4.1 Apply Triangle Sum Properties

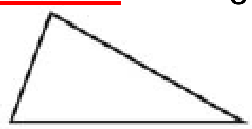
Obj.: **Classify triangles and find measures of their angles.**

Key Vocabulary

- **Triangle -** A **triangle** is a **polygon** with **three** sides. A triangle with vertices *A*, *B*, and *C* is called "triangle *ABC*" or "**▲*ABC***."

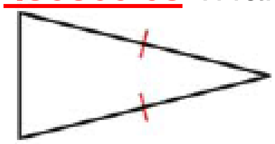
Classifying Triangles by Sides

Scalene Triangle



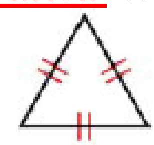
No congruent sides

Isosceles Triangle



At least **2** congruent sides

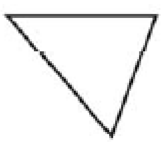
Equilateral Triangle



3 congruent sides

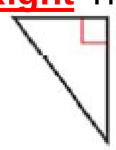
Classifying Triangles by Angles

Acute Triangle



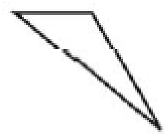
3 acute angles

Right Triangle



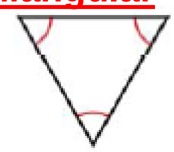
1 right angle

Obtuse Triangle



1 obtuse angle

Equiangular Triangle

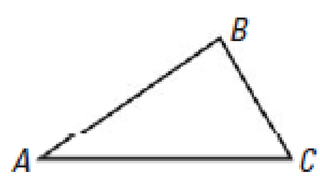


3 congruent angles

- **Interior angles -** The **original** angles are the **interior angles**.
- **Exterior angles -** When the sides of a polygon are extended, **other** angles are formed. The angles that form **linear** pairs with the interior angles are the **exterior angles**.
- **Corollary to a theorem -** A **corollary to a theorem** is a **statement** that can be proved easily **using** the **theorem**.

Triangle Sum Theorem

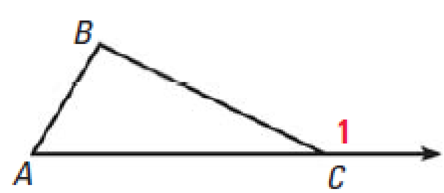
The **sum** of the measures of the interior **angles** of a triangle is **180°.**



$m\angle A + m\angle B + m\angle C = \mathbf{180^\circ}$

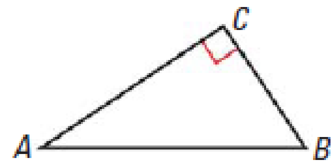
Exterior Angle Theorem

The measure of an **exterior** angle of a triangle is **equal** to the **sum** of the measures of the two nonadjacent **interior** angles.



$m\angle 1 = \mathbf{m\angle A + m\angle B}$

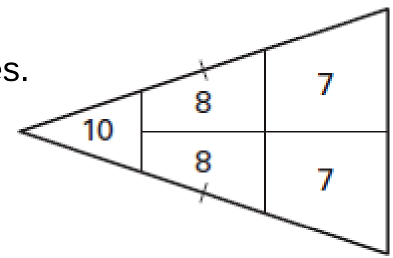
Corollary to the Triangle Sum Theorem
The **acute** angles of a right triangle are **complementary**.



$m\angle A + m\angle B =$ **90^0**

EXAMPLE 1 Classify triangles by sides and by angles

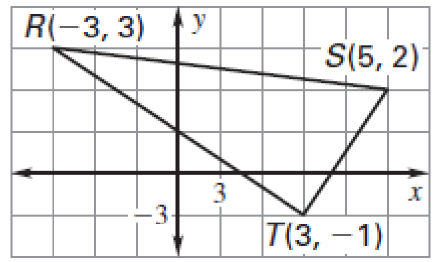
Shuffleboard Classify the triangular shape of the shuffleboard scoring are in the diagram by its sides and by measuring its angles.



Solution The triangle has a pair of congruent sides, so it is isosceles. By measuring, the angles are about 72°, 72°, and 36°. It is an acute isosceles triangle.

EXAMPLE 2 Classify a triangle in a coordinate plane

Classify $\blacktriangle RST$ by its sides. Then determine if the triangle is a right triangle.



Solution **Step 1** **Use** the distance formula $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ to find the side lengths.

$RT = \sqrt{(3 - (-3))^2 + (-1 - 3)^2} = \sqrt{52}$

$RS = \sqrt{(5 - (-3))^2 + (2 - 3)^2} = \sqrt{65}$

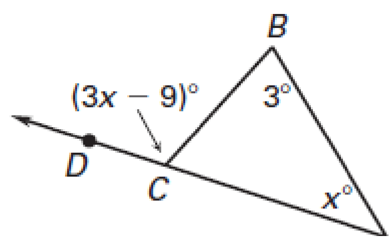
$ST = \sqrt{(3 - 5)^2 + (-1 - 2)^2} = \sqrt{13}$

Step 2 **Check** for right angles. The slope of $\overline{RT}$ is $\frac{-1-3}{3-(-3)} = -\frac{2}{3}$. The slope of $\overline{ST}$ is $\frac{-1-2}{3-5} = \frac{3}{2}$. The product of the slopes is -1, so $\overline{RT} \perp \overline{ST}$ and $\angle RTS$ is a right angle.

Therefore, $\triangle RST$ is a right scalene triangle.

EXAMPLE 3 Find an angle measure

ALGEBRA Find $m\angle DCB$.



Solution

Step 1 **Write** and solve an equation to find the value of x.

$(3x - 9)° = 73° + x°$ — **Exterior Angle Theorem**

$x = 41°$ — **Solve for x.**

Step 2 **Substitute** 41 for x in $3x - 9$ to find $m\angle DCB$.

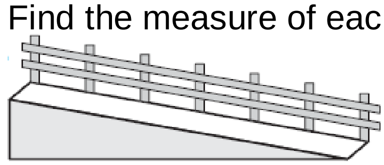
$3x - 9 = 3 \cdot 41 - 9 = 114$

The measure of $\angle DCB$ is 114°.

EXAMPLE 4 Find angle measures from a verbal description

Ramps The front face of the wheelchair ramp shown forms a right triangle. The measure of one acute angle in the triangle is eight times themeasure of the other. Find the measure of each acute angle.

Solution



First, sketch a diagram of the situation. Let the measure of the smaller acute angle be $x°$. Then the measure of the larger acute angle is $8x°$.

Use the Corollary to the Triangle Sum Theorem to set up and solve an equation.

$x° + 8x° = 90°$ — **Corollary to the Triangle Sum Theorem**

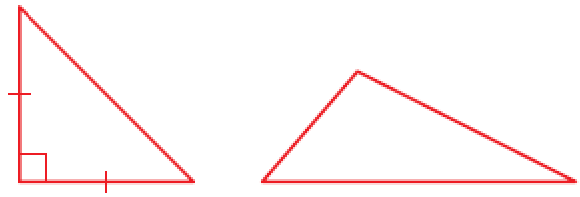
$x = 10$ — **Solve for x.**

So, the measures of the acute angles are 10° and 80°.

