

3. The relationship is not a direct variation. The ratios of °C to °F are not constant: $\frac{41}{5} = 8.2$, $\frac{50}{10} = 5$, $\frac{59}{15} \approx 3.9$, $\frac{68}{20} = 3.4$.

4. The relationship is not a direct variation. Sample table:

Distance Traveled (mi)	Cost (\$)	Cost/Distance Ratio
0	\$2.00	
$\frac{1}{4}$	\$2.20	\$8.80 per mile
$\frac{1}{2}$	\$2.40	\$4.80 per mile
$\frac{3}{4}$	\$2.60	\$3.47 per mile

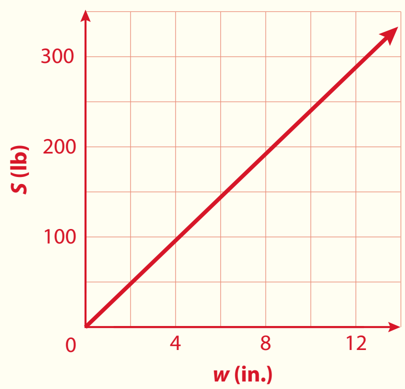
The cost-to-distance ratio is not constant.

CONNECTION

Exercise 5 The English scientist Robert Hooke discovered the law named after him in the late 17th century. The constant ratio k in the equation $F = kx$ is called a *spring constant*.

5. This is a direct variation, since the graph is a straight line that passes through the origin. The equation must have the form $F = kx$, and $k = \frac{F}{x}$ for every point on the line. Using the given point, $k = \frac{7}{2.5} = 2.8$, so $F = 2.8x$.

6a. Sample answer: The ratios of strength to width are all very close to 24. Also, a scatter plot of strength vs width is approximately a straight line that passes through the origin.
6b. Sample answer: $S = 24w$
6c. Sample answer:



6d. $S = 24(7) = 168$ lb

TEACHING TIP

Exercise 7 Students need to recognize the patterns in the table in order to see that the length of a sheet of A7 paper is 105 mm.

7a. Sample answer: Yes, the ratio of width to length is very close to 0.71 for all the papers listed.
7b. $w = 0.71l$
7c. Sample answer: 105 mm long, 75 mm wide

6. The table contains data relating the breaking strength of pine boards to their widths. The boards all have the same thicknesses and lengths.

Width (in.)	Breaking Strength (lb)
4	94
6	145
8	196
10	241
12	289

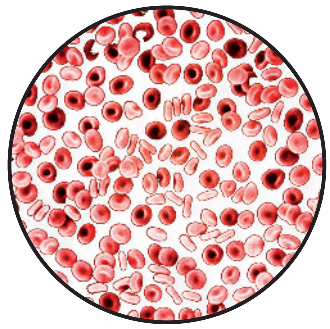
a. Verify that strength is directly proportional to width for these boards.
b. Write an equation that models strength S as a function of width w.
c. Draw a graph of strength vs width.
d. Use your equation to determine the strength of a board that is 7 inches wide.

7. Outside the United States, most countries use "metric" paper. International paper sizes are listed in the table.

Paper Size	Width (mm)	Length (mm)
A0	841	1,189
A1	594	841
A2	420	594
A3	297	420
A4	210	297
A5	148	210
A6	105	148

a. Does width vary directly with length for these papers? Explain.
b. Write an equation that models width w as a function of length l.
c. Find the dimensions of the next smaller size, the A7 paper.

8. On the average, 1 pound of recycled aluminum contains 33 cans.
   a. Write an equation expressing the number of cans N as a function of the weight w (in pounds) of recycled aluminum.
   b. In a recent year, 1,938,000,000 pounds of cans were recycled. How many cans were recycled?
9. A person's red blood cell count can be estimated by looking at a drop of blood under a microscope. The number of cells inside the circular field of the microscope is counted.



If the area of the circle is known, then area can be used as a measure of the number of blood cells. The number of cells varies directly with area.
   a. Assume that a 0.01 mm^2 viewing field contains 23 red blood cells. Find an equation for the number N of red blood cells as a function of area A.
   b. Use your equation from Part (a) to determine how many red blood cells are contained in an area of 50 mm^2.
10. Variable y is directly proportional to x, and y is 28 when x is 6.
   a. Write an equation that gives y as a function of x.
   b. Find the value of y when x is 15.
11. Variable r is directly proportional to t, and r is 12.84 when t is 42.8.
   a. Write an equation that gives r as a function of t.
   b. Find the value of r when t is 21.7.
   c. Find the value of t when r is 15.9.

8a. $N = 33w$
8b. $33(1{,}938{,}000{,}000) =$ 63,954,000,000 cans
9a. $N = kA$; $N = 23$ for $A = 0.01$, so $k = 23/0.01 = 2{,}300$ cells/mm^2, and $N = 2{,}300A$.
9b. $N = 2{,}300(50) = 115{,}000$
10a. $y = \frac{14}{3}x$
10b. $\frac{14}{3}(15) = 70$
11a. $r = 0.3t$
11b. $(0.3)(21.7) = 6.51$
11c. 53

TEACHING TIP

Exercises 10 and 11 Remind students that the equation $y = kx$ can be used to express a direct proportion. Students must first find the value of k in order to write a correct equation.

LESSON 2.4

5e Evaluate

Lesson Objective

- solve problems that require previously learned concepts and skills.

Exercise Reference

Exercise 1: Lesson 2.3
Exercise 2: Lesson 2.2
Exercises 3-4: Lesson 2.3
Exercise 5: Appendix C
Exercise 6: Lesson 1.2
Exercises 7–12: Appendix B
Exercises 13–15: Appendix G
Exercises 16–18: Appendix P
Exercises 19–21: Appendix D
Exercise 22: Appendix O
Exercises 23: Lesson 2.3
Exercises 24–25: Lesson 2.2

Lesson 2.4 R.A.P. Answers

1. directly proportional
2. constant of proportionality
3. function
4. straight line
5. C
6. B
7. $\frac{2}{5}$
8. $\frac{3}{20}$
9. $\frac{11}{50}$
10. 0.625
11. 0.28
12. $0.\overline{27}$
13. 10
14. −2
15. −16
16. 5 and 6
17. 8 and 9
18. 10 and 11

Lesson 2.4 R.A.P.

