5.1 Midsegment Theorem and Coordinate Proof

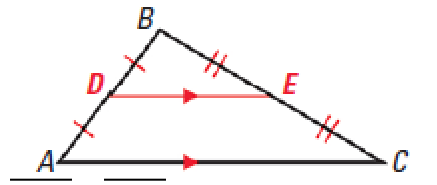
Obj.: <u>Use properties of midsegments and write coordinate proofs.</u>

Key Vocabulary

- **Midsegment of a triangle -** A **midsegment of a triangle** is a **<u>segment</u>** that **<u>connects</u>** the **<u>midpoints</u>** of two sides of the triangle.
- **Coordinate proof –** A **coordinate proof** involves **<u>placing</u>** geometric **<u>figures</u>** in a coordinate **<u>plane</u>**.

Midsegment Theorem

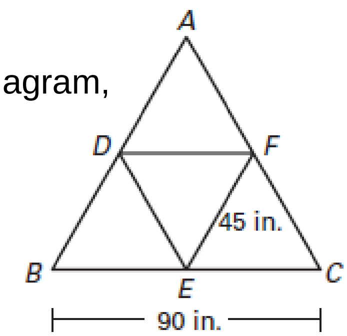
The segment connecting the **<u>midpoints</u>** of two sides of a triangle is **<u>parallel</u>** to the third side and is **<u>half</u>** as long as that side.



$\overline{DE}$ **<u>//</u>** $\overline{AC}$ and DE = **<u>½ AC</u>**

EXAMPLE 1 Use the Midsegment Theorem to find lengths

Windows A large triangular window is segmented as shown. In the diagram, $\overline{DF}$ and $\overline{EF}$ are midsegments of ▲ABC. Find DF and AB.



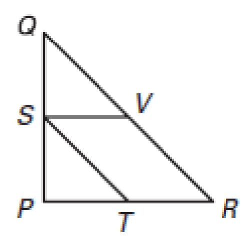
Solution

$DF = \frac{1}{2} \cdot BC = \frac{1}{2}(\underline{90 \text{ in.}}) = \underline{45 \text{ in.}}$

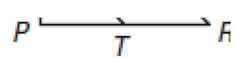
$AB = \underline{2} \cdot FE = \underline{2}(\underline{45 \text{ in.}}) = \underline{90 \text{ in.}}$

EXAMPLE 2 Use the Midsegment Theorem

In the diagdram at the right, QS = SP and PT = TR. Show that $\overline{QR} \parallel \overline{ST}$



Solution



Because $QS = SP$ and $PT = TR$, S is the <u>midpoint</u> of $\overline{QP}$ and T is the <u>midpoint</u> of $\overline{PR}$ by definition. Then $\overline{ST}$ is a <u>midsegment</u> of $\triangle PQR$ by definition and $\overline{QR} \parallel \overline{ST}$ by the <u>Midsegment Theorem</u>.

EXAMPLE 3 Place a figure in a coordinate plane

(5.1 cont.)

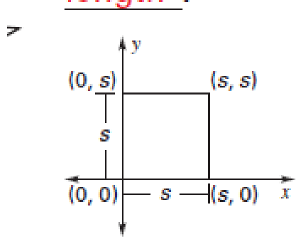
Place each figure in a coordinate plane in a way that is convenient for finding side lengths. Assign coordinates to each vertex.

a. A square **b.** An acute triangle

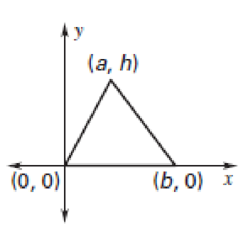
Solution

It is easy to find lengths of horizontal and vertical segments and distances from <u>(0, 0)</u>, so place one vertex at the <u>origin</u> and one or more sides on an <u>axis</u>.

a. Let s represent the <u>side length</u>.



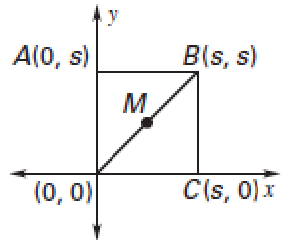
b. You need to use <u>three</u> different variables.



EXAMPLE 4 Apply variable coordinates

In Example 3 part (a), find the length and midpoint of a diagonal of the square.

Solution



Draw a diagonal and its midpoint. Assign letters to the points.

Use the distance formula to find *BD*.

$BD = \sqrt{(s-0)^2 + (s-0)^2}$

$= \sqrt{s^2 + s^2} = \sqrt{2s^2} = s\sqrt{2}$

Use the midpoint formula to find the midpoint *M*.

$M\left(\frac{s+0}{2}, \frac{s+0}{2}\right) = M\left(\frac{s}{2}, \frac{s}{2}\right)$

5.1 Cont.

Checkpoint **Complete the following exercise.**

1. In Example 1, consider $\triangle ADF$. What is the length of the midsegment opposite $\overline{DF}$?

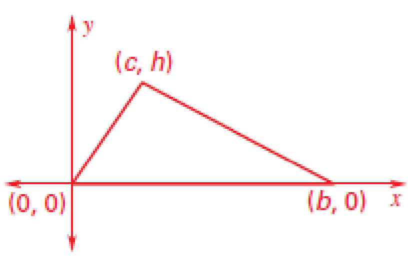
22.5 in.