4.1 Cont.

Checkpoint **Complete the folowing exercise.**

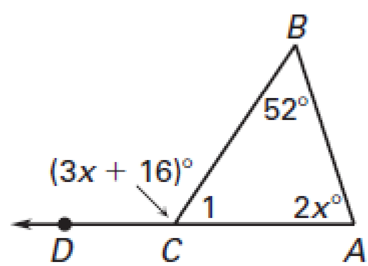
1. Draw an isosceles right triangle and an obtuse scalene triangle.

Sample Drawings: 

2. Triangle JKL has vertices $J(-2, -1)$, $K(1, 3)$, and $L(5, 0)$. Classify it by its sides. Then determine if it is a right triangle.

isosceles triangle; right triangle

3. Find the measure of $\angle 1$ in the diagram shown.



56°

4. In Example 4, what is the measure of the obtuse angle formed between the ramp and a segment extending from the horizontal leg?

170°

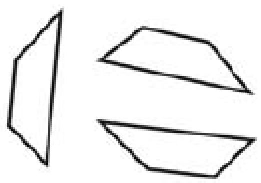
4.2 Apply Congruence and Triangles

Obj.: Identify congruent figures.

Key Vocabulary

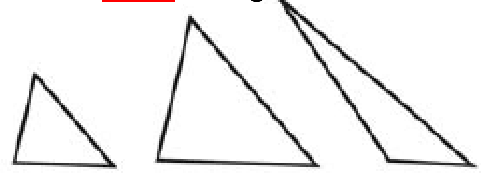
- **Congruent figures -** In two **congruent figures**, all the parts of one figure are congruent to the **corresponding parts** of the other figure.
- **Corresponding parts -** In congruent polygons, this means that the *corresponding* ***sides*** and the *corresponding* ***angles*** are **congruent**.

Congruent



Same size and shape

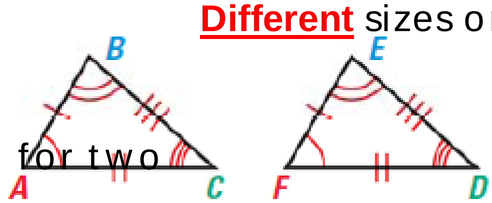
Not congruent



Different sizes or shapes

CONGRUENCE STATEMENTS

When you write a congruence statement for two polygons, always **list** the corresponding **vertices** in the same **order.**



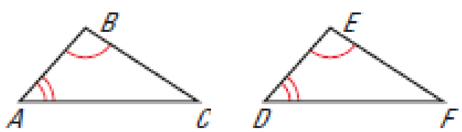
▲***ABC*** ≅ ▲***FED*** or ▲***BCA*** ≅ ▲***EDF***.

Corresponding angles: $\angle A \cong \angle F$, $\angle B \cong \angle E$, $\angle C \cong \angle D$

Corresponding sides: $\overline{AB} \cong \overline{FE}$, $\overline{BC} \cong \overline{ED}$, $\overline{AC} \cong \overline{FD}$

Third Angles Theorem

If **two angels** of one triangle are congruent to two angles of another triangle, then the **third** angles are also **congruent**.



If $\angle A \cong \angle D$ and $\angle B \cong \angle E$, then $\angle C \cong \angle D$

♦♦♦Properties of Congruent Triangles♦♦♦

Reflexive Property of Congruent Triangles

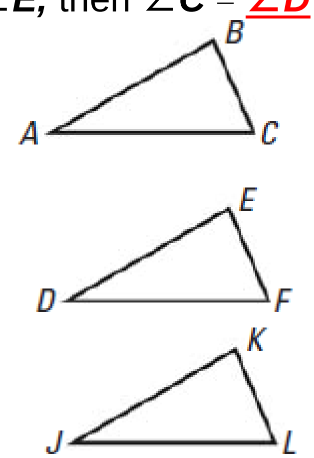
For any triangle ABC, ▲ABC ≅ ▲**ABC**

Symmetric Property of Congruent Triangles

If ▲ABC ≅ ▲**DEF** , then ▲DEF ≅ ▲**ABC**

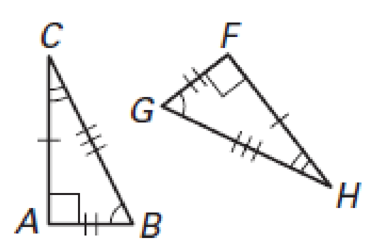
Transitive Property of Congruent Triangles

If ▲ABC ≅ ▲**DEF** and ▲DEF ≅ ▲**JKL**, then ▲ABC ≅ ▲**JKL**.



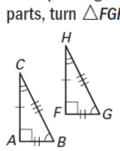
EXAMPLE 1 Identify congruent parts

Write a **congruence statement** for the triangles. Identify all pairs of congruent corresponding parts.



Solution The diagram indicates that $\triangle ABC \cong \triangle$ FGH.

To help you identify corresponding parts, turn $\triangle FGH$.

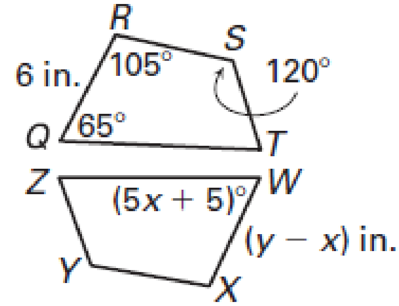


Corresponding angles $\angle A \cong$ $\angle F$, $\angle B \cong$ $\angle G$, $\angle C \cong$ $\angle H$

Corresponding sides $\overline{AB} \cong$ $\overline{FG}$, $\overline{BC} \cong$ $\overline{GH}$, $\overline{CA} \cong$ $\overline{HF}$

EXAMPLE 2 Use properties of congruent figures

In the diagram, *QRST* $\cong$ *WXYZ*.



a. Find the value of x.

Solution a. You know $\angle Q \cong \angle W$.

$m\angle Q = m\angle W$

$65° = (5x + 5)°$

$60 = 5x$

$12 = x$

b. Find the value of y.

b. You know $\overline{QR} \cong \overline{WX}$.

$QR = WX$

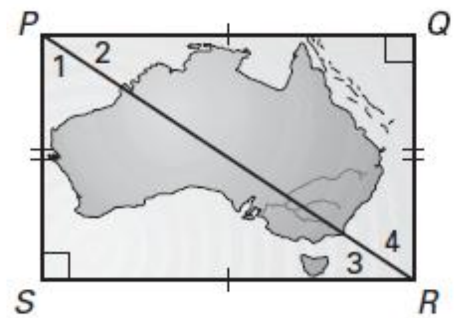
$6 = y - x$

$6 = y - 12$

$18 = y$

EXAMPLE 3 Show that figures are congruent

Maps If you cut the map in half along $\overline{PR}$, will the sections of the wall be the same size and shape? Explain.



Solution

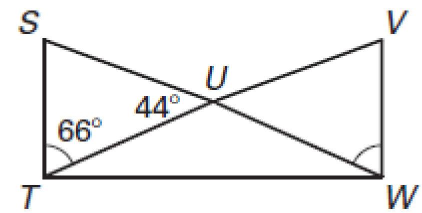
From the diagram, $\angle S \cong \angle Q$ because all right angles are congruent. Also, by the Lines Perpendicular to a Transversal Theorem, $\overline{PQ} \parallel \overline{RS}$. Then $\angle 1 \cong \angle 4$ and $\angle 2 \cong \angle 3$ by the Alternate Interior Angles Theorem. So, all pairs of corresponding angles are congruent.

