1.1 Identify Points, Lines, and Planes

Objective: Name and sketch geometric figures.

Key Vocabulary
• Undefined terms - These words do not have formal definitions, but there is agreement about what they mean.
  - Point - A point has no dimension. It is represented by a dot.
  - Line - A line has one dimension. It is represented by a line with two arrowheads, but it extends without end. Through any two points, there is exactly one line. You can use any two points on a line to name it. Line AB (written as $\overline{AB}$) and points A and B are used here to define the terms below.
  - Plane - A plane has two dimensions. It is represented by a shape that looks like a floor or a wall, but it extends without end. Through any three points not on the same line, there is exactly one plane. You can use three points that are not all on the same line to name a plane.
  - Collinear points - Collinear points are points that lie on the same line.
  - Coplanar points - Coplanar points are points that lie in the same plane.
• Defined terms - In geometry, terms that can be described using known words such as point or line are called defined terms.
  - Line segment, Endpoints - The line segment $\overline{AB}$, or segment $\overline{AB}$, (written as $\overline{AB}$) consists of the endpoints A and B and all points on $\overline{AB}$ that are between A and B. Note that $\overline{AB}$ can also be named $\overline{BA}$.
  - Ray - The ray $\overrightarrow{AB}$ (written as $\overrightarrow{AB}$) consists of the endpoint A and all points on $\overrightarrow{AB}$ that lie on the same side of A as B. Note that $\overrightarrow{AB}$ and $\overrightarrow{BA}$ are different rays.
  - Opposite rays - If point C lies on $\overrightarrow{AB}$ between A and B, then $\overrightarrow{CA}$ and $\overrightarrow{CB}$ are opposite rays.
  - Intersection - The intersection of the figures is the set of points the figures have in common.

The intersection of two different lines is a point. The intersection of two different planes is a line.

EXAMPLE 1 Name points, lines, and planes
a. Give two other names for $\overline{LN}$ and for plane Z.
b. Name three points that are collinear. Name four points that are coplanar.

Solution:

a. Other names for $\overline{LN}$ are $\overline{LM}$ and line b. Other names for plane Z are plane $\overline{LMP}$ and $\overline{LNP}$.
b. Points L, M, and N lie on the same line, so they are collinear. Points L, M, N, and P lie on the same plane, so they are coplanar.
EXAMPLE 2 Name segments, rays, and opposite rays

a. Give another name for \( \overline{VX} \).

b. Name all rays with endpoint \( W \). Which of these rays are opposite rays?

Solution

a. Another name for \( \overline{VX} \) is \( \overline{XV} \).

b. The rays with endpoint \( W \) are \( \overline{WV}, \overline{WY}, \overline{WX}, \) and \( \overline{WZ} \).

The opposite rays with endpoint \( W \) are \( \overline{WV} \) and \( \overline{WX} \), and \( \overline{WY} \) and \( \overline{WZ} \).

EXAMPLE 3 Sketch intersections of lines and planes

a. Sketch a plane and a line that is in the plane.

b. Sketch a plane and a line that does not intersect the plane.

c. Sketch a plane and a line that intersects the plane at a point.

Solution

a. 

b. 

c. 

EXAMPLE 4 Sketch intersections of planes Sketch two planes that intersect in a line.

Solution

STEP 1 Draw a vertical plane. Shade the plane.

STEP 2 Draw a second plane that is horizontal. Shade this plane a different color. Use dashed lines to show where one plane is hidden.

STEP 3 Draw the line of intersection.
1.1 Cont.

**Checkpoint** Use the diagram in Example 1.

1. Give two other names for $\overrightarrow{MQ}$. Name a point that is not coplanar with points $L$, $N$, and $P$.
   $\overrightarrow{QM}$ and line $a$; point $Q$.

**Checkpoint** Use the diagram in Example 2.

2. Give another name for $\overrightarrow{YW}$.
   $\overrightarrow{WY}$

3. Are $\overrightarrow{VX}$ and $\overrightarrow{XV}$ the same ray? Are $\overrightarrow{VW}$ and $\overrightarrow{VX}$ the same ray? Explain.
   No, the rays do not have the same endpoint; Yes, the rays have a common endpoint, are collinear, and consist of the same points.

**Checkpoint** Complete the following exercises.

4. Sketch two different lines that intersect a plane at different points.

5. Name the intersection of $\overrightarrow{MX}$ and line $a$.
   point

6. Name the intersection of plane $C$ and plane $D$.
   line $a$
2.4 Use Postulates and Diagrams

Obj.: **Use postulates involving points, lines, and planes.**

Key Vocabulary
- **Line perpendicular to a plane** - A line is a **line perpendicular to a plane** if and only if the line intersects the plane in a **point** and is **perpendicular** to every line in the **plane** that intersects it at that point.
- **Postulate** - In geometry, **rules** that are accepted **without proof** are called **postulates** or **axioms**.

**POSTULATES**
**Point, Line, and Plane Postulates**
- **POSTULATE 5** - Through any **two** points there exists exactly one **line**.
- **POSTULATE 6** - A **line** contains at least two **points**.
- **POSTULATE 7** - If **two lines** intersect, then their intersection is exactly one **point**.
- **POSTULATE 8** - Through any **three** noncollinear **points** there exists exactly one **plane**.
- **POSTULATE 9** - A **plane** contains at least three noncollinear **points**.
- **POSTULATE 10** - If **two points** lie in a **plane**, then the **line** containing them lies in the **plane**.
- **POSTULATE 11** - If **two planes** intersect, then their intersection is a **line**.