2. In Example 2, if *V* is the midpoint of $\overline{QR}$, what do you know about $\overline{SV}$?

$\overline{SV}$ is a midsegment of $\triangle PQR$ and $\overline{SV} \parallel \overline{PR}$.

3. Place an obtuse scalene triangle in a coordinate plane that is convenient for finding side lengths. Assign coordinates to each vertex.

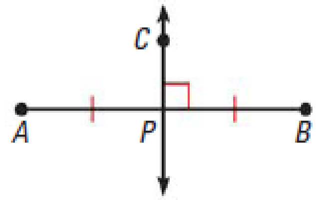
Sample answer:



4. In Example 4, find the length and midpoint of diagonal $\overline{AC}$. What do you notice? *Explain* why this is true for all squares.

5.2 Use Perpendicular Bisectors

Obj.: **Use perpendicular bisectors to solve problems.**



$\overleftrightarrow{CP}$ is a $\perp$ bisector of $\overline{AB}$.

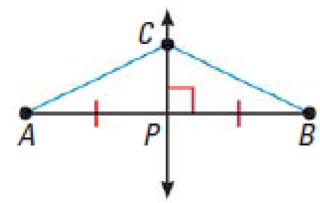
Key Vocabulary

- **Perpendicular bisector -** A segment, ray, line, or plane that is **perpendicular** to a segment at its **midpoint** is called a **perpendicular** **bisector**.
- **Equidistant -** A point is **equidistant** from **two** figures if the point is the ***same distance*** from each figure.
- **Concurrent -** When **three** or more lines, rays, or segments **intersect** in the same **point**, they are called **concurrent** lines, rays, or segments.
- **Point of concurrency -** The **point** of **intersection** of the lines, rays, or segments is called the **point of** **concurrency**.
- **Circumcenter -** The **point** of concurrency of the **three** perpendicular **bisectors** of a triangle is called the **circumcenter** of the triangle.

Perpendicular Bisector Theorem **$\perp$ bis Th.**

In a plane, if a point is on the **perpendicular bisector** of a segment, then it is **equidistant** from the endpoints of the segment.

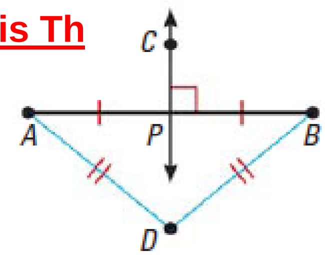
If $\overleftrightarrow{CP}$ is the **$\perp$ bisector** of $\overline{AB}$, then ***CA = CB.***



Converse of the Perpendicular Bisector Theorem **Conv. $\perp$ bis Th**

In a plane, if a **point** is equidistant from the **endpoints** of a segment, then it is **on** the perpendicular **bisector** of the segment.

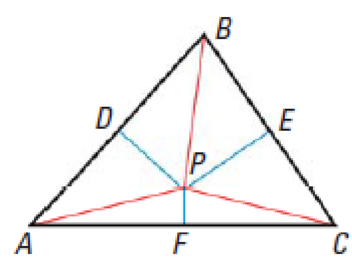
If $DA = DB$, then ***D* lies on** the $\perp$ bisector of $\overline{AB}$.

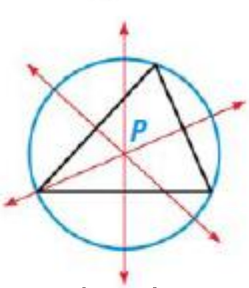


Concurrency of Perpendicular Bisectors of a Triangle

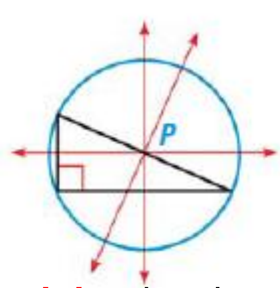
The perpendicular bisectors of a triangle intersect at a point that is **equidistant** from the **vertices** of the triangle.

If ***PD***, ***PE***, and ***PF*** are perpendicular **bisectors**, then ***PA = PB = PC***.

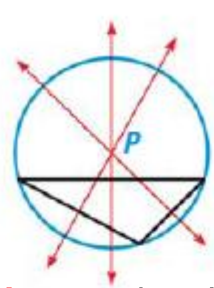




Acute triangle
P is **inside** triangle.



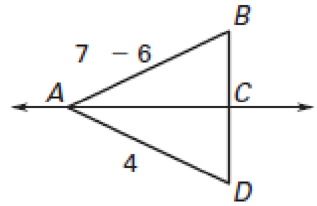
Right triangle
P is **on** triangle



Obtuse triangle
P is **outside** triangle.

EXAMPLE 1 Use the Perpendicular Bisector Theorem

ALGEBRA $\overleftrightarrow{AC}$ is the perpendicular bisector of $\overline{BD}$. Find *AD*.