Fill in the blank.

1. For two variables x and y, if the ratio $\frac{y}{x}$ is constant, then y is ________________ to x.

2. In the equation $y = kx$, k is called the ________________.

3. An input/output relationship in which each input value has exactly one output value is called a(n) ________________.

4. The shape of the graph of a direct variation function is a(n) ________________.

Choose the correct answer.

5. Which of the following is *not* a correct way of writing the ratio "3 to 4"?

A. 3 : 4 **B.** $\frac{3}{4}$ **C.** 3×4 **D.** 3 to 4

6. Given that 1 kilogram weighs approximately 2.2 pounds, rewrite 10 pounds as an equivalent number of kilograms.

A. 22 kilograms **B.** 4.5 kilograms
C. 7.8 kilograms **D.** 0.22 kilograms

Change the decimal to a fraction in lowest terms.

7. 0.4 **8.** 0.15 **9.** 0.22

Change the fraction to a decimal.

10. $\frac{5}{8}$ **11.** $\frac{7}{25}$ **12.** $\frac{3}{11}$

Evaluate the expression.

13. $|-8| + 2$ **14.** $-|-2|$ **15.** $4 - 2|-10|$

The value of the square root is between which two integers?

16. $\sqrt{30}$ **17.** $\sqrt{78}$ **18.** $\sqrt{108}$

Solve the proportion.

19. $\frac{5}{6} = \frac{x}{42}$ **20.** $\frac{1.2}{n} = \frac{60}{45}$ **21.** $\frac{7}{19} = \frac{14}{a}$

22. The table shows the temperature of a glass of juice at different times after being removed from a refrigerator.

Time (min)	Temperature (°F)
0	40
5	48
10	54
15	59
20	62

Make a well-labeled scatter plot of temperature versus time.

23. The table shows the pressure due to water at various depths below the surface.

Depth (feet)	Pressure (pounds per square foot)
0	0
2	125
5	312
10	624
20	1,248

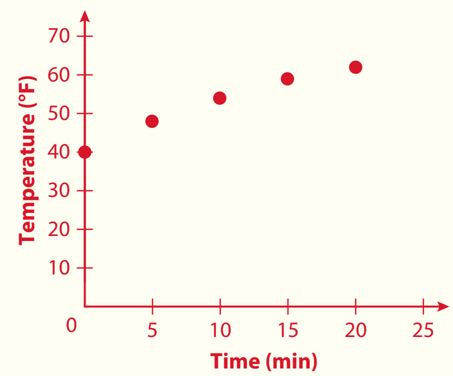
Make a scatter plot of pressure versus depth and draw a smooth line through the points.

State whether there is a proportional relationship between the variables. Explain.

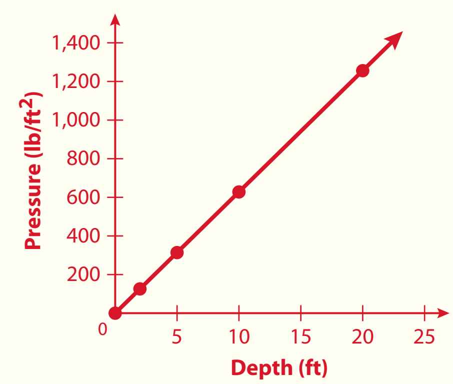
24. temperature and time in Exercise 22

25. pressure and depth in Exercise 23

19. 35
20. 0.9
21. 38
22. Sample answer:



23. Sample answer:



24. No, the ratios are not constant, and the scatter plot has a curved shape.

25. Yes, the ratio of pressure to depth is approximately constant, and the graph is a straight line that passes through the origin.

LESSON 2.5

5e Explain

Lesson Objectives

- find the slope of a line given rise and run.
- find the slope of a line given two points on the line.
- interpret the slope of a line as a rate of change.

Vocabulary

- rate of change
- slope

Description

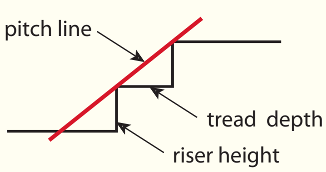
In this lesson students explore the concept of slope as a measure of the steepness of a line and relate it to the concept of rate of change.

TEACHING TIP

To help students connect the concept of slope to their real world, ask them to give other examples (besides staircases) for which the word slope is used to describe steepness. These may include roofs, streets, ski trails, etc.

CONNECTION

The line used to define the slope of a staircase is not the same as the *pitch* of the staircase. Pitch refers to the ratio of tread depth to riser height for an individual stair.



ADDITIONAL EXAMPLE 1

Find the slope of a staircase that rises 90 inches over a run of 120 inches.

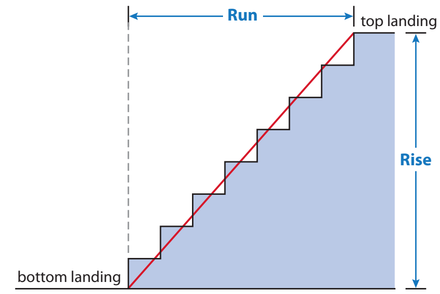
$\frac{3}{4}$ or 0.75

Lesson 2.5 Slope

You have seen that the graph of a direct variation function is a straight line. The slope of a line is a measure of its steepness. In this lesson, you will learn how to find the slope of a line and to interpret it as a rate of change.

SLOPE OF A LINE

The *rise* of a staircase is defined as the vertical distance between landings. The horizontal distance from the edge of the bottom step to the top landing is called the *run*.



The steepness of the red line in the figure above can be described by a ratio known as the **slope** of the line.

$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

EXAMPLE 1

Find the slope of a staircase that rises 64 inches over a run of 80 inches.

Solution:

The slope of the staircase is the ratio of rise to run.

$$\begin{aligned}\text{Slope of staircase} &= \frac{\text{rise}}{\text{run}} \\ &= \frac{64 \text{ in.}}{80 \text{ in.}} \\ &= \frac{4}{5}\end{aligned}$$

The slope is $\frac{4}{5}$ or 0.8.