The diagram shows $\overline{PQ} \cong \overline{RS}$ and $\overline{QR} \cong \overline{SP}$. By the Reflexive Property, $\overline{PR} \cong \overline{RP}$. All corresponding parts are congruent, so $\triangle PQR \cong \triangle RSP$.

Yes, the two sections will be the same size and shape.

EXAMPLE 4 Use the Third Angles Theorem

Find $m\angle V$.



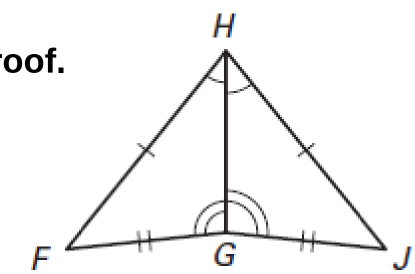
$\angle SUT \cong \angle VUW$ by the Vertical Angles Theorem. The diagram shows that $\angle STU \cong \angle VWU$, so by the Third Angles Theorem, $\angle S \cong \angle V$. By the Triangle Sum Theorem, $m\angle S = 180° - 66° - 44° = 70°$. So, $m\angle S = m\angle V = 70°$ by the definition of congruent angles.

EXAMPLE 5 Prove that triangles are congruent Write a proof.

Given: $\overline{FH} \cong \overline{JH}$, $\overline{FG} \cong \overline{JG}$,

$\angle FHG \cong \angle JHG$, $\angle FGH \cong \angle JGH$

Prove: $\triangle FGH \cong \triangle JGH$



Plan for Proof

a. Use the Reflexive Property to show $\overline{HG} \cong \overline{HG}$.

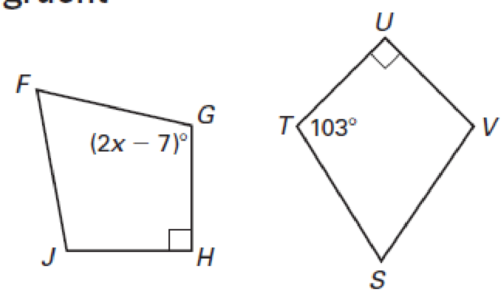
b. Use the Third Angles Theorem to show $\angle F \cong \angle J$.

Solution

	Statements	Reasons
	1. $\overline{FH} \cong \overline{JH}$, $\overline{FG} \cong \overline{JG}$	1. Given
a.	2. $\overline{HG} \cong \overline{HG}$	2. Reflexive Property of Congruence
	3. $\angle FHG \cong \angle JHG$, $\angle FGH \cong \angle JGH$	3. Given
b.	4. $\angle F \cong \angle J$	4. Third Angles Theorem
	5. $\triangle FGH \cong \triangle JGH$	5. Definition of $\cong$ $\triangle$s

Checkpoint In Exercises 1 and 2, use the diagram shown in which $FGHJ \cong STUV$.

1. Identify all pairs of congruent corresponding parts.



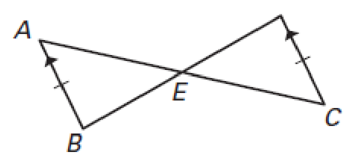
Corresponding angles: $\angle F \cong \angle S$, $\angle G \cong \angle T$, $\angle H \cong \angle U$, $\angle J \cong \angle V$

Corresponding sides: $\overline{FG} \cong \overline{ST}$, $\overline{GH} \cong \overline{TU}$, $\overline{HJ} \cong \overline{UV}$, $\overline{JF} \cong \overline{VS}$

2. Find the value of x and find $m\angle G$.

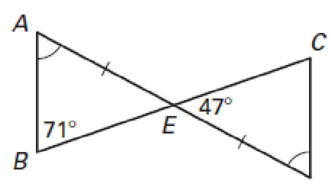
$x = 55$; $m\angle G = 103°$

3. In the diagram at the right, E is the midpoint of $\overline{AC}$ and $\overline{BD}$. Show that $\triangle ABE \cong \triangle CDE$.



4. In the diagram, what is the measure of $\angle D$?

62°



5. By the definition of congruence, what additional information is needed to know that $\triangle ABE \cong \triangle DCE$ in Exercise 4?

You must know that $\overline{AB} \cong \overline{DC}$ and $\overline{BE} \cong \overline{CE}$ to conclude that $\triangle ABE \cong \triangle DCE$. The remaining information can be inferred from the graph.

4.3 Prove Triangles Congruent by SSS

Obj.: Use the side lengths to prove triangles are congruent.

Key Vocabulary

- **Congruent figures -** In two **congruent figures**, all the parts of one figure are congruent to the **corresponding parts** of the other figure.
- **Corresponding parts -** In congruent polygons, this means that the *corresponding* ***sides*** and the *corresponding* ***angles*** are congruent.

Side-Side-Side (SSS) Congruence Postulate

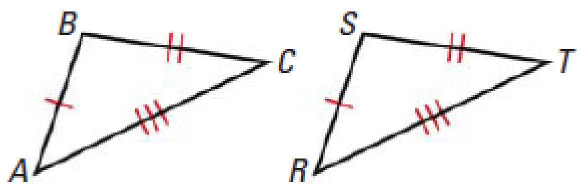
If three sides of one triangle are congruent to three sides of a second triangle, then the two triangles are congruent.

If Side $\overline{AB} \cong \overline{RS}$,

Side $\overline{BC} \cong \overline{ST}$, and

Side $\overline{AC} \cong \overline{TR}$,

then $\blacktriangle ABC \cong \blacktriangle RST$.



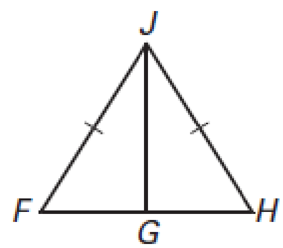
EXAMPLE 1 Use the SSS Congruence Postulate

Write a proof.

GIVEN ♦ $\overline{FJ} \cong \overline{HJ}$,

G is the midpoint of $\overline{FH}$

PROVE♦ $\blacktriangle FGJ \cong \blacktriangle HGJ$.

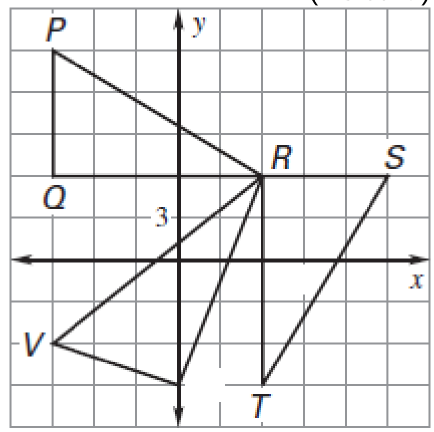


Statements	**Reasons**
1.	**1.**
2.	**2.**
3.	**3.**
4.	**4.**

Proof It is given that $\overline{FJ} \cong$ $\overline{J}$. Point G is the midpoint of $\overline{FH}$, so $\overline{FG} \cong \overline{G}$. By the Reflexive Property, $\overline{GJ} \cong \overline{JG}$. So, by the SSS Congruence Postulate, $\triangle FGJ \cong \triangle HGJ$.