Solution

$AD = \underline{AB}$		**Perpendicular Bisector Theorem**
$\underline{4x} = \underline{7x - 6}$		**Substitute.**
$x = \underline{2}$		**Solve for *x*.**

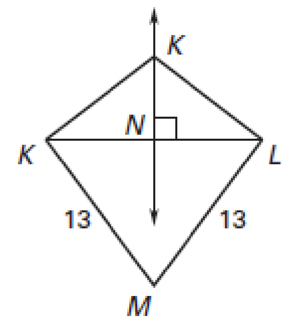
$AD = \underline{4x} = \underline{4(2)} = \underline{8}$.

EXAMPLE 2 Use perpendicular bisectors

In the diagram, $\overleftrightarrow{KN}$ is the perpendicular bisector of $\overline{JL}$.

a. What segment lengths in the diagram are equal?

b. Is *M* on $\overleftrightarrow{KN}$?



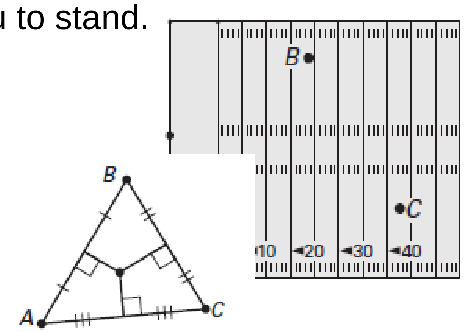
Solution

a. $\overleftrightarrow{KN}$ bisects $\overline{JL}$, so $\underline{NJ} = \underline{NL}$. Because *K* is on the perpendicular bisector of $\overline{JL}$, $\underline{KJ} = \underline{KL}$ by Theorem 5.2. The diagram shows that $\underline{MJ} = \underline{ML} = 13$.

b. Because $MJ = ML$, *M* is <u>equidistant</u> from *J* and *L*. So, by the <u>Converse of the Perpendicular Bisector Theorem</u>, *M* is on the perpendicular bisector of $\overline{JL}$, which is $\overleftrightarrow{KN}$.

EXAMPLE 3 Use the concurrency of perpendicular bisectors

Football Three friends are playing catch. You want to join and position yourself so that you are the same distance from your friends. Find a location for you to stand.



Solution Theorem 5.4 shows you that you can find a point equidistant from three points by using the <u>perpendicular bisectors</u> of the triangle formed by those points. Copy the positions of points *A*, *B*, and *C* and connect those points to draw $\triangle ABC$. Then use a ruler and a protractor to draw the three <u>perpendicular bisectors</u> of $\triangle ABC$. The point of concurrency *D* is a location for you to stand.

ACTIVITY FOLD THE PERPENDICULAR BISECTORS OF A TRIANGLE

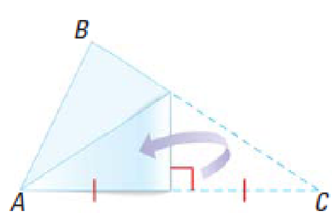
QUESTION Where do the perpendicular bisectors of a triangle meet?

Materials:
- paper
- scissors
- ruler

Follow the steps below and answer the questions about perpendicular bisectors of triangles.

STEP 1 Cut four large acute scalene triangles out of paper. Make each one different.

STEP 2 Choose one triangle. Fold it to form the perpendicular bisectors of the sides. Do the three bisectors intersect at the same point?



STEP 3 Repeat the process for the other three triangles. Make a conjecture about the perpendicular bisectors of a triangle.

STEP 4 Choose one triangle. Label the vertices *A*, *B*, and *C*. Label the point of intersection of the perpendicular bisectors as *P*. Measure $\overline{AP}$, $\overline{BP}$, and $\overline{CP}$. What do you observe?

5.3 Use Angle Bisectors of Triangles

Obj.: Use angle bisectors to find distance relationships.

Key Vocabulary

• **Incenter -** The point of concurrency of the three angle bisectors of a triangle is called the **incenter** of the triangle. The incenter always lies inside the triangle.

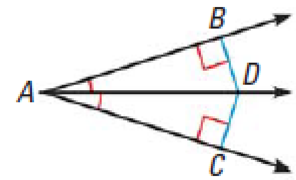
• **Angle bisector -** An *angle bisector* is a ray that divides an angle into two congruent adjacent angles.

• **Distance from a point to a line –** The *distance from a point to a line* is the length of the perpendicular segment from the point to the line.

Angle Bisector Theorem ∠ bis Th

If a point is on the bisector of an angle, then it is equidistant from the two sides of the angle.

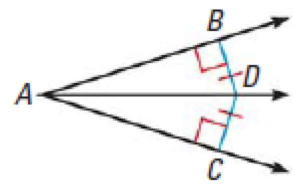
If $\overrightarrow{AD}$ bisects $\angle BAC$ and $\overline{DB} \perp \overline{AB}$ and $\overline{DC} \perp \overline{AC}$, then DB = DC.