**CONCEPT SUMMARY** - Interpreting a Diagram
When you interpret a diagram, you can only **assume** information about size or measure if it is **marked**.

**YOU CAN ASSUME**
- All points shown are **coplanar**.
- \(\angle AHB\) and \(\angle BHD\) are a **linear pair**.
- \(\angle AHF\) and \(\angle BHD\) are **vertical** angles.
- \(A, H, J,\) and \(D\) are collinear.
- \(AD\) and \(BF\) intersect at \(H\).

**YOU CANNOT ASSUME**
- \(G, F,\) and \(E\) are **collinear**.
- \(BF\) and \(CE\) **intersect**.
- \(BF\) and \(CE\) **do not** intersect.
- \(\angle BHA \neq \angle CJA\)
- \(AD \perp BF\) or \(m\angle AHB = 90^\circ\)

**EXAMPLE 1** Identify a postulate illustrated by a diagram
State the postulate illustrated by the diagram.

If \(\triangle ABC\) then \(\triangle DEF\)

**Solution**
- **Postulate 8** Through any three noncollinear points there exists exactly one **plane**.
EXAMPLE 2 Identify postulates from a diagram
Use the diagram to write examples of Postulates 9 and 11.
Solution:

Postulate 9  Plane \( Q \) contains at least three noncollinear points, \( W, V, \) and \( Y \).

Postulate 11  The intersection of plane \( P \) and plane \( Q \) is \( \text{line} \ b \).

EXAMPLE 3 Use given information to sketch a diagram
Sketch a diagram showing \( \overline{RS} \) perpendicular to \( TV \), intersecting at point \( X \).
Solution:

Step 1  Draw \( \overline{RS} \) and label points \( R \) and \( S \).

Step 2  Draw a point \( X \) \underline{between} \( R \) and \( S \).

Step 3  Draw \( \overrightarrow{TV} \) through \( X \) so that it is \underline{perpendicular} to \( \overline{RS} \).

EXAMPLE 4 Interpret a diagram in three dimensions
Which of the following statements cannot be assumed from the diagram?

- \( E, D, \) and \( C \) are collinear.
- The intersection of \( \overrightarrow{BD} \) and \( \overrightarrow{EC} \) is \( D \).
- \( \overrightarrow{BD} \perp \overrightarrow{EC} \)
- \( \overrightarrow{EC} \perp \text{plane} \ G \)

Solution: With no right angles marked, you cannot assume that \( \overrightarrow{BD} \perp \overrightarrow{EC} \) or \( \overrightarrow{EC} \perp \text{plane} \ G \).
2.4 Cont.

**Checkpoint** Use the diagram in Example 2 to complete the following exercises.

1. Which postulate allows you to say that the intersection of line \(a\) and line \(b\) is a point?
   
   Postulate 7

2. Write examples of Postulates 5 and 6.
   
   Line \(a\) passes through \(X\) and \(Y\); line \(a\) contains points \(X\) and \(Y\).

**Checkpoint** Complete the following exercises.

3. In Example 3, if the given information indicated that \(RX\) and \(XS\) are congruent, how would the diagram change?
   
   Point \(X\) would be drawn as the midpoint of \(RS\) and the congruent segments would be marked.

4. In the diagram for Example 4, can you assume that \(BD\) is the intersection of plane \(F\) and plane \(G\)?
   
   Yes
1.2 Use Segments and Congruence

Obj.: Use segment postulates to identify congruent segments.

Key Vocabulary

• Postulate, axiom - In Geometry, a rule that is accepted without proof is called a postulate or axiom.
• Coordinate - The points on a line can be matched one to one with the real numbers. The real number that corresponds to a point is the coordinate of the point.
• Distance - The distance between points A and B, written as AB, is the absolute value of the difference of the coordinates of A and B.
• Between- When three points are collinear, you can say that one point is between the other two.
• Congruent segments - Line segments that have the same length are called congruent segments.

POSTULATE 1 Ruler Postulate

POSTULATE 2 Segment Addition

If B is between A and C, then \( AB + BC = AC \).
If \( AB + BC = AC \), then B is between A and C.

EXAMPLE 1 Apply the Ruler Postulate

Measure the length of CD to the nearest tenth of a centimeter.

Solution

Lengths are equal. \( AB = CD \) \( \iff \) “is equal to”

EXAMPLE 2 Apply the Segment Addition Postulate

MAPS The cities shown on the map lie approximately in a straight line. Use the given distances to find the distance from Lubbock, Texas, to St. Louis, Missouri.

Solution

Because Tulsa, Oklahoma, lies between Lubbock and St. Louis, you can apply the Segment Addition Postulate.

\( LS = LT + TS = 380 + 360 = 740 \)

The distance from Lubbock to St. Louis is about 740 miles.
EXAMPLE 3 Find a length
Use the diagram to find $KL$.

Solution
Use the Segment Addition Postulate to write an equation. Then solve the equation to find $KL$.

\[
\frac{JL}{38} = \frac{JK}{15} + KL \quad \text{Segment Addition Postulate}
\]

\[
\frac{23}{23} = \frac{15}{15} + KL \\
\frac{15}{15} = KL
\]

EXAMPLE 4 Compare segments for congruence
Plot F(4, 5), G(−1, 5), H(3, 3), and J(3, −2) in a coordinate plane. Then determine whether FG and HJ are congruent.