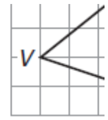
EXAMPLE 2 Congruence in the coordinate plane

Determine whether PQR is congruent to the other triangles shown at the right.



Solution

By counting, $PQ = 3$ and $QR = 5$.
Use the distance formula to find PR.



$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$PR = \sqrt{\underline{(2-(-3))^2 + (2-5)^2}} = \sqrt{\underline{34}}$$

By the SSS Congruence Postulate, any triangle with side lengths __3__, __5__, and __$\sqrt{34}$__ will be congruent to $\triangle PQR$. The distance from R to S is __3__. The distance from R to T is __5__. The distance from S to T is $\sqrt{\underline{(2-5)^2 + (-3-2)^2}} = \sqrt{\underline{34}}$. So,

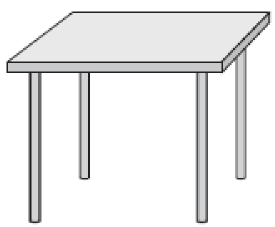
$\triangle PQR \cong$ __$\triangle\ RT$__.

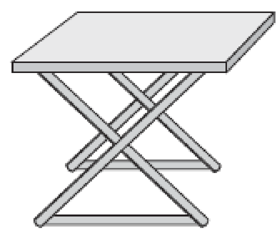
The distance from W to V is $\sqrt{\underline{(-3-0)^2 + (-2-(-3))^2}} = \sqrt{\underline{10}}$. No side of $\triangle PQR$ has a length of $\sqrt{\underline{10}}$, so $\triangle PQR$ __$\not\cong$__ $\triangle VWR$.

EXAMPLE 3 Solve a real-world problem

Stability Explain why the table with the diagonal legs is stable, while the one without the diagonal legs can collapse.

Solution



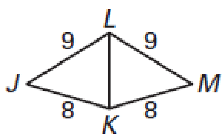


The table with the diagonal legs forms triangles with __fixed__ side lengths. By the SSS Congruence Postulate, these triangles __cannot change shape__, so the table is __stable__. The table without the diagonal legs is __not stable__ because there are many possible quadrilaterals with the given side lengths.

4.3 Cont.

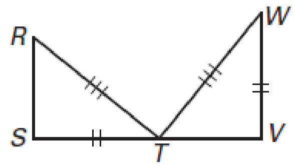
Checkpoint **Decide whether the congruence statement is true. *Explain* your reasoning.**

1. $\triangle JKL \cong \triangle MKL$



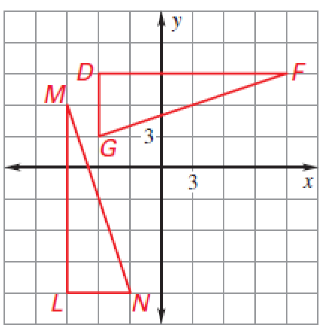
True; all corresponding sides are congruent.

2. $\triangle RST \cong \triangle TVW$



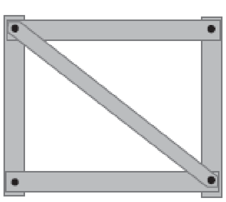
False; $\overline{RS} \ncong \overline{TV}$

3. $\triangle DFG$ has vertices $D(-2, 4)$, $F(4, 4)$, and $G(-2, 2)$. $\triangle LMN$ has vertices $L(-3, -3)$, $M(-3, 3)$, and $N(-1, -3)$. Graph the triangles in the same coordinate plane and show that they are congruent.



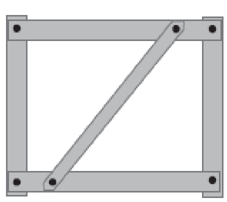
$DG = LN = 2$, $DF = LM = 6$, and $FG = MN = \sqrt{40}$, so $\triangle DFG \cong \triangle LMN$ by the SSS Congruence Postulate.

4.



Yes, the figure is stable. By the SSS Congruence Postulate, the triangles formed cannot change shape, so it is stable.

5.



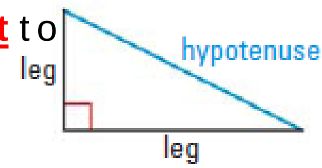
No, the figure is not stable. There are many possible quadrilaterals with the given side lengths.

4.4 Prove Triangles Congruent by SAS and HL

Obj.: Use sides and angles to prove congruence.

Key Vocabulary

- **Leg of a right triangle -** In a right triangle, the sides **adjacent** to the right **angle** are called the **legs**.
- **Hypotenuse –** The side **opposite** the **right angle** is called the **hypotenuse** of the right triangle.



Side-Angle-Side (SAS) Congruence Postulate

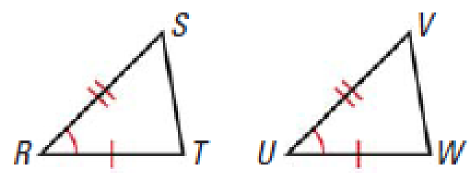
If **two** sides and the **included** angle of one triangle are **congruent** to two sides and the included angle of a **second** triangle, then the two triangles **are** congruent.

If **Side** $\overline{RS} \cong \overline{UV}$,

Angle $\angle R \cong \angle U$, and

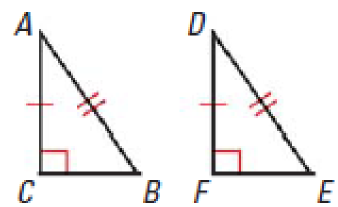
Side $\overline{RT} \cong \overline{UW}$,

then $\blacktriangle RST \cong \blacktriangle UVW$.



Hypotenuse-Leg (HL) Congruence Theorem

If the **hypotenuse** and a **leg** of a right triangle are **congruent** to the hypotenuse and a leg of a second **right** triangle, then the **two** triangles are congruent.



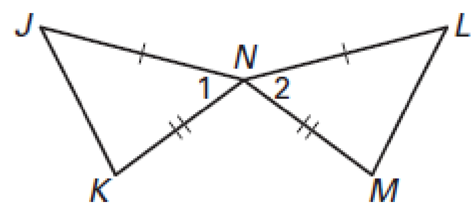
$\blacktriangle ABC \cong$ **$\blacktriangle DEF$**

EXAMPLE 1 Use the SAS Congruence Postulate

Write a proof.

GIVEN ♦ $\overline{JN} \cong \overline{LN}$, $\overline{KN} \cong \overline{MN}$

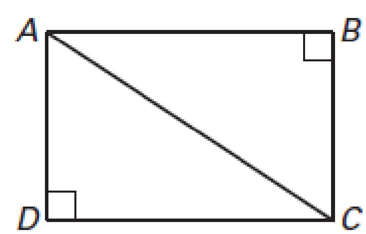
PROVE ♦ $\blacktriangle JKN \cong \blacktriangle LMN$



Statements	Reasons
1. $\overline{JN} \cong \overline{LN}$, $\overline{KN} \cong \overline{MN}$	1. Given
2. $\angle 1 \cong \angle 2$	2. Vertical Angles Theorem
3. $\triangle JKN \cong \triangle LMN$	3. SAS Congruence Postulate

EXAMPLE 2 Use SAS and properties of shapes

In the diagram, ABCD is a rectangle. What can you conclude about $\blacktriangle ABC$ and $\blacktriangle CDA$?