Converse of the Angle Bisector Theorem Conv. ∠ bis Th

If a point is in the interior of an angle and is equidistant from the sides of the angle, then it lies on the bisector of the angle.

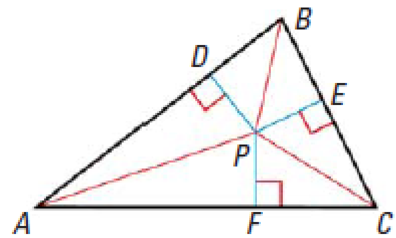
If $\overline{DB} \perp \overrightarrow{AB}$ and $\overline{DC} \perp \overrightarrow{AC}$ and DB = DC, then $\overrightarrow{AD}$ bisects ∠BAC.



Concurrency of Angle Bisectors of a Triangle

The angle bisectors of a triangle intersect at a point that is equidistant from the sides of the triangle.

If $\overline{AP}$, $\overline{BP}$, and $\overline{CP}$ are angle bisectors of ▲ABC, then PD = PE = PF.

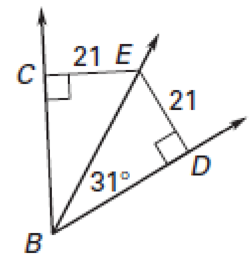


EXAMPLE 1 Use the Angle Bisector Theorems

Find the measure of ∠CBE.

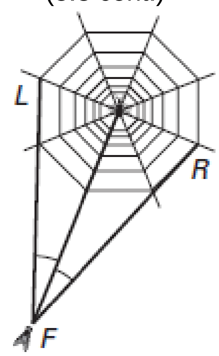
Solution

Because $\overline{EC} \perp \overrightarrow{BC}$ and $\overline{ED} \perp \overrightarrow{BD}$ and $EC = ED = 21$, $\overrightarrow{BE}$ bisects $\angle CBD$ by the Converse of the Angle Bisector Theorem. So, $m\angle CBE = m\angle DBE = 31°$.



EXAMPLE 2 Solve a real-world problem

Web A spider's position on its web relative to an approaching fly and the opposite sides of the web forms congruent angles, as shown. Will the spider have to move farther to reach a fly toward the right edge or the left edge?



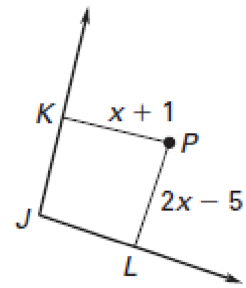
Solution

The congruent angles tell you that the spider is on the bisector of $\angle LFR$. By the Angle Bisector Theorem, the spider is equidistant from $\overrightarrow{FL}$ and $\overrightarrow{FR}$.

So, the spider must move the same distance to reach each edge.

EXAMPLE 3 Use algebra to solve a problem

ALGEBRA For what value of x does P lie on the bisector of $\angle J$?



Solution

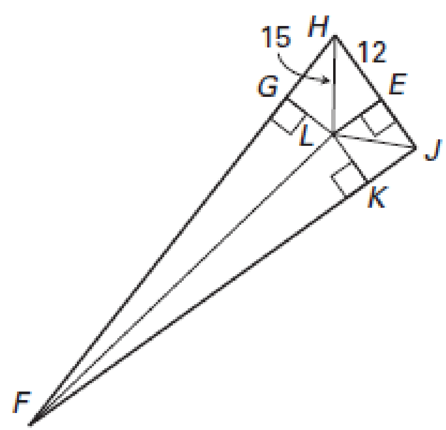
From the Converse of the Angle Bisector Theorem, you know that P lies on the bisector of $\angle J$ if P is equidistant from the sides of $\angle J$, so when $PK = PL$.

$PK = PL$		**Set segment lengths equal.**
$x + 1 = 2x - 5$		**Substitute expressions for segment lengths.**
$6 = x$		**Solve for x.**

Point P lies on the bisector of $\angle J$ when $x = 6$.

EXAMPLE 4 Use the concurrency of angle bisectors

In the diagram, L is the incenter of ▲FHJ. Find LK.



Solution

By the Concurrency of Angle Bisectors of a Triangle Theorem, the incenter L is equidistant from the sides of $\triangle FHJ$. So, to find LK, you can find LI in $\triangle LHI$. Use the Pythagorean Theorem.

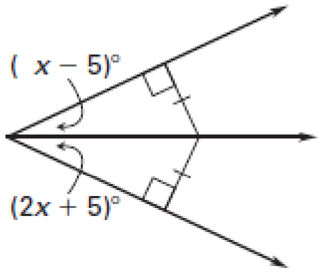
$c^2 = a^2 + b^2$	**Pythagorean Theorem**
$15^2 = LI^2 + 12^2$	**Substitute known values.**
$81 = LI^2$	**Simplify.**
$9 = LI$	**Take the positive square root of each side.**

Because $LI = LK$, $LK = 9$.