Solution: Horizontal segment: Subtract the $x$-coordinates of the endpoints.

\[
FG = |4 - (-1)| = 5
\]

Vertical segment: Subtract the $y$-coordinates of the endpoints.

\[
HJ = |3 - (-2)| = 5
\]

$FG$ and $HJ$ have the same length.

Checkpoint Complete the following exercises.

1. Find the length of $\overline{AB}$ to the nearest $\frac{1}{8}$ inch.

\[
\frac{17}{8} \text{ inches}
\]

2. Find $QS$ and $PQ$.

\[
61; 24
\]

3. Consider the points $A(-2, -1), B(4, -1), C(3, 0),$ and $D(3, 5)$. Are $\overline{AB}$ and $\overline{CD}$ congruent?

No
1.3 Use Midpoint and Distance Formulas

Obj.: Find lengths of segments in the coordinate plane.

Key Vocabulary
- **Midpoint** - The midpoint of a segment is the point that divides the segment into two congruent segments.
- **Segment bisector** - A segment bisector is a point, ray, line, line segment, or plane that intersects the segment at its midpoint.

The Midpoint Formula

**KEY CONCEPT**
The coordinates of the midpoint of a segment are the averages of the x-coordinates and of the y-coordinates of the endpoints.

If \(A(x_1, y_1)\) and \(B(x_2, y_2)\) are points in a coordinate plane, then the midpoint \(M\) of \(AB\) has coordinates

\[
M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)
\]

The Distance Formula

**KEY CONCEPT**
If \(A(x_1, y_1)\) and \(B(x_2, y_2)\) are points in a coordinate plane, then the distance between \(A\) and \(B\) is

\[
AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\]

**EXAMPLE 1** Find segment lengths

Find RS.

Solution: Point \(T\) is the midpoint of \(RS\). So, \(RT = \frac{TS}{2} = 21.7\).

\[
RS = \frac{RT}{2} + \frac{TS}{2} = 21.7 + 21.7 = 43.4
\]

The length of \(RS\) is 43.4.

**EXAMPLE 2** Use algebra with segment lengths

ALGEBRA Point C is the midpoint of \(BD\).

Find the length of \(BC\).

Solution:

Step 1 Write and solve an equation.

\[
BC = CD \\
3x - 2 = 2x + 3 \\
x - 2 = 3 \\
x = 5
\]

Step 2 Evaluate the expression for \(BC\) when \(x = 5\).

\[
BC = 3x - 2 = 3(5) - 2 = 13
\]

So, the length of \(BC\) is 13.
EXAMPLE 3 Use the Midpoint Formula

a. FIND MIDPOINT The endpoints of $PR$ are $P(-2, 5)$ and $R(4, 3)$. Find the coordinates of the midpoint $M$.

Solution:

a. Use the Midpoint Formula.

$$M = \left( \frac{-2 + 4}{2}, \frac{5 + 3}{2} \right) = M(1, 4)$$

The coordinates of the midpoint of $PR$ are $M(1, 4)$.

b. FIND ENDPOINT The midpoint of $AC$ is $M(3, 4)$. One endpoint is $A(1, 6)$. Find the coordinates of endpoint $C$.

Solution:

Step 1 Find $x$. Step 2 Find $y$. The coordinates of endpoint $C$ are $(5, 2)$.

$$\frac{1 + x}{2} = 3 \quad \frac{6 + y}{2} = 4$$

$$1 + x = 6 \quad 6 + y = 8$$

$$x = 5 \quad y = 2$$

The Distance Formula is based on the Pythagorean Theorem, which you will see again when you work with right triangles in Chapter 7.

EXAMPLE 4 Use the Distance Formula

What is the approximate length of $TR$, with endpoints $T(-4, 3)$ and $R(3, 2)$?

Solution:

Use the Distance Formula.

$$RT = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(-4 - 3)^2 + (3 - 2)^2}$$

$$= \sqrt{(-7)^2 + (1)^2}$$

$$= \sqrt{49 + 1}$$

$$= \sqrt{50}$$

$$\approx 7.07$$

The length of $RT$ is about 7.07.
1.3 Cont.

**Checkpoint** Complete the following exercise.

1. Find \( AB \).

![Diagram](image)

\[ \frac{41}{2} \]

**Checkpoint** Complete the following exercise.

2. Point \( K \) is the midpoint of \( JL \). Find the length of \( KL \).

![Diagram](image)

\[ 6\frac{1}{5} \]

**Checkpoint** Complete the following exercises.

3. The endpoints of \( CD \) are \( C(-8, -1) \) and \( D(2, 4) \). Find the coordinates of the midpoint \( M \).

\( M\left(-3, \frac{3}{2}\right) \)

4. The midpoint of \( XZ \) is \( M(5, -6) \). One endpoint is \( X(-3, 7) \). Find the coordinates of endpoint \( Z \).

\( (13, -19) \)

**Checkpoint** Complete the following exercise.

5. What is the approximate length of \( GH \), with endpoints \( G(5, -1) \) and \( H(-3, 6) \)?

about 10.63
1.4 Measure and Classify Angles

Obj.: Name, measure, and classify angles.

Key Vocabulary

- **Angle** - An angle consists of two different rays with the same endpoint.

- **Sides, vertex of an angle** - The rays are the sides of the angle. The endpoint is the vertex of the angle.

- **Measure of an angle** - A protractor can be used to approximate the measure of an angle. An angle is measured in units called degrees (°).

  - **Words** The measure of \( \angle WXZ \) is 32°. **Symbols** \( m\angle WXZ = 32° \)

- **Congruent angles** - Two angles are congruent angles if they have the same measure.