Connection

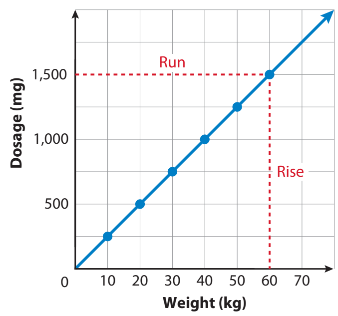
Staircases are usually built with slopes between 0.6 and 0.9.

SLOPE AND RATE OF CHANGE

For a direct variation function of the form $y = kx$, the constant of proportionality k is equal to the slope of the graph of the function.

EXAMPLE 2

The graph below shows the medication dosage function $d = 25w$ from Lesson 2.3.



Find the slope of the line.

Solution:

As the weight increases from 0 to 60 kg, the dosage rises from 0 to 1,500 mg. The slope of the line is

$$\frac{\text{rise}}{\text{run}} = \frac{1{,}500 \text{ mg}}{60 \text{ kg}}$$
$$= 25 \text{ mg/kg}$$

Notice that in this case, the slope has units of milligrams per kilogram.

The ratio of the *change* in dosage to the *change* in weight for any two points on the graph of dosage vs weight is also equal to the slope of the line.

- Consider the points (20, 500) and (30, 750).

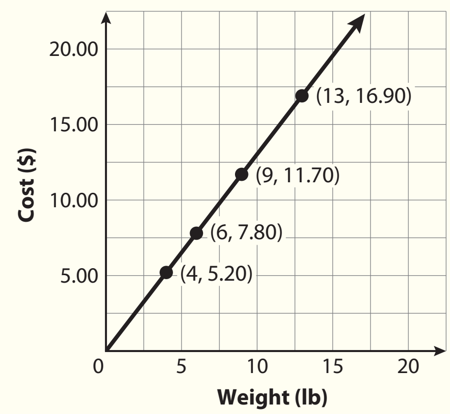
$$\frac{\text{change in dosage}}{\text{change in weight}} = \frac{750 \text{ mg} - 500 \text{ mg}}{30 \text{ kg} - 20 \text{ kg}} = 25 \text{ mg/kg}$$

- Also, consider the points (20, 500) and (50, 1,250).

$$\frac{\text{change in dosage}}{\text{change in weight}} = \frac{1{,}250 \text{ mg} - 500 \text{ mg}}{50 \text{ kg} - 20 \text{ kg}} = 25 \text{ mg/kg}$$

ADDITIONAL EXAMPLE 2

The graph below shows the cost of having various amounts of clothes cleaned at a drop-off laundry. Find the slope of the line.



$$\frac{11.70 - 5.20}{9 - 4} = 1.30 \frac{\text{dollars}}{\text{pound}}$$

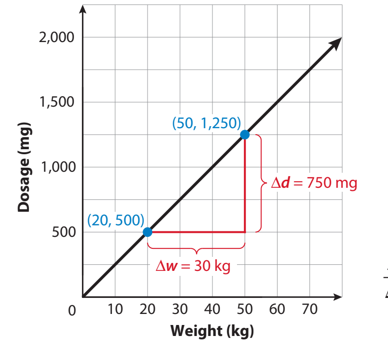
COMMON ERROR

Students may mistakenly think that rates of change always involve time. Hence, it is important to include both variables in the definition of a rate, as in "the rate of change of dosage *with respect to weight*."

Note

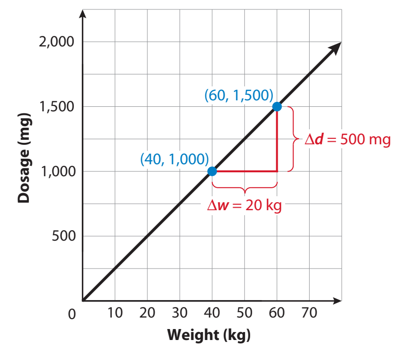
The symbol Δ (pronounced "delta") is the equivalent of a capital *D* in the Greek alphabet. It stands for a change or *difference* between two values of a variable.

In these calculations, the change in dosage can be represented by Δd, read "change in d." The change in weight can be represented by Δw, read "change in w." So, the slope of the line can also be thought of as the ratio of these changes $\frac{\Delta d}{\Delta w}$, or the **rate of change** of dosage with respect to weight.



$$\frac{\Delta d}{\Delta w} = \frac{750 \text{ mg}}{30 \text{ kg}} = 25 \text{ mg/kg}$$

And again, any two points on the line can be used to determine the slope.



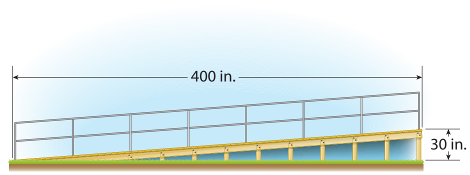
$$\frac{\Delta d}{\Delta w} = \frac{500 \text{ mg}}{20 \text{ kg}} = 25 \text{ mg/kg}$$

The dosage increases by 25 milligrams for each kilogram of increase in weight.

In general, the slope of the graph of a direct variation function of the form $y = kx$ can be symbolized as $\frac{\Delta y}{\Delta x}$, read as "the change in y divided by the change in x."

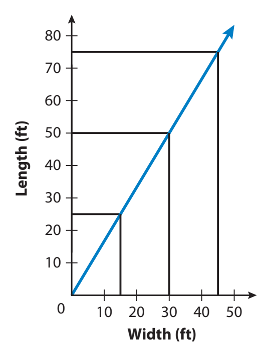
Practice for Lesson 2.5

1. A school staircase has a total vertical rise of 96 inches and a horizontal run of 150 inches.
   a. For this staircase, what is the rise when the run is 50 inches?
   b. What is the slope of the staircase?
2. The Americans with Disabilities Act (ADA) requires that a straight ramp have a slope no greater than $\frac{1}{12}$.
   a. What is the slope of the ramp in the figure? Does it meet the ADA requirement?



   b. Find the coordinates of a point that is halfway up the ramp.
   c. Find the slope from the halfway point to the top of the ramp.
   d. What can you say about the slope between any two points along a straight ramp?
3. Roofers describe the steepness of a roof by its *pitch*, the number of inches of rise for each 12 inches of run. A "3 in 12" roof rises 3 inches for every 12 inches of run.
   a. What is the slope of a "3 in 12" roof?
   b. A roof rises 100 inches over a run of 20 feet. Find the slope of the roof.
4. In Lesson 2.3, you learned that architects often use a regulating line to unify a design. The corners of three similar rectangles are on the regulating line shown here.