5.3 Cont.

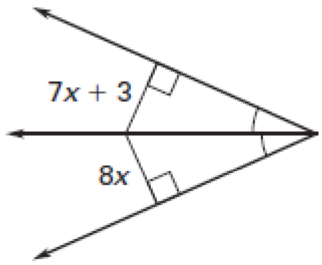
Checkpoint In Exercises 1 and 2, find the value of x.

1.



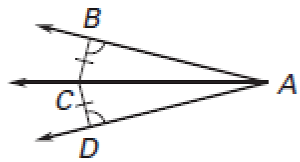
$x = 10$

2.



$x = 3$

3. Do you have enough information to conclude that $\overrightarrow{AC}$ bisects $\angle DAB$? *Explain.*



No, you must know that $m\angle ABC = m\angle ADC = 90°$ before you can conclude that $\overrightarrow{AC}$ bisects $\angle DAB$.

4. In Example 4, suppose you are not given HL or HI, but you are given that $JL = 25$ and $JI = 20$. Find LK.

$LK = 15$

5.4 Use Medians and Altitudes

Obj.: Use medians and altitudes of triangles.

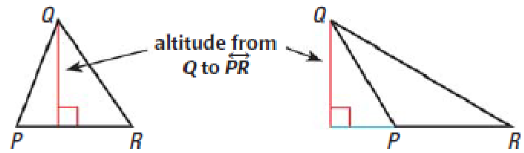
Key Vocabulary

• **Median of a triangle -** A **median of a triangle** is a segment **from** a **vertex** to the **midpoint** of the opposite **side**.

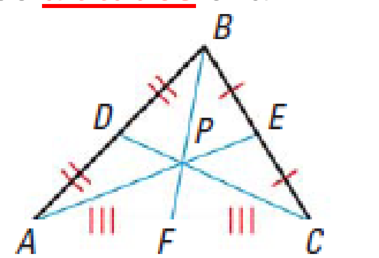
Three medians meet at the centroid.

• **Centroid -** The **point** of concurrency, called the **centroid**, is **inside** the triangle.

• **Altitude of a triangle -** An **altitude of a triangle** is the **perpendicular** segment from a **vertex** to the opposite **side** or to the line that **contains** the opposite side.

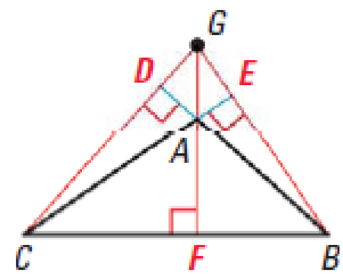


• **Orthocenter -** The **point** at which the lines containing the three **altitudes** of a triangle **intersect** is called the **orthocenter** of the triangle.



Concurrency of Medians of a Triangle

The **medians** of a triangle intersect at a point that is **two thirds** of the distance from each **vertex** to the midpoint of the opposite **side**. The medians of ▲ABC meet at P and **AP** = **⅔ AE**, **BP** = **⅔ BF**, and **CP** = **⅔ CD**.



Concurrency of Altitudes of a Triangle

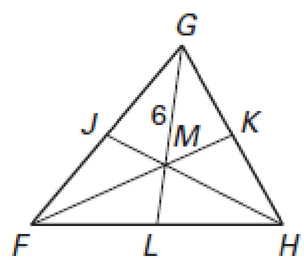
The lines containing the **altitudes** of a triangle are concurrent.

The lines containing $\overline{AF}$, $\overline{BE}$, and $\overline{CD}$ meet at **G**.

EXAMPLE 1 Use the centroid of a triangle

In ▲FGH, *M* is the centroid and *GM* = 6. Find *ML* and *GL*.

Solution



$GM = \frac{2}{3}GL$ **Concurrency of Medians of a Triangle Theorem**

$6 = \frac{2}{3}GL$ **Substitute 6 for *GM*.**

$9 = GL$ **Multiply each side by the reciprocal, $\frac{3}{2}$.**

Then $ML = GL - GM = 9 - 6 = 3$.

So, $ML = 3$ and $GL = 9$.

EXAMPLE 2 Use the centroid of a triangle

The vertices of ▲JKL are $J(1, 2)$, $K(4, 6)$, and $L(7, 4)$. Find the coordinates of the centroid P of ▲JKL.

Sketch ▲JKL. Then use the Midpoint Formula to find the midpoint M of $\overline{JL}$ and sketch median $\overline{KM}$.

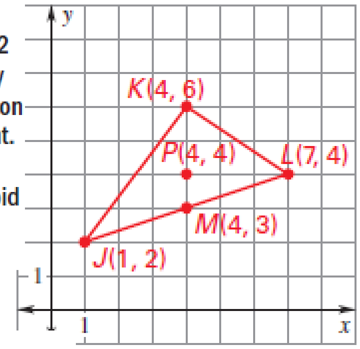
$$M\left(\frac{\boxed{1+7}}{2}, \frac{\boxed{2+4}}{2}\right) = \underline{M(4, 3)}$$

The centroid is _two thirds_ of the distance from each vertex to the midpoint of the opposite side.