- **Angle bisector** - An angle bisector is a ray that divides an angle into two angles that are congruent.

**POSTULATE 3 - Protractor Postulate**

Consider \( \overline{OB} \) and a point \( A \) on one side of \( \overline{OB} \). The rays of the form \( \overline{OA} \) can be matched one to one with the real numbers from 0 to 180. The measure of \( \angle AOB \) is equal to the absolute value of the difference between the real numbers for \( \overline{OA} \) and \( \overline{OB} \).

**CLASSIFYING ANGLES** Angles can be classified as acute, right, obtuse, and straight, as shown below.

**POSTULATE 4 - Angle Addition Postulate (AAP)**

**Words** If \( P \) is in the interior of \( \angle RST \), then the measure of \( \angle RST \) is equal to the sum of the measures of \( \angle RSP \) and \( \angle PST \).

**Symbols** If \( P \) is in the interior of \( \angle RST \), then

\[
m\angle RST = m\angle RSP + m\angle PST.
\]

The arcs show that the angles are congruent

\[
m\angle A = m\angle B \quad \text{“equal to”}
\]

Angles are congruent.

\[
\angle A \cong \angle B \quad \text{“is congruent to”}
\]
EXAMPLE 1 Name angles
Name the three angles in the diagram.
Solution:
\[ \angle ABC \text{, or } \angle CBA \]
\[ \angle CBD \text{, or } \angle DBC \]
\[ \angle ABD \text{, or } \angle DBA \]

EXAMPLE 2 Measure and classify angles
Use the diagram to find the measure of the indicated angle. Then classify the angle.
a. \( \angle WSR \)
b. \( \angle TSW \)
c. \( \angle RST \)
d. \( \angle VST \)
Solution:
a. \( \overline{SR} \) is lined up with the 0° on the \text{outer} scale of the protractor. \( \overline{SW} \) passes through 65° on the \text{outer} scale. So, \( \angle WSR = 65° \). It is \text{an acute} angle.
b. \( \overline{ST} \) is lined up with the 0° on the \text{inner} scale of the protractor. \( \overline{SW} \) passes through 115° on the \text{inner} scale. So, \( \angle TSW = 115° \). It is \text{an obtuse} angle.

c. \( \angle RST = 180° \). It is \text{a straight} angle.
d. \( \angle VST = 90° \). It is \text{a right} angle.

EXAMPLE 3 Find angle measures
ALGEBRA Given that \( m \angle GFJ = 155° \), find \( m \angle GFH \) and \( m \angle HFJ \).
Solution:
Step 1 Write and solve an equation to find the value of \( x \).
\[
m \angle GFJ = m \angle GFH + m \angle HFJ \quad \text{Angle Addition Postulate}
\]
\[
155° = (4x + 4)° + (4x - 1)°
\]
\[
155 = 8x + 3
\]
\[
152 = 8x
\]
\[
19 = x
\]
Step 2 Evaluate the given expressions when \( x = 19 \).
\[
m \angle GFH = (4x + 4)° = (4 \cdot 19 + 4)° = 80°
\]
\[
m \angle HFJ = (4x - 1)° = (4 \cdot 19 - 1)° = 75°
\]
So, \( m \angle GFH = 80° \) and \( m \angle HFJ = 75° \).

EXAMPLE 4 Identify congruent angles
Identify all pairs of congruent angles in the diagram.
If \( m \angle P = 120° \), what is \( m \angle N \)?
Solution:
There are two pairs of congruent angles:
\[ \angle P \cong \angle N \text{ and } \angle L \cong \angle M \]
Because \( \angle P \cong \angle N \), \( m \angle P = m \angle N \).
So, \( m \angle N = 120° \).

EXAMPLE 5 Double an angle measure
In the diagram at the right, \( \overline{WY} \) bisects \( \angle XWZ \), and \( m \angle XWY = 29° \).
Find \( m \angle XWZ \)
Solution:
By the Angle Addition Postulate,
\( m \angle XWZ = m \angle XWY + m \angle YWZ \).
Because \( \overline{WY} \) bisects \( \angle XWZ \), you know
\[ \angle XWY = \angle YWZ \]
So,
\[ m \angle XWY = m \angle YWZ \]
and you can write
\[ m \angle XWZ = m \angle XWY + m \angle YWZ \]
\[ = 29° + 29° = 58° \]
1.4 Cont.

**Checkpoint** Complete the following exercises.

1. Name all the angles in the diagram at the right.

   \( \angle FGH \) or \( \angle HGF \), \( \angle FGJ \) or \( \angle JGF \), \( \angle JGH \) or \( \angle HGJ \)

2. What type of angles do the x-axis and y-axis form in a coordinate plane?
   
   right angles

**Checkpoint** Complete the following exercise.

3. Given that \( \angle VRS \) is a right angle, find \( m \angle VRT \) and \( \angle TRS \).

   \( m \angle VRT = 19^\circ \), \( m \angle TRS = 71^\circ \)

**Checkpoint** Complete the following exercises.

4. Identify all pairs of congruent angles in the diagram. If \( m \angle B = 135^\circ \), what is \( m \angle D \)?

   \( \angle B \equiv \angle D \) and \( \angle A \equiv \angle C \); 135°

5. In the diagram below, \( \overrightarrow{KM} \) bisects \( \angle LKN \) and \( m \angle LKM = 78^\circ \). Find \( m \angle LKN \).

   156°
2.6 Prove Statements about Segments and Angles

Obj.: Write proofs using geometric theorems.