   Find the slopes of line segments joining two different pairs of points to show that the slope of the regulating line is constant.



Practice for Lesson 2.5 Answers

1a. $\frac{96}{150} = \frac{\text{rise}}{50}$; rise $=$ 32 inches

1b. $\frac{16}{25}$ or 0.64

2a. $\frac{30}{400} = \frac{3}{40}$ or 0.075; Yes, since $\frac{1}{12} \approx 0.083$ and $0.075 < 0.083$.

2b. (200, 15)

2c. $\frac{(30 - 15)}{(400 - 200)} = \frac{15}{200} = \frac{3}{40}$ or 0.075

2d. Sample answer: The slope is the same between any two points.

TEACHING TIP

Exercise 3 If students give the slope of the roof as "5 inches per foot," point out that, for comparison to the roof in Part (a), 20 feet must be converted to 240 inches.

3a. $\frac{1}{4}$ or 0.25

3b. $\frac{100 \text{ inches}}{240 \text{ inches}} = \frac{5}{12}$ or about 0.42

4. Sample answer: $\frac{75 - 0}{45 - 0} = \frac{5}{3}$; $\frac{50 - 25}{30 - 15} = \frac{5}{3}$; The slopes are equal.

LESSON 2.5

TEACHING TIP

Make sure that **Exercises 5 and 7** are assigned to all students and discussed in class later as new content is taught in these exercises.

TEACHING TIP

Exercise 6 Point out that it is appropriate to state a slope as a number without units only if the rise and run are measured in the same units. The slope of the gutter can be given as $\frac{1}{20}$ only if accompanied by the units "inches per foot."

TEACHING TIP

Exercise 7 Point out that subscripts are used to identify and distinguish between variables that represent similarly-named quantities.

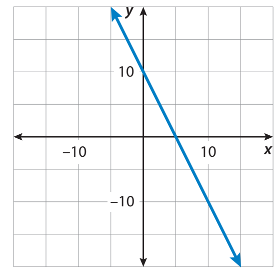
COMMON ERROR

Exercise 7 Students occasionally reverse the order of the subscripted variables in either the numerator or the denominator, resulting in an incorrect slope value. Part (c) is intended to focus attention on this possible source of error.

5. $\dfrac{-136}{189} \approx -0.72$

6. $\dfrac{1\text{ inch}}{240\text{ inches}} = \dfrac{1}{240}$ or about 0.0042; also $\dfrac{1}{20}$ or 0.05 inches per foot

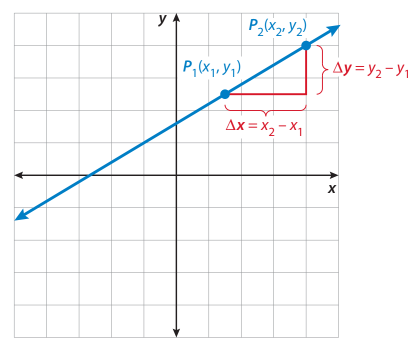
5. If a line slopes downward from left to right, its rise is negative, and so is its slope.



For example, the line in the figure above falls from 10 units to 0 as x increases from 0 to 5 units. It has a slope of $\frac{-10}{5} = -2$.
The White Heat trail at Sunday River Ski Resort in Maine is one of the steepest ski trails in the eastern United States. It falls 136 feet over a horizontal distance of 189 feet. What is the slope of White Heat?

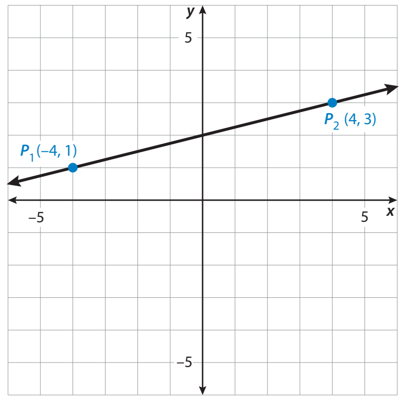
6. A roof gutter should be installed so that it slopes about 1 inch in 20 feet. What is the slope of such a gutter?

7. The general expression $\frac{\Delta y}{\Delta x}$ for slope can be written another way. For two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ on a line, the change in x, or Δx, equals the difference $(x_2 - x_1)$ of the x-coordinates. Similarly, $\Delta y = (y_2 - y_1)$.



So, the formula $\dfrac{\Delta y}{\Delta x} = \dfrac{y_2 - y_1}{x_2 - x_1}$ can be used to calculate the slope for a line that passes through any two points P_1 and P_2.

a. Use the formula to find the slope of the line containing points $P_1(-4, 1)$ and $P_2(4, 3)$.

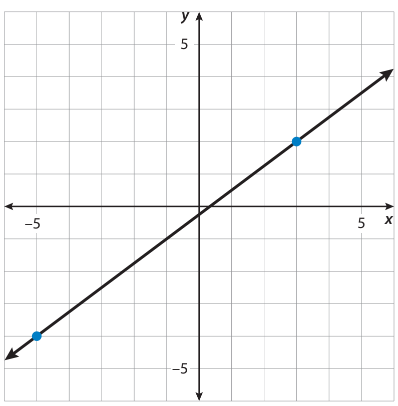


b. Now switch the names of the points. Let the point on the left be $P_2(-4, 1)$ and let the point on the right be $P_1(4, 3)$. Once again, use the formula to calculate the slope.

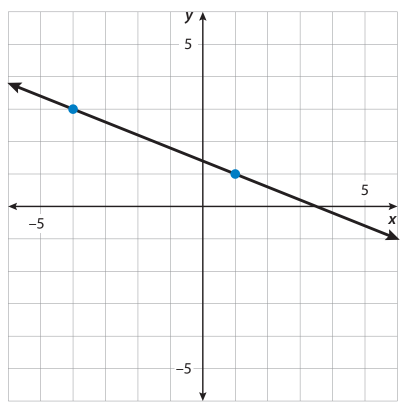
c. Explain what the results of Parts (a) and (b) suggest about the use of the slope formula.