The distance from vertex K to point M is $6 - \underline{3} = \underline{3}$ units. So, the centroid is $\frac{2}{3}(\underline{3}) = \underline{2}$ units down from K on $\overline{KM}$.

The coordinates of the centroid P are $(4, 6 - \underline{2})$, or $(\underline{4, 4})$.

Median $\overline{KM}$ was used in Example 2 because it is easy to find distances on a vertical segment. You can check by finding the centroid using a different median.

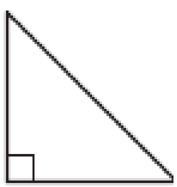


EXAMPLE 3 Find the orthocenter

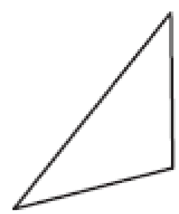
Find the orthocenter P in the triangle.

Solution

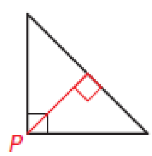
a.



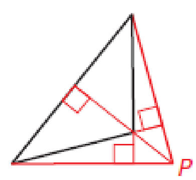
b.



a.



b.



Notice that in a right triangle the legs are also altitudes. The altitudes of the obtuse triangle are extended to find the orthocenter.

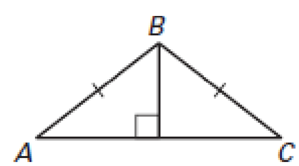
EXAMPLE 4 Prove a property of isosceles triangles

Prove that the altitude to the base of an isosceles triangle is median.

Solution

Given: ▲ABC is isosceles, with base $\overline{AC}$.
$\overline{BD}$ is the altitude to base $\overline{AC}$.

Prove: $\overline{BD}$ is a median of ▲ABC



Proof Legs $\overline{AB}$ and $\overline{CD}$ of $\triangle ABC$ are congruent. _$\angle ADB$_ and _$\angle CDB$_ are congruent right angles because $\overline{BD}$ is the _altitude_ to $\overline{AC}$. Also, $\overline{BD} \cong \overline{BD}$. Therefore, $\triangle ADB \cong \triangle CDB$ by the _HL Congruence Theorem_.

$\overline{AD} \cong$ _$\overline{CD}$_ because corresponding parts of congruent triangles are congruent. So, D is the _midpoint_ of $\overline{AC}$ by definition. Therefore, $\overline{BD}$ intersects $\overline{AC}$ at its _midpoint_, and $\overline{BD}$ is a median of $\triangle ABC$.

5.4 Cont.

Checkpoint **Complete the following exercise.**

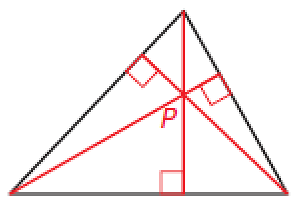
1. In Example 1, suppose $FM = 10$. Find MK and FK.

   $MK = 5$, $FK = 15$

2. In Example 2, where do you need to move point K so that the centroid is $P(4, 5)$?

   Point K should be moved to $(4, 9)$.

3. Find the orthocenter P in the triangle.

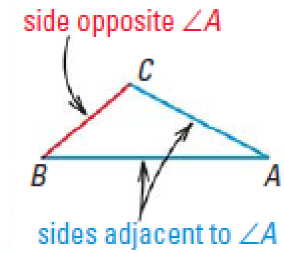


4. Prove that the altitude $\overline{BD}$ in Example 4 is also an angle bisector.

   Proof Legs $\overline{AB}$ and $\overline{CB}$ of $\triangle ABC$ are congruent. $\angle ADB$ and $\angle CDB$ are congruent right angles because $\overline{BD}$ is the altitude to $\overline{AC}$. Also, $\overline{BD} \cong \overline{BD}$. Therefore, $\triangle ADB \cong \triangle CDB$ by the HL Congruence Theorem. $\angle ABD \cong \angle CBD$ because corresponding parts of congruent triangles are congruent. Therefore, $\overline{BD}$ bisects $\angle ABC$, and $\overline{BD}$ is an angle bisector.

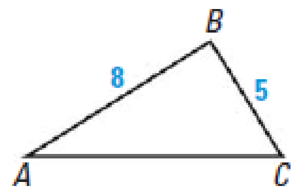
5.5 Use Inequalities in a Triangle

Obj.: Find possible side lengths of a triangle.



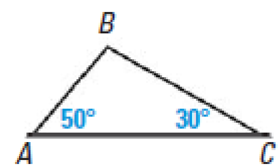
Key Vocabulary
• **Side opposite -** The side not adjacent to the angle is *opposite* the angle.
• **Inequality -** A mathematical sentence built from expressions using one or more of the symbols **<, >, ≤, or ≥.**

▲ side-∠ inequal. Th.
If one side of a triangle is longer than another side, then the angle opposite the longer side is larger than the angle opposite the **shorter** side.



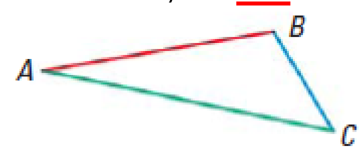
AB > BC, so m∠C > m∠ A.