Key Vocabulary
• Proof - A proof is a logical argument that shows a statement is true. There are several formats for proofs.
• Two-column proof - A two-column proof has numbered statements and corresponding reasons that show an argument in a logical order.
• Theorem - The reasons used in a proof can include definitions, properties, postulates, and theorems. A theorem is a statement that can be proven.

THEOREMS

Congruence of Segments
Segment congruence is reflexive, symmetric, and transitive.

Reflexive For any segment \( AB \), \( AB \cong AB \).
Symmetric If \( AB \cong CD \), then \( CD \cong AB \).
Transitive If \( AB \cong CD \) and \( CD \cong EF \), then \( AB \cong EF \).

Congruence of Angles
Angle congruence is reflexive, symmetric, and transitive.

Reflexive For any angle \( \angle A \), \( \angle A \cong \angle A \).
Symmetric If \( \angle A \cong \angle B \), then \( \angle B \cong \angle A \).
Transitive If \( \angle A \cong \angle B \) and \( \angle B \cong \angle C \), then \( \angle A \cong \angle C \).

EXAMPLE 1 Write a two-column proof
Use the diagram to prove \( m\angle 1 = m\angle 4 \).

Given: \( m\angle 2 = m\angle 3 \), \( m\angle AXD = m\angle AXC \)

Prove: \( m\angle 1 = m\angle 4 \)

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( m\angle AXC = m\angle AXD )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( m\angle AXD = m\angle 1 + m\angle 2 )</td>
<td>2. Angle Addition Postulate</td>
</tr>
<tr>
<td>3. ( m\angle AXC = m\angle 3 + m\angle 4 )</td>
<td>3. Angle Addition Postulate</td>
</tr>
<tr>
<td>4. ( m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4 )</td>
<td>4. Substitution Property of Equality</td>
</tr>
<tr>
<td>5. ( m\angle 2 = m\angle 3 )</td>
<td>5. Given</td>
</tr>
<tr>
<td>6. ( m\angle 1 + m\angle 3 = m\angle 3 + m\angle 4 )</td>
<td>6. Substitution Property of Equality</td>
</tr>
<tr>
<td>7. ( m\angle 1 = m\angle 4 )</td>
<td>7. Subtraction Property of Equality</td>
</tr>
</tbody>
</table>

EXAMPLE 2 Name the property shown
Name the property illustrated by the statement.

If \( \angle 5 \cong \angle 3 \), then \( \angle 3 \cong \angle 5 \)

Solution: Symmetric Property of Angle Congruence
EXAMPLE 3 Use properties of equality
If you know that $BD$ bisects $\angle ABC$, prove that $m \angle ABC$ is two times $m \angle 1$.

**Given:** $BD$ bisects $\angle ABC$.

**Prove:** $m \angle ABC = 2 \cdot m \angle 1$

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $BD$ bisects $\angle ABC$.</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. $\angle 1 \equiv \angle 2$</td>
<td>2. Definition of angle bisector</td>
</tr>
<tr>
<td>3. $m \angle 1 = m \angle 2$</td>
<td>3. Definition of congruent angles</td>
</tr>
<tr>
<td>4. $m \angle 1 + m \angle 2 = m \angle ABC$</td>
<td>4. Angle Addition Postulate</td>
</tr>
<tr>
<td>5. $m \angle 1 + m \angle 1 = m \angle ABC$</td>
<td>5. Substitution Property of Equality</td>
</tr>
<tr>
<td>6. $2 \cdot m \angle 1 = m \angle ABC$</td>
<td>6. Distributive Property</td>
</tr>
</tbody>
</table>

EXAMPLE 4 Solve a multi-step problem
**Interstate** There are two exits between rest areas on a stretch of interstate. The Rice exit is halfway between rest area A and Mason exit. The distance between rest area B and Mason exit is the same as the distance between rest area A and the Rice exit. Prove that the Mason exit is halfway between the Rice exit and rest area B.

**Solution:**

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $R$ is the midpoint of $AB$, $MB = AR$.</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. $AR \equiv RM$</td>
<td>2. Definition of midpoint</td>
</tr>
<tr>
<td>3. $AR = RM$</td>
<td>3. Definition of congruent segments</td>
</tr>
<tr>
<td>4. $MB = RM$</td>
<td>4. Transitive Property of Congruence</td>
</tr>
<tr>
<td>5. $MB \equiv RM$</td>
<td>5. Definition of congruent segments</td>
</tr>
<tr>
<td>6. $M$ is the midpoint of $RB$.</td>
<td>6. Definition of midpoint</td>
</tr>
</tbody>
</table>

CONCEPT SUMMARY - Writing a Two-Column Proof

In a proof, you make one statement at a time, until you reach the conclusion. Because you make statements based on facts, you are using deductive reasoning. Usually the first statement-and-reason pair you write is given information.

**Proof of the Symmetric Property of Angle Congruence**

**GIVEN** $\angle 1 \equiv \angle 2$

**PROVE** $\angle 2 \equiv \angle 1$

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $\angle 1 \equiv \angle 2$</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. $m \angle 1 = m \angle 2$</td>
<td>2. Definition of congruent angles</td>
</tr>
<tr>
<td>3. $m \angle 2 = m \angle 1$</td>
<td>3. Symmetric Property of Equality</td>
</tr>
<tr>
<td>4. $\angle 2 \equiv \angle 1$</td>
<td>4. Definition of congruent angles</td>
</tr>
</tbody>
</table>

Copy or draw diagrams and label given information to help develop proofs.