Solution

By the Right Angles Congruence Theorem, $\angle B \cong \angle D$. Opposite sides of a rectangle are congruent, so $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{DA}$.

$\triangle ABC$ and $\triangle CDA$ are congruent by the SAS Congruence Postulate.

EXAMPLE 3 Use the Hypotenuse-Leg Congruence Theorem (4.4 cont.)

Write a proof.

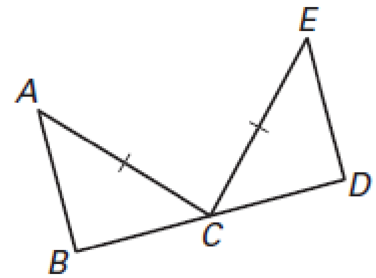
GIVEN ♦ $\overline{AC} \cong \overline{EC}$,

$\overline{AB} \perp \overline{BD}$

$\overline{ED} \perp \overline{BD}$

$\overline{AC}$ is a bisector of $\overline{BD}$

PROVE ♦ ▲*ABC* ≅ ▲*EDC*

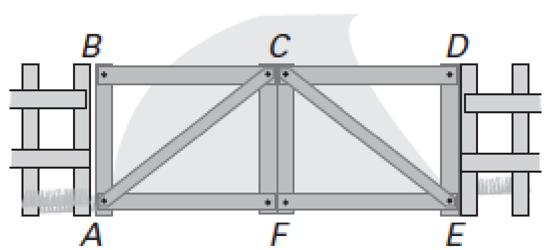


ons

	Statements	Reasons
H	1. $\overline{AC} \cong \overline{EC}$	1. Given
	2. $\overline{AB} \perp \overline{BD}$, $\overline{ED} \perp \overline{BD}$	2. Given
	3. $\angle B$ and $\angle D$ are right angles.	3. Definition of $\perp$ lines
	4. $\triangle ABC$ and $\triangle EDC$ are right triangles.	4. Definition of a right triangle
	5. $\overline{AC}$ is a bisector of $\overline{BD}$.	5. Given
L	6. $\overline{BC} \cong \overline{DC}$	6. Definition of segment bisector
	7. $\triangle ABC \cong \triangle EDC$	7. HL Congruence Theorem

EXAMPLE 4 Choose a postulate or theorem

Gate The entrance to a ranch has a rectangular gate as shown in the diagram. You know that ▲*AFC* ≅ ▲*EFC*. What postulate or theorem can you use to conclude that ▲*ABC* ≅ ▲*EDC.*

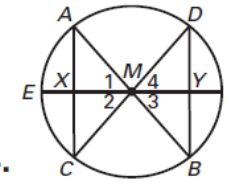


Solution You are given that *ABDE* is a rectangle, so $\angle B$ and $\angle D$ are right angles. Because opposite sides of a rectangle are congruent, $\overline{AB} \cong \overline{DE}$. You are also given that $\triangle AFC \cong \triangle EFC$, so $\overline{AC} \cong \overline{E}$. The hypotenuse and a leg of each triangle is congruent.

You can use the HL Congruence Theorem to conclude that $\triangle ABC \cong \triangle EDC$.

4.5 Cont.

Checkpoint **In the diagram, $\overline{AB}$, $\overline{CD}$, and $\overline{EF}$ pass through the center M of the circle. Also, $\angle 1 \cong \angle 2 \cong \angle 3 \cong \angle 4$.**



1. Prove that $\triangle DMY \cong \triangle BMY$.

Statements	Reasons
1. $\angle 3 \cong \angle 4$	1. Given
2. $\overline{DM} \cong \overline{BM}$	2. Definition of a circle
3. $\overline{MY} \cong \overline{MY}$	3. Reflexive Property of Congruence
4. $\triangle DMY \cong \triangle BMY$	4. SAS Congruence Postulate

2. What can you conclude about $\overline{AC}$ and $\overline{BD}$?

Because they are vertical angles, $\angle AMC \cong \angle BMD$. All points on a circle are the same distance from the center, so $AM = BM = CM = DM$. By the SAS Congruence Postulate, $\triangle AMC \cong \triangle BMD$. Corresponding parts of congruent triangles are congruent, so you know $\overline{AC} \cong \overline{BD}$.

3. *Explain* why a diagonal of a rectangle forms a pair of congruent triangles.

A diagonal of a rectangle will be the hypotenuse of each triangle formed. Because the hypotenuse is congruent to itself, and because opposite sides of a rectangle are congruent, you can use the HL Congruence Theorem to conclude the triangles are congruent.

4. In Example 4, suppose it is given that *ABCF* and *EDCF* are squares. What postulate or theorem can you use to conclude that $\triangle ABC \cong \triangle EDC$? *Explain*.

It is given that $AB\ \ F$ and $ED\ \ F$ are squares, so $\angle B$ and $\angle D$ are right angles, $\overline{AB} \cong \overline{DE}$, and $\overline{B\ \ } \cong \overline{D\ \ }$. You can use the SAS Congruence Postulate to conclude that $\triangle AB\ \ \cong \triangle ED\ \ $.

4.5 Prove Triangles Congruent by ASA and AAS

Obj.: Use two more methods to prove congruences.

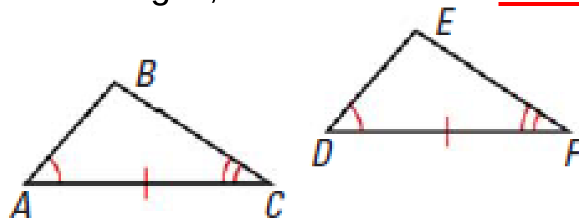
Key Vocabulary

• **Flow proof -** A **flow proof** uses **arrows** to show the **flow** of a logical argument. Each reason is written **below** the statement it justifies.

Angle-Side-Angle (ASA) Congruence Postulate

If **two** angles and the included **side** of one triangle are **congruent** to two angles and the **included** side of a second triangle, then the two **triangles** are congruent.

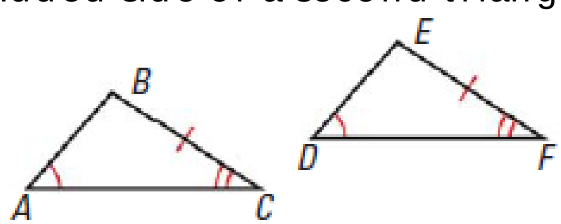
If **Angle** $\angle A \cong \angle D$,
Side $\overline{AC} \cong \overline{DF}$, and
Angle $\angle C \cong \angle F$,
then $\blacktriangle ABC \cong \blacktriangle DEF$.



Angle-Angle-Side (AAS) Congruence Theorem

If two **angles** and a **non**-included **side** of one triangle are congruent to **two** angles and the corresponding non-included side of a second triangle, then the two triangles are **congruent**.