▲ ∠-side inequal. Th.
If one angle of a triangle is larger than another angle, then the side opposite the larger angle is longer than the side opposite the smaller angle.



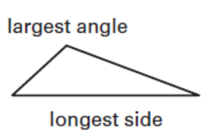
m∠A > m∠C, so BC > AB.

Triangle Inequality Theorem ▲ sum th.
The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

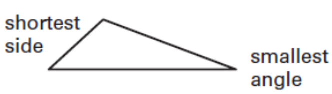


AB + **BC** > **AC**   **AC** + **BC** > **AB**   **AB** + **AC** > **BC**

EXAMPLE 1 Relate side length and angle measure
Mark the largest angle, longest side, smallest angle, and shortest side of the triangle shown. What do you notice?
Solution



The longest side and largest angle are opposite each other.

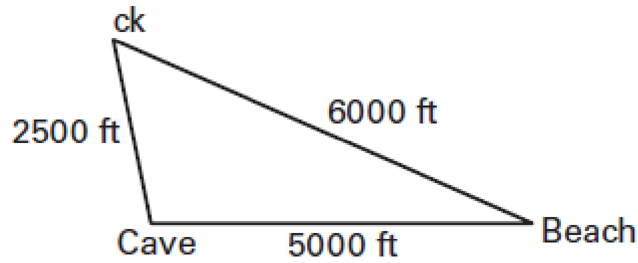


The shortest side and smallest angle are opposite each other.

EXAMPLE 2 Find angle measures

Boating A long-tailed boat leaves a dock and travels 2500 feet to a cave, 5000 feet to a beach, then 6000 feet back to the dock as shown below. One of the angles in the path is about 55^{O} and one is about 24^{O}. What is the angle measure of the path made at the cave?

Solution



The cave is opposite the longest side so, by Theorem 5.10, the cave angle is the largest angle.

The angle measures sum to 180°, so the third angle measure is $180° - (55° + 24°)$ = $101°$.

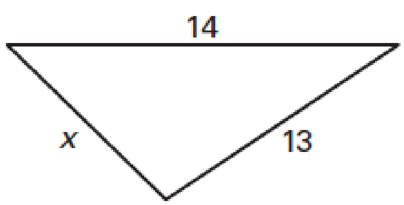
The angle measure made at the cave is $101°$.

EXAMPLE 3 Find possible side lengths

ALGEBRA A triangle has one side of length 14 and another of length 10. Describe the possible lengths of the third side.

Solution

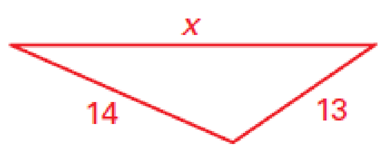
Small values of x



$x + 10 > 14$

$x > 4$

Large values of x



$10 + 14 > x$

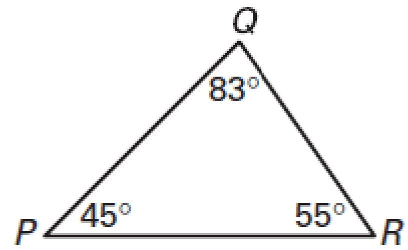
$24 > x$, or $x < 24$

The length of the third side must be greater than 4 and less than 24.

5.5 Cont.

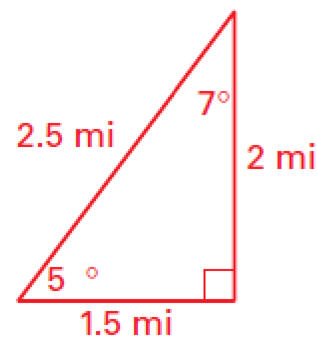
Checkpoint **Complete the following exercises.**

1. List the sides of $\triangle PQR$ in order from shortest to longest.



$\overline{QR}$, $\overline{PQ}$, $\overline{PR}$

2. Another boat makes a trip whose path has sides of 1.5 miles, 2 miles, and 2.5 miles long and angles of 90°, about 53°, and about 37°. Sketch and label a diagram with the shortest side on the bottom and the right angle at the right.



3. A triangle has one side of 23 meters and another of 17 meters. *Describe* the possible lengths of the third side.

The length of the third side must be greater than 6 meters and less than 40 meters.

5.6 Inequalities in Two Triangles and Indirect Proof

Obj.: Use inequalities to make comparisons in two triangles.

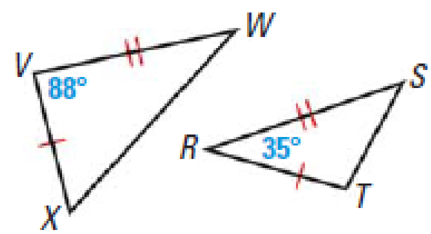
Key Vocabulary

• **Indirect proof -** In an **indirect proof**, you start by making the **temporary** assumption that the desired **conclusion** is **false**. By then showing that this assumption leads to a logical **impossibility**, you prove the original statement **true** *by **contradiction**.*

• **Included angle -** The **included angle** is the **angle** made by two **sides** of a polygon.