For Exercises 8–11, find the slope of each line. Use the two points with integer coordinates indicated on each graph.

8.



9.



7a. $\dfrac{\Delta y}{\Delta x} = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{3 - 1}{4 - (-4)} = \dfrac{2}{8} = \dfrac{1}{4}$

7b. $\dfrac{\Delta y}{\Delta x} = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{1 - 3}{(-4) - 4}$
$= \dfrac{-2}{-8} = \dfrac{1}{4}$

7c. Sample answer: The order of the subtractions in the formula is not important, as long as x_1 and y_1 are coordinates of the same point (and similarly for x_2 and y_2).

8. $\dfrac{3}{4}$

9. $-\dfrac{2}{5}$

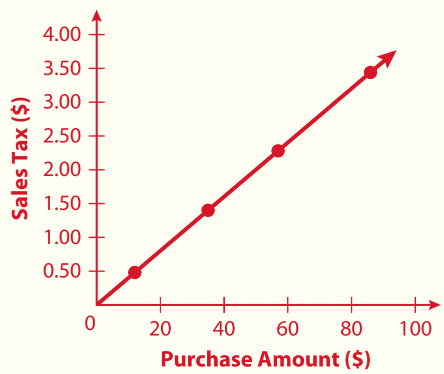
10. 3

11. 0

12. Sample answer: All points on a horizontal line have the same y-coordinate. Therefore, the rise is always 0, and $\frac{0}{\Delta x} = 0$.

13. Sample answer: All points on a vertical line have the same x-coordinate. Therefore, the denominator in the slope formula is always 0, and division by 0 is undefined.

14a. Sample graph:



14b. Sample answer: Yes, the ratio of sales tax to purchase amount is always 0.04, and the graph is a straight line that passes through the origin.

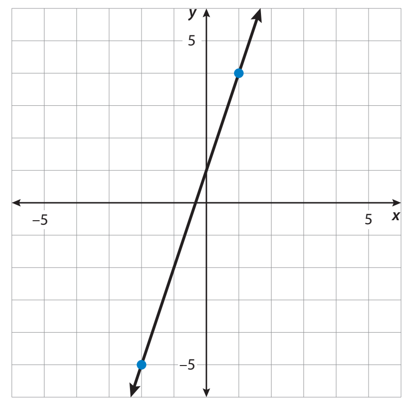
14c. Sample answer:
$\frac{\$1.40 - \$0.48}{\$35 - \$12} = \frac{\$0.92}{\$23} = 0.04$

14d. Sample answer: For each dollar spent, the sales tax is $0.04. So, the sales tax rate in this state is 4 percent.

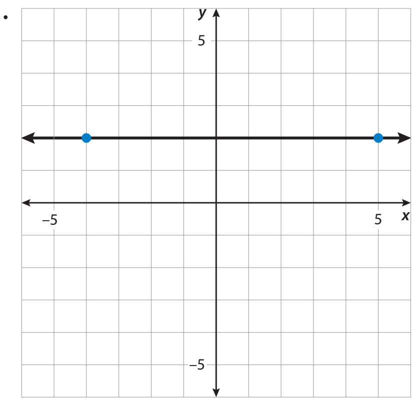
15a. $\frac{760 \text{ kilobytes}}{8 \text{ seconds}} = 95$ kilobytes per second

15b. The computer can download files at a rate of 95 kilobytes per second.

10.



11.



12. Explain the statement "Every horizontal line has the same slope."

13. Explain why the slope of any vertical line is undefined.

14. The table shows the amount of sales tax paid on various purchases in a particular state.

a. Make a graph of sales tax vs purchase amount.

b. Is this a direct variation function? Why or why not?

c. Find the slope of the line on the graph.

d. Write a sentence that interprets the meaning of the slope.

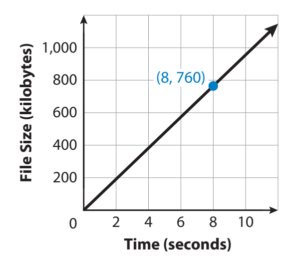
Purchase Amount	Sales Tax
$12	$0.48
$35	$1.40
$57	$2.28
$86	$3.44

For Exercises 15–17, answer Parts (a) and (b).

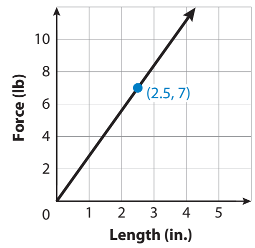
a. Find the slope of the graph, including units.

b. Interpret the meaning of the slope as a rate of change.

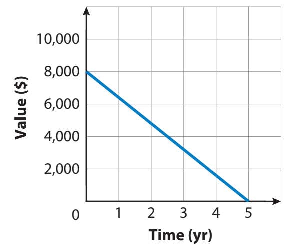
15. The graph shows the size of a file that can be downloaded from the Internet by a particular computer in a given number of seconds.



16. The graph below shows the force needed to stretch a spring to different lengths.



17. The graph below shows the *depreciation*, or decrease in value over time, of a piece of office equipment.



18. The table shows the data for the stretching of a bungee cord from Exercise 5 of Lesson 2.1.

Total Weight (pounds)	Total Length (feet)	Length of Stretch (feet)
0	60	0
64	76	16
144	96	36
176	104	44
280	130	70

a. Make a graph of stretch vs weight.
b. Is this a direct variation function? Why or why not?
c. Find the slope of the line, including units.
d. Write a sentence that interprets the slope as a rate of change.

19. Find the slope of the line that passes through the points (–3, 2) and (1, 0).

20. A line passes through the point (1, 5) and has a slope of $\frac{2}{3}$.
a. Find another point on the line.
b. Graph the line.

LESSON 2.5

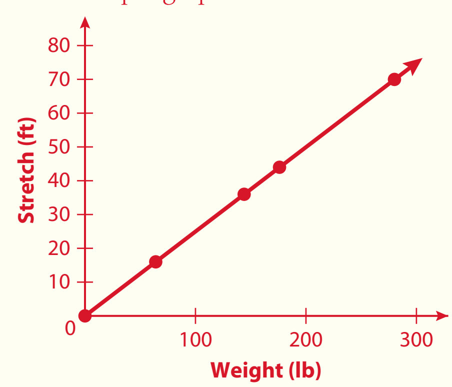
16a. $\frac{7 \text{ pounds}}{2.5 \text{ inches}} = 2.8$ lb/in.