The number of statements will vary for the last statement.
Definitions, postulates

Theorems. Statements based on facts that you know or on conclusions from deductive reasoning.

2.6 Cont.

**Checkpoint** Complete the following exercises.

1. Three steps of a proof are shown. Give the reasons for the last two steps.

   Given \( BC = AB \)
   
   Prove \( AC = AB + AB \)

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( BC = AB )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( AC = AB + BC )</td>
<td>2. Segment Addition Postulate</td>
</tr>
<tr>
<td>3. ( AC = AB + AB )</td>
<td>3. Substitution Property of Equality</td>
</tr>
</tbody>
</table>

2. Name the property illustrated by the statement.
   If \( \angle H \cong \angle T \) and \( \angle T \cong \angle B \), then \( \angle H \cong \angle B \).

   Transitive Property of Angle Congruence

**Checkpoint** Complete the following exercise.

3. In Example 4, there are rumble strips halfway between the Rice and Mason exits. What other two places are the same distance from the rumble strips?

   Rest area A and rest area B
1.5 Describe Angle Pair Relationships

**Obj.:** Use special angle relationships to find angle measure.

**Key Vocabulary**

- **Complementary angles** - Two angles are **complementary angles** if the **sum** of their measures is $90^\circ$.
- **Supplementary angles** - Two angles are **supplementary angles** if the sum of their measures is $180^\circ$.
- **Adjacent angles** - **Adjacent angles** are two angles that share a **common vertex** and **side**, but have no common interior points.
- **Linear pair** - Two adjacent angles are a **linear pair** if their noncommon sides are **opposite rays**.
- **Vertical angles** - Two angles are **vertical angles** if their sides form **two pairs** of opposite rays.

\[ \angle 1 \text{ and } \angle 2 \text{ are a } \text{linear pair}. \quad \angle 3 \text{ and } \angle 6 \text{ are } \text{vertical angles}. \quad \angle 4 \text{ and } \angle 5 \text{ are vertical angles}. \]

**EXAMPLE 1 Identify complements and supplements**

In the figure, name a pair of complementary angles, a pair of supplementary angles, and a pair of adjacent angles.

**Solution:**

- Because $52^\circ + 38^\circ = 90^\circ$, $\angle ABD$ and $\angle CDB$ are **complementary** angles.
- Because $52^\circ + 128^\circ = 180^\circ$, $\angle ABD$ and $\angle EDB$ are **supplementary** angles.
- Because $\angle CDB$ and $\angle BDE$ share a common vertex and side, they are **adjacent** angles.

**EXAMPLE 2 Find measures of a complement and a supplement**

**a.** Given that $\angle 1$ is a complement of $\angle 2$ and $m \angle 2 = 57^\circ$, find $m \angle 1$.

**Solution:**

\[ m \angle 1 = 90^\circ - m \angle 2 = 90^\circ - 57^\circ = 33^\circ \]

**b.** Given that $\angle 3$ is a supplement of $\angle 4$ and $m \angle 4 = 41^\circ$, find $m \angle 3$.

**Solution:**

\[ m \angle 3 = 180^\circ - m \angle 4 = 180^\circ - 41^\circ = 139^\circ \]
EXAMPLE 3 Find angle measures
SPORTS The basketball pole forms a pair of supplementary angles with the ground. Find $m\angle BCA$ and $m\angle DCA$.
Solution:

Step 1 Use the fact that 180° is the sum of the measures of supplementary angles.

$$m\angle BCA + m\angle DCA = 180°$$

Substitute.

$$\frac{3x + 8}{7x + 5} = 180$$

Combine like terms.

$$\frac{7x}{175} = 25$$

Subtract.

$$x = 25$$

Divide.

Step 2 Evaluate the original expressions when $x = 25$.

$$m\angle BCA = \frac{3x + 8}{3 \cdot 25 + 8} = 83°.$$  

$$m\angle DCA = \frac{4x - 3}{4 \cdot 25 - 3} = 97°.$$  

The angle measures are 83° and 97°.

EXAMPLE 4 Identify angle pairs
Identify all of the linear pairs and all of the vertical angles in the figure at the right.
Solution:

To find vertical angles, look for angles formed by intersecting lines.  

\[ \angle 1 \text{ and } \angle 3 \text{ are vertical angles.} \]

To find linear pairs, look for adjacent angles whose noncommon sides are opposite rays.

\[ \angle 1 \text{ and } \angle 2 \text{ are a linear pair. } \angle 2 \text{ and } \angle 3 \text{ are a linear pair.} \]

EXAMPLE 5 Find angle measures in a linear pair
ALGEBRA Two angles form a linear pair. The measure of one angle is 4 times the measure of the other. Find the measure of each angle.
Solution:

Let $x°$ be the measure of one angle. The measure of the other angle is $4x°$. Then use the fact that the angles of a linear pair are supplementary to write an equation.

$$x° + 4x° = 180°$$

Combine like terms.

$$5x = 180$$

Divide each side by 5.

$$x = 36$$

The measures of the angles are 36° and 144°.

CONCEPT SUMMARY Interpreting a Diagram
There are some things you can conclude from a diagram, and some you cannot. For example, here are some things that you can conclude from the diagram at the right:

- All points shown are coplanar.
- Points $A$, $B$, and $C$ are collinear, and $B$ is between $A$ and $C$.
- $\overline{AC}$, $\overline{BD}$, and $\overline{BE}$ intersect at point $B$.
- $\angle DBE$ and $\angle EBC$ are adjacent angles, and $\angle ABC$ is a straight angle.
- Point $E$ lies in the interior of $\angle DBC$.