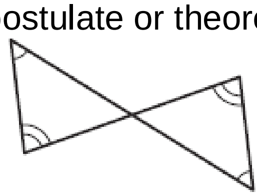
If **Angle** $\angle A \cong \angle D$,
Angle $\angle C \cong \angle F$, and
Side $\overline{BC} \cong \overline{EF}$,
then $\blacktriangle ABC \cong \blacktriangle DEF$.



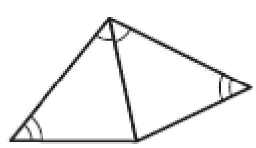
EXAMPLE 1 Identify congruent triangles

Can the triangles be proven congruent with the information given in the diagram? If so, state the postulate or theorem you would use.

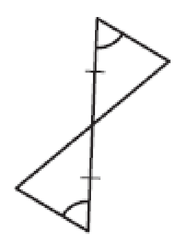
a.



b.



c.



Solution

a. There is not enough information to prove the triangles are congruent, because no sides are known to be congruent.

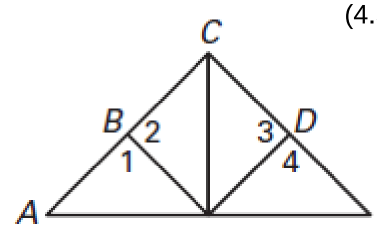
b. Two pairs of angles and a non-included pair of sides are congruent. The triangles are congruent by the AAS Congruence Theorem.

c. The vertical angles are congruent, so two pairs of angles and their included sides are congruent. The triangles are congruent by the ASA Congruence Postulate.

(4.5 cont.)

EXAMPLE 2 Write a flow proof

In the diagram $\angle 1 \cong \angle 4$ $\overline{CF}$ bisects and $\angle ACE$. Write a flow proof to show $\blacktriangle CBF \cong \blacktriangle CDF$.



Solution

Given ♦ $\angle 1 \cong \angle 4$ $\overline{CF}$ bisects and $\angle ACE$.

Prove ♦ $\blacktriangle CBF \cong \blacktriangle CDF$

$\angle 1 \cong \angle 4$ — Given

$\angle 1$ and $\angle 2$ are supplements. $\angle 3$ and $\angle 4$ are supplements. — Def. of supplementary angles

$\overline{CF}$ bisects. $\angle ACE$. — Given

$\angle 2 \cong \angle 3$ — Congruent Supps. Thm.

$\overline{CF} \cong \overline{CF}$ — Reflexive Prop.

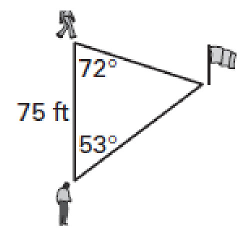
$\angle ACF \cong \angle$ — Def. of $\angle$ bisector

$\angle CBF \cong \angle CDF$ — AAS Congruence Theorem

EXAMPLE 3 Choose a postulate or theorem

Games You and a friend are trying to find a flag hidden in the woods. Your friend is standing 75 feet away from you. When facing each other, the angle from you to the flag is 72° and the angle from your friend to the flag is 53°. Is there enough information to locate the flag?

Solution



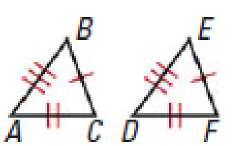
The locations of you, your friend, and the flag form a triangle. The measures of two angles and an included side of the triangle are known.

By the ASA Congruence Postulate, all triangles with these measures are congruent. So, the triangle formed is unique and the flag location is given by the third vertex.

Triangle Congruence Postulates and Theorems

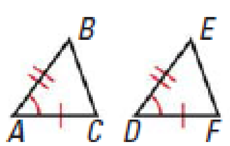
You have learned five methods for proving that triangles are congruent.

SSS



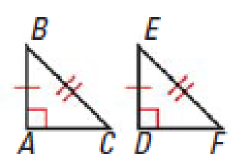
All three sides are congruent.

SAS



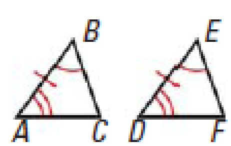
Two sides and the included angle are congruent.

HL **(right ▲s only)**



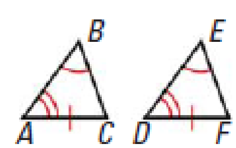
The hypotenuse and one of the legs are congruent.

ASA



Two angles and the included side are congruent.

AAS

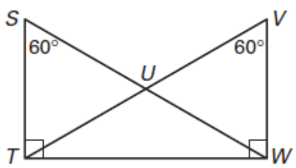


Two angles and a (non-included) side are congruent.

4.5 Cont.

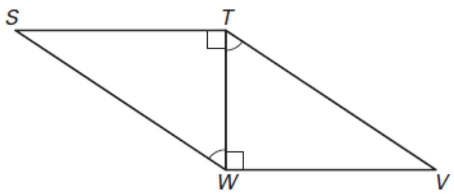
✔ *Checkpoint* Can $\triangle STW$ and $\triangle VWT$ be proven congruent with the information given in the diagram? If so, state the postulate or theorem you would use.

1.



Yes; AAS Congruence Theorem

2.

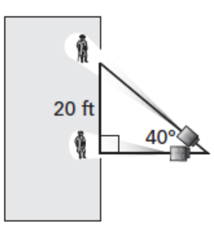


Yes; ASA Congruence Postulate

3. In Example 2, suppose it is given that $\overline{CF}$ bisects $\angle ACE$ and $\angle BFD$. Write a flow proof to show $\triangle CBF \cong \triangle CDF$.

bisects $\angle A$. Given		bisects $\angle B$ D. Given
$\angle A \cong \angle$ Def. of $\angle$ bisector	$\cong$ Reflexive Prop.	$\angle B \cong \angle D$ Def. of $\angle$ bisector
	$\angle B \cong \angle D$ ASA Congruence Postulate	

4. Theater You are working two spotlights for a play. Two actors are standing apart from each other on the end of the stage. The spotlights are located and pointed as shown in the diagram. Can one of the actors move without requiring the spotlight to move and without changing the distance between the other actor?



4.6 Use Congruent Triangles

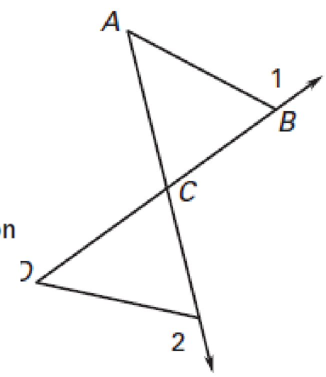
Obj.: Use congruent triangles to prove corresponding parts congruent.

Key Vocabulary

• **Corresponding parts –** In congruent polygons, this means that the *corresponding* ***sides*** and the *corresponding* ***angles*** are ***congruent***.

EXAMPLE 1 Use congruent triangles

Explain how you can use the given information to prove that the triangles are congruent.