16b. It requires 2.8 pounds of force to stretch a spring for each inch of stretch. The rate of change of force with respect to length is 2.8 lb/in.

17a. $\frac{-\$8{,}000}{5 \text{ years}} = -1{,}600$ dollars per year

17b. The equipment depreciates at a rate of 1,600 dollars per year. The rate of change of value with respect to time is −1,600 dollars per year.

18a. Sample graph:



18b. Sample answer: Yes, the ratios of stretch to weight are all equal to $\frac{1}{4}$, and the graph is a straight line that passes through the origin.

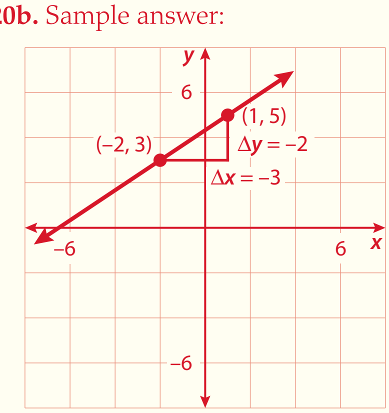
18c. $\frac{1}{4}$ ft/lb or 0.25 ft/lb

18d. Sample answer: The bungee cord stretches one-fourth of a foot for every pound of weight suspended from it. The rate of change of stretch with respect to weight is 0.25 ft/lb.

19. $-\frac{1}{2}$

20a. Sample answer: (–2, 3)

20b. Sample answer:



MODELING PROJECT

5e Explore, Elaborate

Materials List

- bathroom scale
- large pot
- measuring cup
- water

Description

In this project students examine the relationship between the volume of water added to a pot and the total weight of pot plus water. They are asked to find a way of analyzing the relationship that involves direct variation. This project works well when students work together in small groups of two or three.

Sample Answers

The data in the students' reports will vary, but the conclusions should all be similar. The relationship between the weight of the pot plus water and the volume of the water is a linear relationship, but it is not direct variation.

Students should recognize that by subtracting the weight of the pot from the total weight of pot plus water, they can model the relationship between the weight of the water and the volume of the water with a direct variation function. The ratio of weight to volume, which equals the slope of a graph of weight of water vs volume of water, provides an experimental value for the density of water. For comparison purposes, the true density of water is 62.4 pounds per cubic foot, or about 0.13 pounds per fluid ounce.

If students have trouble seeing how to analyze the data to find a direct variation, suggest they look at the treatment of the data in the Activity in Lesson 2.1. There, the heights of the can were subtracted from the original height to find the amount of stretch.

CHAPTER 2

Modeling Project

It's Only Water Weight

In this chapter, you have learned to identify proportional relationships and to model them with direct variation functions. You are now ready to construct and analyze a model beginning with data that you collect.

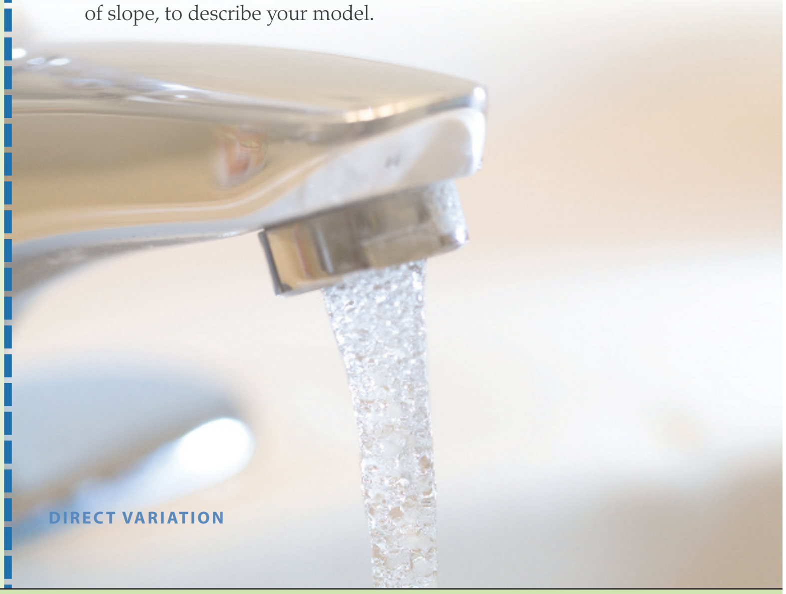
You will need a scale, such as a bathroom scale, a large pot, a measuring cup, and a source of water.

- Place the pot on the scale and record its weight.
- Add measured volumes of water to the pot. After each added amount, record the total volume of water and the weight of the pot and water.
- Continue adding water until the pot is almost full.

Conduct an investigation of the weight vs volume relationship.

1. Make a table and graph comparing the total weight of the pot and water with the total volume of water in the pot.
2. Determine whether a direct variation function provides a good model for the relationship, and explain how you know. If so, find a direct variation function that models your data. If not, decide how you can analyze the data in a different way so that a direct variation model is appropriate.

Prepare a report of your findings. Use what you have learned about proportions, as well as the calculation and interpretation of slope, to describe your model.



Chapter 2 Review

You Should Be Able to:

Lesson 2.1
- collect experimental data.
- make a scatter plot of experimental data.

Lesson 2.2
- identify proportional relationships.
- identify independent and dependent variables.
- find a constant of proportionality.
- write an equation that expresses a proportional relationship.

Lesson 2.3
- identify direct variation.
- write direct variation equations.

Lesson 2.4
- solve problems that require previously learned concepts and skills.

Lesson 2.5
- find the slope of a line given rise and run.
- find the slope of a line given two points on the line.
- interpret the slope of a line as a rate of change.

Key Vocabulary

dependent variable (p. 26)
independent variable (p. 26)
proportional relationship (p. 26)
constant of proportionality (p. 26)
direct proportion (p. 32)
variation (p. 32)
direct variation (p. 32)
varies directly (p. 32)
origin (p. 33)
function (p. 33)
input value (p. 33)
output value (p. 33)
direct variation function (p. 35)
slope (p. 40)
rate of change (p. 42)

Chapter 2 Test Review

Fill in the blank.