In the diagram above, you cannot conclude that $AB \cong BC$, that $\angle DBE \cong \angle EBC$, or that $\angle ABD$ is a right angle. This information must be indicated by marks, as shown at the right.
1.5 Cont.

**Checkpoint** Complete the following exercises.

1. In the figure, name a pair of complementary angles, a pair of supplementary angles, and a pair of adjacent angles.
   - complementary: $\angle DEF$ and $\angle ABC$
   - supplementary: $\angle FEG$ and $\angle ABC$
   - adjacent: $\angle DEF$ and $\angle FEG$

2. Given that $\angle 1$ is a complement of $\angle 2$ and $m\angle 1 = 73^\circ$, find $m\angle 2$.
   - $17^\circ$

3. Given that $\angle 3$ is a supplement of $\angle 4$ and $m\angle 4 = 37^\circ$, find $m\angle 3$.
   - $143^\circ$

**Checkpoint** Complete the following exercise.

4. In Example 3, suppose the angle measures are $(5x + 1)^\circ$ and $(6x + 3)^\circ$. Find $m\angle BCA$ and $m\angle DCA$.
   - $81^\circ$ and $99^\circ$

**Checkpoint** Complete the following exercise.

5. Identify all of the linear pairs and all of the vertical angles in the figure.
   - linear pairs: none; vertical angles: $\angle 1$ and $\angle 4$, $\angle 2$ and $\angle 5$, $\angle 3$ and $\angle 6$

**Checkpoint** Complete the following exercise.

6. Two angles form a linear pair. The measure of one angle is 3 times the measure of the other. Find the measure of each angle.
   - $45^\circ$ and $135^\circ$
2.7 Prove Angle Pair Relationships

Obj.: Use properties of special pairs of angles.

Key Vocabulary
• Complementary angles - Two angles whose measures have the sum 90°.
• Supplementary angles - Two angles whose measures have the sum 180°.
• Linear pair - Two adjacent angles whose noncommon sides are opposite rays.
• Vertical angles – Two angles whose sides form two pairs of opposite rays.

Right Angles Congruence Theorem
All right angles are congruent.

Congruent Supplements Theorem
If two angles are supplementary to the same angle (or to congruent angles), then they are congruent.
If \( \angle 1 \) and \( \angle 2 \) are supplementary and \( \angle 3 \) and \( \angle 2 \) are supplementary, then \( \angle 1 \cong \angle 3 \).

Congruent Complements Theorem
If two angles are complementary to the same angle (or to congruent angles), then they are congruent.
If \( \angle 4 \) and \( \angle 5 \) are complementary and \( \angle 6 \) and \( \angle 5 \) are complementary, then \( \angle 4 \cong \angle 6 \).

Linear Pair Postulate
If two angles form a linear pair, then they are supplementary. \( \angle 1 \) and \( \angle 2 \) form a linear pair, so \( \angle 1 \) and \( \angle 2 \) are supplementary and \( m\angle 1 + m\angle 2 = 180^\circ \).

Vertical Angles Congruence Theorem
Vertical angles are congruent.
\( \angle 1 \cong \angle 3, \angle 2 \cong \angle 4 \)

EXAMPLE 1 Use right angle congruence
Write a proof.
GIVEN: \( JK \perp KL, ML \perp KL \)
PROVE: \( \angle K \cong \angle L \)

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( JK \perp KL, ML \perp KL )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( \angle K ) and ( \angle L ) are right angles.</td>
<td>2. Definition of perpendicular lines</td>
</tr>
<tr>
<td>3. ( \angle K \cong \angle L )</td>
<td>3. Right Angles Congruence Theorem</td>
</tr>
</tbody>
</table>
EXAMPLE 2 Use Congruent Supplements Theorem
Write a proof.

**GIVEN:** \( \angle 1 \) and \( \angle 2 \) are supplements.
\( \angle 1 \) and \( \angle 4 \) are supplements.
\( m\angle 2 = 45^\circ \)

**PROVE:** \( m\angle 4 = 45^\circ \)

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \angle 1 ) and ( \angle 2 ) are supplements. ( \angle 1 ) and ( \angle 4 ) are supplements.</td>
<td>1. <strong>Given</strong></td>
</tr>
<tr>
<td>2. ( \angle 2 \cong \angle 4 )</td>
<td>2. Congruent Supplements Theorem</td>
</tr>
<tr>
<td>3. ( m\angle 2 = m\angle 4 )</td>
<td>3. <strong>Definition of congruent angles</strong></td>
</tr>
<tr>
<td>4. ( m\angle 2 = 45^\circ )</td>
<td>4. <strong>Given</strong></td>
</tr>
<tr>
<td>5. ( m\angle 4 = 45^\circ )</td>
<td>5. Substitution Property of Equality</td>
</tr>
</tbody>
</table>

EXAMPLE 3 Use the Vertical Angles Congruence Theorem
Prove vertical angles are congruent.