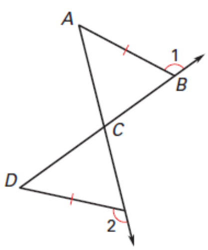
Given ♦ $\angle 1 \cong \angle 2$, $\overline{AB} \cong \overline{DE}$

Prove ♦ $\overline{DC} \cong \overline{AC}$

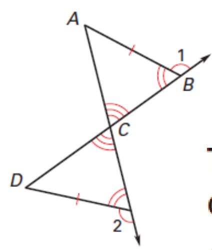
Solution

If you can show that $\triangle AB$ ≅ $\triangle D$, you will know that $\overline{DC} \cong \overline{AC}$. First, copy the diagram and mark the given information. Then add the information that you can deduce. In this case, $\angle ABC$ and $\angle DEC$ are supplementary to congruent angles, so $\angle$ AB ≅ $\angle$ D. Also, $\angle ACB \cong$ $\angle D$.

Mark given information.



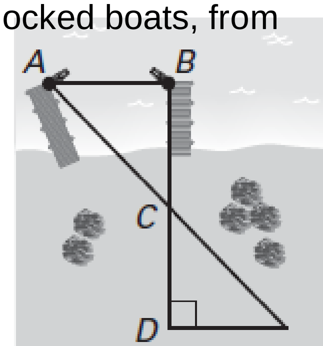
Add deduced information.



Two angle pairs and a non-included side are congruent, so by the AAS Congruence Theorem, $\triangle ABC \cong \triangle DEC$. Because corresponding parts of congruent triangles are congruent, $\overline{DC} \cong \overline{AC}$.

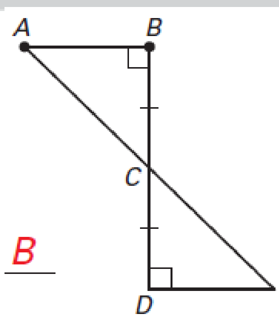
EXAMPLE 2 Use congruent triangles for measurement

Boats Use the following method to find the distance between two docked boats, from point *A* to point *B*.



- Place a marker at D so that $\overline{AB} \perp \overline{BD}$.
- Find C, the midpoint of $\overline{BD}$.
- Locate the point E so that $\overline{BD} \perp \overline{DE}$ and A, C, and E are collinear.
- Explain how this plan allows you to find the distance.

Solution



Because $\overline{AB} \perp \overline{BD}$ and $\overline{BD} \perp \overline{DE}$, $\angle B$ and $\angle D$ are congruent right angles. Because C is the midpoint of $\overline{BD}$, B ≅ D. The vertical angles $\angle A$ B and $\angle$ D are congruent. So,

B

$\triangle CBA \cong$ $\triangle$ D by the ASA Congruence Postulate. Then, because corresponding parts of congruent triangles are congruent, $BA =$ D. So, you can find the distance *AB* between the boats by measuring D.

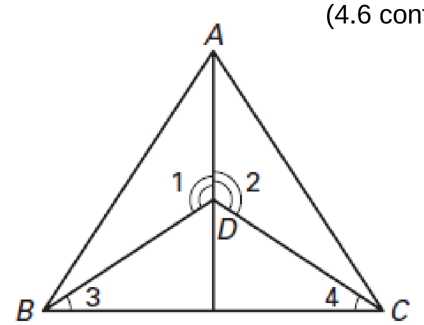
EXAMPLE 3 Plan a proof involving pairs of triangles

(4.6 cont.)

Use the given information to write a plan for proof.

Given ♦ $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$

Prove ♦ $\blacktriangle ABD \cong \blacktriangle ACD$



Solution

In $\triangle ABD$ and $\triangle ACD$, you know that $\angle 1 \cong$ $\angle 2$ and $\overline{AD} \cong \overline{AD}$. If you can show that $\overline{BD} \cong \overline{CD}$, you can use the SAS Congruence Postulate.

To prove that $\overline{BD} \cong \overline{CD}$, you can first prove that $\triangle BED \cong$ $\triangle$ D. You are given $\angle 1 \cong \angle 2$ and $\angle 3 \cong \angle 4$. $\overline{ED} \cong \overline{ED}$ by the Reflexive Property and $\angle BDE \cong$ $\angle$ D by the Congruent Supplements Theorem. You can use the AAS Congruence Theorem to prove that $\triangle BED \cong$ $\triangle$ D.

Plan for Proof Use the AAS Congruence Theorem to prove that $\triangle BED \cong$ $\triangle$ D. Then state that $\overline{BD} \cong \overline{CD}$. Use the SAS Congruence Postulate to prove that $\triangle ABD \cong \triangle ACD$.

EXAMPLE 4 Prove a construction

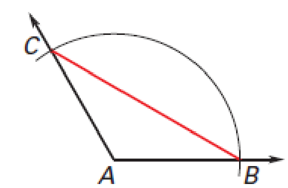
Write a proof to verify that the construction for copying an obtuse angle is valid.

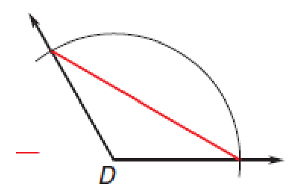
Solution

Add $\overline{BC}$ and $\overline{EF}$ to the diagram. In the construction, $\overline{AB}$, $\overline{D}$, $\overline{A}$, and $\overline{D}$ are determined by the same compass setting, as are $\overline{BC}$ and ____. So, you can assume the following as given statements.

Given $\overline{AB} \cong$ $\overline{D}$, $\overline{AC} \cong$ $\overline{D}$, $\overline{BC} \cong$ ____

Prove $\angle D \cong$ $\angle A$





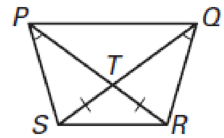
Plan for Proof Show that $\triangle CAB \cong$ $\triangle$ D, so you can conclude that the corresponding parts $\angle D$ and $\angle A$ are congruent.

Statements	Reasons
1. $\overline{AB} \cong$ $\overline{D}$, $\overline{AC} \cong$ $\overline{D}$, $\overline{BC} \cong$ ____	1. Given
2. $\triangle CAB \cong$ $\triangle$ D	2. SSS Congruence Postulate
3. $\angle D \cong$ $\angle A$	3. Corresp. parts of $\cong$ triangles are $\cong$.

4.6 Cont.

Checkpoint **Complete the following exercises.**

1. *Explain* how you can prove that $\overline{PR} \cong \overline{QS}$.



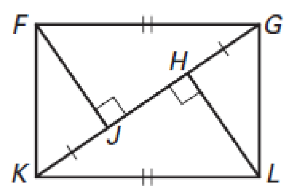
Use the AAS Congruence Theorem to show $\triangle PTS \cong \triangle QTR$. Because corresponding pairs of congruent triangles are congruent, $\overline{PT} \cong \overline{QT}$. Then $\overline{PR} \cong \overline{QS}$ because $\overline{ST} \cong \overline{RT}$.

2. In Example 2, does it matter how far away from point *B* you place a marker at point *D*? *Explain*.

Point *D* should be placed far enough away from point *B* so that it is on land. This allows $\overline{D}$ to be easily measured. However, the method will work regardless of how far *D* is from *B*.

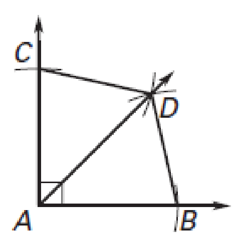
3. **Given** $\overline{GH} \cong \overline{KJ}$, $\overline{FG} \cong \overline{LK}$, $\angle FJG$ and $\angle LHK$ are rt. ∆s.