1. A relationship between two variables such that there is exactly one output value for each input value is called a(n) ___________.
2. The slope of a line is equal to the ratio of the ___________ to the ___________ between any two points on the line.

CHAPTER REVIEW

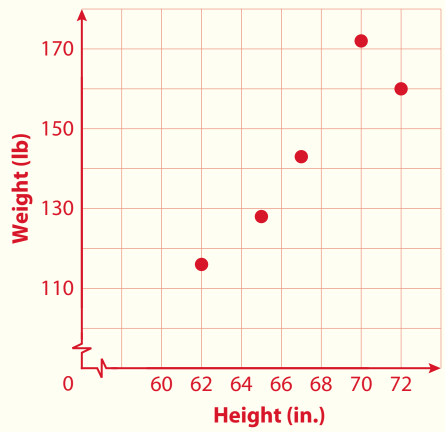
5e Evaluate

Chapter 2 Test Review Answers

1. function
2. rise, run

CHAPTER REVIEW

3. x or horizontal, y or vertical
4. ratio
5. Sample plot:



6. The number scales on the axes are not uniformly increasing.

3. On the graph of a function, the independent variable is on the ________ axis and the dependent variable is on the _________ axis.

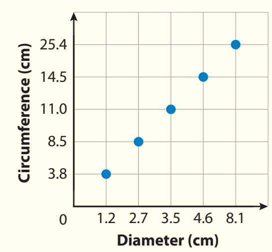
4. For a direct variation function, the _________ of the variables is constant.

5. The table lists heights and weights for the starters on a high school girls' basketball team.

Height (inches)	Weight (pounds)
67	143
62	116
65	128
72	160
70	172

Make a scatter plot of weight vs height for the data. Label the axes appropriately.

6. In a geometry class, students measured the diameters and circumferences of a variety of circular objects. What is wrong with the scatter plot of their data?



For Exercises 7–9, state whether the relationship between the variables is proportional. If it is, complete Parts (a), (b), (c), and (d). If it is not, explain how you know.

a. Identify the independent and dependent variables.

b. Make a graph of the relationship.

c. Find the constant of proportionality, including units if appropriate.

d. Write an equation for a direct variation function that models the data.

7. The table shows the speed of a ball after falling for various times.

Time (seconds)	Speed (meters per second)
0.25	2.45
0.50	4.90
0.75	7.35
1.00	9.80
1.25	12.25

8. The table shows the period (time between bounces) for people of different weights on a bungee cord.

Weight (lb)	Period (s)
100	8.0
140	9.5
180	10.7
200	11.3

9. The table shows the weight of different volumes of water in a pitcher.

Volume (oz)	Weight (lb)
16	2.1
20	2.6
30	3.9
42	5.5
60	7.8

10. Variable y is directly proportional to variable x, and y is 128 when x is 5.

a. Write an equation that gives y as a function of x.

b. Find the value of y when x is 12.

11. A house sewer drain must fall at least $\frac{1}{4}$ inch for each foot of length. What is the slope of a house sewer drain?

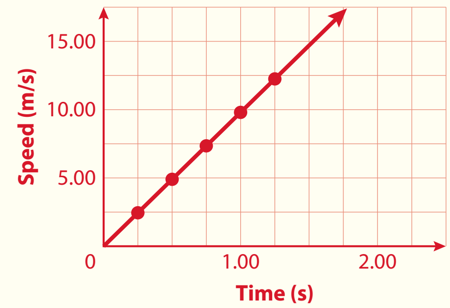
12. Find the slope of a line that passes through the points $(-2, 1)$ and $(3, 8)$.

CHAPTER REVIEW

7. proportional

7a. independent variable: time; dependent variable: speed

7b. Sample graph:



7c. 9.80 meters per second per second

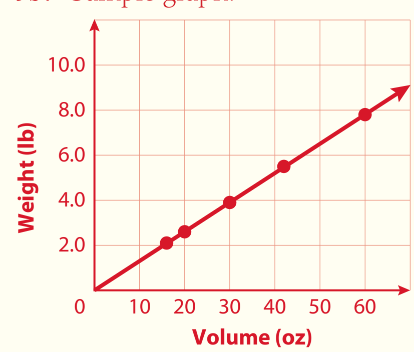
7d. Sample equation: $s = 9.80t$

8. not proportional; The ratios of period to weight are not constant.

9. proportional

9a independent variable: volume; dependent variable: weight

9b. Sample graph:



9c. 0.13 lb/oz

9d. $W = 0.13V$

10a. $y = 25.6x$

10b. 307.2

11. $\dfrac{-\frac{1}{4}\text{ inch}}{12\text{ inch}} = -\dfrac{1}{48}$ or about -0.021

12. $\dfrac{7}{5}$

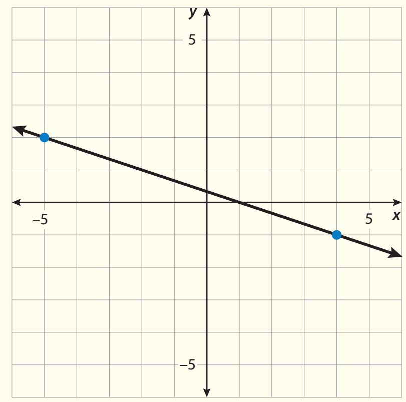
CHAPTER REVIEW

13. $-\frac{1}{3}$

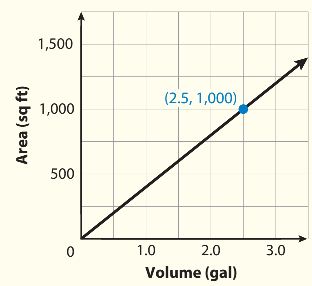
14a. 400 square feet per gallon

14b. Each gallon of paint covers 400 square feet of wall area. The rate of change of painted area with respect to paint volume is 400 square feet per gallon.

13. Find the slope of the line. The coordinates of the indicated points are integers.



14. The graph shows the relationship between wall area covered and volume of paint from a certain manufacturer.



a. Find the slope of the graph, including units.

b. Write a sentence that interprets the slope as a rate of change.