Hinge Theorem Hinge th

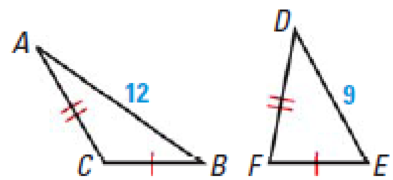
If **two** sides of one triangle are **congruent** to two **sides** of another **triangle**, and the **included** angle of the first is **larger** than the included **angle** of the second, then the **third** side of the first is **longer** than the third **side** of the **second**.



WX > ST

Converse of the Hinge Theorem Conv Hinge th

If two **sides** of one **triangle** are congruent to **two** sides of **another** triangle, and the **third** side of the **first** is longer than the third side of the second, then the **included** angle of the **first** is larger than the included angle of the **second**.



$m\angle C > m\angle F$

KEY CONCEPT ◆◆**How to Write an Indirect Proof** ◆◆

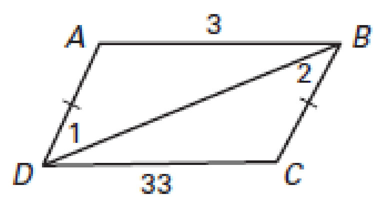
STEP 1 Identify the statement you want to prove. Assume temporarily that this **statement** is false by assuming that its **opposite** is **true.**

STEP 2 **Reason** logically until you **reach** a **contradiction**.

STEP 3 Point out that the desired **conclusion** must be **true** because the contradiction proves the **temporary** assumption **false**.

EXAMPLE 1 Use the Converse of the Hinge Theorem

Given that $\overline{AD} \cong \overline{BC}$, how does $\angle 1$ compare to $\angle 2$?



Solution

You are given that $\overline{AD} \cong \overline{BC}$ and you know that $\overline{BD} \cong \overline{BD}$ by the Reflexive Property. Because 34 > 33, AB > CD. So, two sides of $\triangle ADB$ are congruent to two sides of $\triangle CBD$ and the third side in $\triangle ADB$ is longer.

By the Converse of the Hinge Theorem, $m\angle 1 > m\angle 2$.

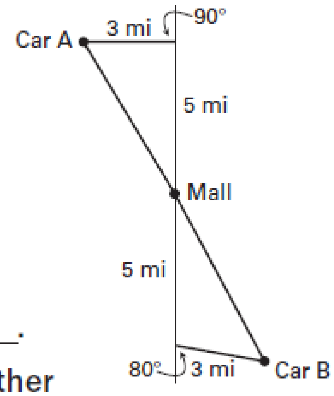
EXAMPLE 2 Solve a multi-step problem

(5.6 cont.)

Travel Car A leaves a mall, heads due north for 5 mi and then turns due west for 3 mi. Car B leaves the same mall, heads due south for 5 mi and then turns 80° toward east for 3 mi. Which car is farther from the mall? Explain your reasoning.

Solution

Draw a diagram. The distance driven and the distance back to the mall form two triangles, with congruent 5 mile sides and congruent 3 mile sides. Add the third side to the diagram.



Use linear pairs to find the included angles of 90° and 100°.

Because 100° > 90°, Car B is farther from the mall than Car A by the Hinge Theorem.

EXAMPLE 3 Write an indirect proof

Write an indirect proof that an odd number is not divisible by 6.

GIVEN: x is an odd number.

PROVE: x is not divisible by 6.

Solution

Step 1 Assume temporarily that x is divisible by 6. This means that $\frac{x}{6} = n$ for some whole number n. So, multiplying both sides by 6 gives $x = 6n$.

Step 2 If x is odd, then, by definition, x cannot be divided evenly by 2. However, $x = 6n$ so $\frac{x}{2} = \frac{6n}{2} = 3n$. We know that $3n$ is a whole number because n is a whole number, so x can be divided evenly by 2. This contradicts the given statement that x is odd.

You have reached a contradiction when you have two statements that cannot both be true at the same time.

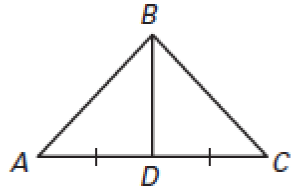
Step 3 Therefore, the assumption that x is divisible by 6 is false, which proves that x is not divisible by 6.

5.6 Cont. (Write these on your paper)

Checkpoint **Complete the following exercises.**

1. If $m\angle ADB > m\angle CDB$ which is longer, $\overline{AB}$ or $\overline{CB}$?

$\overline{AB}$ is longer.



2. In Example 2, car C leaves the mall and goes 5 miles due west, then turns 85° toward south for 3 miles. Write the cars in order from the car closest to the mall to the car farthest from the mall.

car A, car C, car B

3. Suppose you wanted to prove the statement "If $x + y \neq 5$ and $y = 2$, then $x \neq 3$." What temporary assumption could you make to prove the conclusion indirectly?

You can temporarily assume that $x = 3$.