Prove $\triangle FJK \cong \triangle LHG$



Plan for Proof: Use the HL Congruence Theorem to prove that $\triangle\ JG \cong \triangle\ HK$. Then state that $\overline{\ J} \cong \overline{\ H}$. Then show that $\angle\ JK \cong \angle\ HG$ and use the SAS Congruence Postulate to prove that $\triangle\ JK \cong \triangle\ HG$.

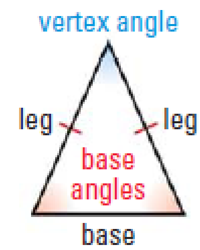
4. Write a paragraph proof to verify that the construction for bisecting a right angle is valid.



You know that $\overline{A\ } \cong \overline{AB}$ and $\overline{BD} \cong \overline{\ D}$ because they are determined by the same compass settings. Also, $\overline{AD} \cong \overline{AD}$ by the Reflexive Property. So, by the SSS Congruence Postulate, $\triangle\ AD \cong \triangle BAD$. Thus, $\angle\ AD \cong \angle BAD$ because corresponding parts of congruent triangles are congruent.

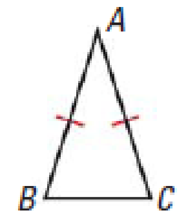
4.7 Use Isosceles and Equilateral Triangles

Obj.: Use theorems about isosceles and equilateral triangles.



Key Vocabulary

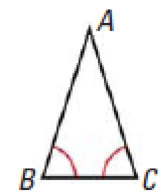
- **Legs -** When an isosceles triangle has exactly **two** congruent **sides**, these two sides are the **legs.**
- **Vertex angle -** The **angle** formed by the **legs** is the **vertex angle**.
- **Base –** The **third** side is the **base** of the isosceles triangle.
- **Base angles -** The **two** angles **adjacent** to the **base** are called **base angles**.



Base Angles Theorem base ∠ Th.

If **two sides** of a triangle are congruent, then the **angles** opposite them are **congruent**.

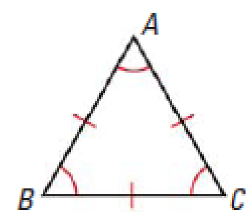
If $\overline{AB} \cong \overline{AC}$, then **$\angle B \cong \angle C$.**



Converse of Base Angles Theorem conv. base ∠ Th.

If **two angles** of a triangle are congruent, then the **sides** opposite them are **congruent**.

If $\angle B \cong \angle C$., then **$\overline{AB} \cong \overline{AC}$** .



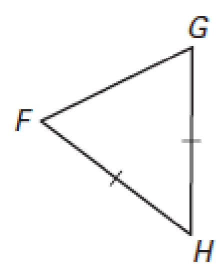
Corollary to the Base Angles Theorem

If a triangle is equilateral, then it is **equiangular**.

Corollary to the Converse of Base Angles Theorem

If a triangle is equiangular, then it is **equilateral**.

EXAMPLE 1 Apply the Base Angles Theorem



In ▲FGH, $\overline{FH} \cong \overline{GH}$. Name two congruent angles.

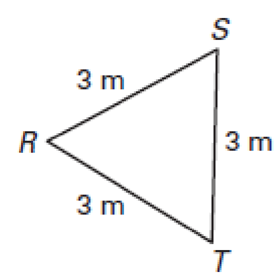
Solution

$\overline{FH} \cong \overline{GH}$, so by the Base Angles Theorem,

$\angle$ F $\cong \angle$ G .

EXAMPLE 2 Find measures in a triangle

Find the measures of $\angle R$, $\angle S$, and $\angle T$.



Solution

The diagram shows that $\triangle RST$ is equilateral . Therefore, by the Corollary to the Base Angles Theorem, $\triangle RST$ is equiangular . So, $m\angle R = m\angle S = m\angle T$.

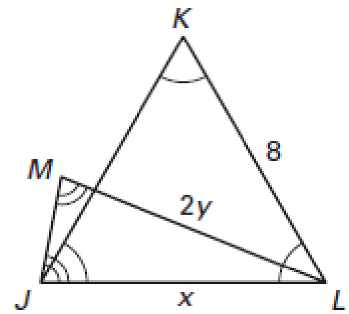
$3(m\angle R) =$ 180° **Triangle Sum Theorem**

$m\angle R =$ 60° **Divide each side by 3.**

The measures of $\angle R$, $\angle S$, and $\angle T$ are all 60° .

EXAMPLE 3 Use isosceles and equilateral triangles

ALGEBRA Find the values of *x* and *y* in the diagram.



Solution

Step 1 **Find** the value of *x*. Because $\triangle JKL$ is equiangular, it is also equilateral and $\overline{KL} \cong \overline{JL}$. Therefore, $x = 8$.

Step 2 Find the value of *y*. Because $\angle JML \cong \angle LJM$, $\overline{LM} \cong \overline{LJ}$, and $\triangle LMJ$ is isosceles. You know that $LJ = 8$.

You cannot use $\angle J$ to refer to $\angle LJM$ because three angles have *J* as their vertex.

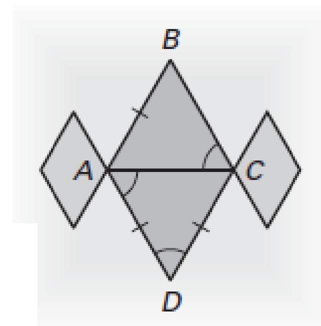
$LM = LJ$	**Definition of congruent segments**
$2y = 8$	**Substitute 2*y* for *LM* and 8 for *LJ*.**
$y = 4$	**Divide each side by 2.**

EXAMPLE 4 Solve a multi-step problem

Quilting The pattern at the right is present in a quilt.

a. Explain why ▲ADC is equilateral.

b. Show that ▲CBA ≅ ▲ADC.



a. By the Base Angles Theorem, $\angle DAC \cong \angle DCA$. So, $\triangle ADC$ is equiangular. By the Corollary to the Converse of Base Angles Theorem, $\triangle ADC$ is equilateral.

b. By the Base Angles Theorem, $\angle ABC \cong \angle ACB$. So, $\triangle CBA \cong \triangle ADC$ by the AAS Congruence Theorem.

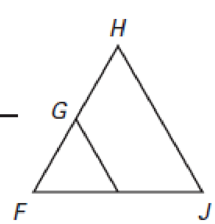
Checkpoint **Complete the following exercises.**

1. Copy and complete the statement: If $\overline{FH} \cong \overline{FJ}$, then $\angle \underline{?} \cong \angle \underline{?}$.

H; *J*

2. Copy and complete the statement: If $\triangle FGK$ is equiangular and $FG = 15$, then $GK = \underline{?}$.

15



Use parts (a) and (b) in Example 4 to show that $m\angle BAD = 120°$.

$\triangle DCA$ is equiangular. So, $m\angle ADC = m\angle DCA = m\angle CAD$.

$3(m\angle CAD) = 180°$	Triangle Sum Theorem
$m\angle CAD = 60°$	Divide each side by 3.

Because $\triangle DCA$ is equiangular and $\triangle CBA \cong \triangle ADC$, you know that $m\angle BAC = 60°$.

$$m\angle BAD = m\angle BAC + m\angle CAD = 60° + 60° = 120°$$