CHAPTER 2

Chapter Extension

Inverse Variation

Variables can be related to each other in many different ways. In this chapter, you have explored direct variation in some detail. Direct variation is one of the most common types of variation between two quantities. This extension consists of an activity that involves another frequently found type of variation.

When you look through a telescope, the amount you can see changes as the length of the telescope changes. The size of the *viewing circle* depends on the length of the telescope.

- Make a simple telescope by cutting a paper towel tube from one end to the other so that a second paper towel tube can fit inside of it. Do the same with a third paper towel tube. Insert the uncut tube inside one of the cut tubes. Then put these two inside the other cut tube. Now you can vary the length of your scope from one tube to nearly three.
- Tape a meter stick to a wall in a horizontal position. Back away from the wall and look at the stick through your scope. If you align the left edge of your viewing circle with the left end of the meter stick, you can estimate the diameter of the viewing circle. If you change the length of your scope, the diameter of your viewing circle should also change.

1. Stand at the farthest distance from the wall that still allows you to read the numbers on the meter stick. Start with the telescope at its shortest length. Measure that length. Then measure the diameter of the viewing circle. Record both measurements in the first row of a table like the one shown.

Telescope Length (cm)	Diameter of Viewing Circle (cm)	

2. Pull on the outer tube of the telescope to increase the telescope's length by 10 centimeters. Again, record the telescope length and the diameter of the viewing circle.

CHAPTER 2 EXTENSION

5e Elaborate

Lesson Objective

- use inverse variation to model experimental data.

Vocabulary

- inverse variation
- vary inversely

Materials List

Per group:

- 3 paper towel tubes
- masking tape
- meter stick
- tape measure
- scissors

Description

Students should work in groups of 2 or 3 on this lesson. They investigate the relationship between the length of a simulated telescope and the diameter of the resulting viewing circle.

TEACHING TIP

Questions 1-3 The validity of the data will be improved if each student in a group estimates the diameters independently and an average is determined for each telescope length.

Chapter 2 Extension Answers

1. See table in Question 9.
2. See table in Question 9.

CHAPTER 2 EXTENSION

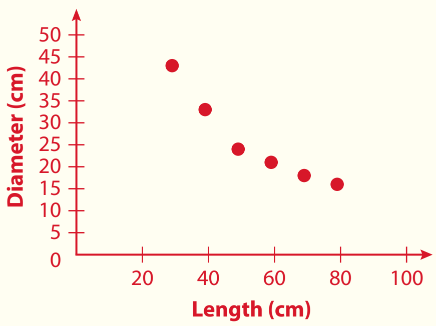
TEACHING TIP

Question 9 The products of diameter and length will not be exactly constant, but they should be approximately constant to the nearest 100 centimeters, which is roughly the accuracy of the calculations.

TEACHING TIP

Question 11 provides an opportunity to anticipate the discussion of the domain of a function in Chapter 5. Ask students to identify the minimum and maximum values for telescope length. Although the domain of an inverse variation function is all positive numbers, the *problem domain* for this experiment includes any length between 0 and the maximum length of students' telescopes.

3. See table in Question 9.
4. Sample graph:



5. Sample answer: The scatter plot has a curved shape, and the diameters decrease as the length of the telescope increases.
6. Sample answer: No, the diameter-to-length ratios are not constant, and the point (0, 0) does not fit the trend of the plot.
7. Sample answer: The diameter of the viewing circle would get smaller but would level off.
8. Sample answer: The diameter would get larger and larger.

3. Continue to increase the telescope's length 10 centimeters at a time. Record each new length and the corresponding diameter of the viewing circle.

4. Make a scatter plot of diameter of viewing circle vs length of telescope.

5. Describe the shape of your scatter plot.

6. Could the relationship between diameter and length be one of direct variation? Explain.

7. What do you expect would happen to the diameter of the viewing circle if the length of the telescope could get longer and longer?

8. What do you expect would happen to the diameter of the viewing circle if the length of the telescope got shorter and shorter?

9. Label the third column of your table "Product of Length and Diameter (cm^2)." Then multiply each length and diameter pair and record the results in this column.

10. Describe any pattern you see in the Product of Length and Diameter column.

Note

Inverse variation is sometimes called *inverse proportion* or *indirect variation.*

11. When the product of two variables is always equal to the same number, the variables are said to **vary inversely** with each other. This type of relationship is called **inverse variation**. In general, if y varies inversely with x, the relationship can be expressed by the equation $y = \frac{k}{x}$. The number k is the constant product of the variables.

Assume that your variables vary inversely with each other. Write an equation that models viewing circle diameter d as a function of telescope length L.

12. Use your equation to predict the diameter of the viewing circle when telescope length is half of the shortest length you recorded in your table. You might even cut one of your tubes in half to check your prediction.

13. In the 17th century, the English scientist Robert Boyle found that the pressure P of a gas varies inversely with the volume V of the gas. This relationship is now known as *Boyle's Law*.

a. Use a constant k to write an equation for Boyle's Law.

b. A particular sample of gas has a volume of 30 cubic inches at a pressure of 0.2 pound per square inch. Use this information along with Boyle's Law to write an equation giving pressure as a function of volume for this gas sample.

c. Use your equation to find the pressure when the gas is compressed to a volume of 10 cubic inches.

9. Sample answer:

Telescope Length (cm)	Diameter of Viewing Circle (cm)	Product of Length and Diameter (cm^2)
29	43	1,247
39	33	1,287
49	24	1,176
59	21	1,239
69	18	1,242
79	16	1,264

10. Sample answer: The products vary, but all are between 1,176 and 1,287.

11. Sample answer: The average product of d and L is 1,242.5. An equation is $(d)(L) = 1{,}242.5$, which is equivalent to the function $d = \frac{1{,}242.5}{L}$.

12. Sample answer: For $L = 14.5$ cm, $d = \frac{1{,}242.5}{14.5} \approx 86$ cm.

13a. $P = \frac{k}{V}$ or $PV = k$

13b. $k = PV = (0.2)(30) = 6$, so $P = \frac{6}{V}$.

13c. $P = \frac{6}{10} = 0.6$ pound per square inch