**GIVEN:** \( \angle 4 \) is a right angle.

**PROVE:** \( \angle 2 \) and \( \angle 4 \) are supplementary.

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \angle 4 ) is a right angle.</td>
<td>1. <strong>Given</strong></td>
</tr>
<tr>
<td>2. ( m\angle 4 = 90^\circ )</td>
<td>2. Definition of a right angle</td>
</tr>
<tr>
<td>3. ( \angle 2 \cong \angle 4 )</td>
<td>3. <strong>Vertical Angles Congruence Theorem</strong></td>
</tr>
<tr>
<td>4. ( m\angle 2 = m\angle 4 )</td>
<td>4. Definition of congruent angles</td>
</tr>
<tr>
<td>5. ( m\angle 2 = 90^\circ )</td>
<td>5. <strong>Substitution Property of Equality</strong></td>
</tr>
<tr>
<td>6. ( \angle 2 ) and ( \angle 4 ) are supplementary.</td>
<td>6. ( m\angle 2 + m\angle 4 = 180^\circ )</td>
</tr>
</tbody>
</table>

EXAMPLE 4 Find angle measures
Write and solve an equation to find \( x \). Use \( x \) to find the \( m\angle FKG \).

**Solution:**
Because \( m\angle FKG \) and \( m\angle GKH \) form a linear pair, the sum of their measures is \( 180^\circ \).

\[
(4x - 1)^\circ + 113^\circ = 180^\circ
\]

Write equation.

\[
4x + \frac{112}{4} = 180
\]
Simplify.

\[
4x = 68
\]
Subtract \( 112 \) from each side.

\[
x = 17
\]
Divide each side by 4.

Use \( x = 17 \) to find \( m\angle FKG \).

\[
m\angle FKG = (4x - 1)^\circ
\]
Write equation.

\[
= [4(17) - 1]^\circ
\]
Substitute \( 17 \) for \( x \).

\[
= [68 - 1]^\circ
\]
Multiply.

\[
= 67^\circ
\]
Simplify.

The measure of \( \angle FKG \) is \( 67^\circ \).
2.7 Cont.

磋 Checkpoint Complete the following exercises.

1. In Example 1, suppose you are given that $\angle K \equiv \angle L$. Can you use the Right Angles Congruence Theorem to prove that $\angle K$ and $\angle L$ are right angles? Explain.

No, you cannot prove that $\angle K$ and $\angle L$ are right angles, because the converse of the Right Angles Congruence Theorem is not always true.

2. Suppose $\angle A$ and $\angle B$ are complements, and $\angle A$ and $\angle C$ are complements. Can $\angle B$ and $\angle C$ be supplements? Explain.

No, $\angle B$ and $\angle C$ are complements by the Congruent Complements Theorem, so they cannot be supplements.

磋 Checkpoint In Exercises 3 and 4, use the diagram.

3. If $m\angle 4 = 63^\circ$, find $m\angle 1$ and $m\angle 2$.

$$m\angle 1 = 117^\circ, \ m\angle 2 = 63^\circ$$

4. If $m\angle 3 = 121^\circ$, find $m\angle 1$, $m\angle 2$, and $m\angle 4$.

$$m\angle 1 = 121^\circ, \ m\angle 2 = 59^\circ, \ m\angle 4 = 59^\circ$$

磋 Checkpoint Complete the following exercise.

5. Find $m\angle AEB$.

$$m\angle AEB = 70^\circ$$
1.6 Classify Polygons

Obj.: Classify polygons.

Key Vocabulary
• **Polygon** - In geometry, a figure that lies in a plane is called a *plane figure*. A polygon is a *closed* plane figure with the following properties:
  • **Side** - 1. It is formed by three or more line *segments* called *sides*.  
    2. Each side intersects exactly two *sides*, one at each *endpoint*, so that no two sides with a common endpoint are collinear.
• **Vertex** - Each *endpoint* of a *side* is a *vertex* of the polygon. The plural of vertex is *vertices*.
• **Convex** - A polygon is convex if no line that contains a side of the polygon contains a point in the *interior* of the polygon.
• **Concave** - A polygon that is not convex is called *nonconvex* or *concave*.
• **n-gon** - The term *n-gon*, where *n* is the number of a polygon’s *sides*, can also be used to name a polygon. For example, a polygon with 14 sides is a 14-gon.
• **Equilateral** - In an *equilateral* polygon, all *sides* are congruent.
• **Equiangular** - In an *equiangular* polygon, all *angles* in the interior of the polygon are congruent.
• **Regular** - A *regular* polygon is a convex polygon that is both equilateral and equiangular.

A polygon can be named by listing the vertices in consecutive order. For example, ABCDE and CDEAB are both correct names for the polygon at the right.

**CLASSIFYING POLYGONS** A polygon is named by the number of its sides.

<table>
<thead>
<tr>
<th># of sides</th>
<th>Type of polygon</th>
<th># of sides</th>
<th>Type of polygon</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>Triangle</td>
<td>8</td>
<td>Octagon</td>
</tr>
<tr>
<td>4</td>
<td>Quadrilateral</td>
<td>9</td>
<td>Nonagon</td>
</tr>
<tr>
<td>5</td>
<td>Pentagon</td>
<td>10</td>
<td>Decagon</td>
</tr>
<tr>
<td>6</td>
<td>Hexagon</td>
<td>12</td>
<td>Dodecagon</td>
</tr>
<tr>
<td>7</td>
<td>Heptagon</td>
<td><em>n</em></td>
<td><em>n-gon</em></td>
</tr>
</tbody>
</table>
EXAMPLE 1 Identify polygons
Tell whether the figure is a polygon and whether it is convex or concave.

\[ \text{a. Some segments intersect more than two segments, so it is not a polygon.} \]
\[ \text{b. The figure is a convex polygon.} \]
\[ \text{c. The figure is a concave polygon.} \]

EXAMPLE 2 Classify polygons
Classify the polygon by the number of sides. Tell whether the polygon is equilateral, equiangular, or regular. Explain your reasoning.

Solution: The polygon has 8 sides. It is equilateral and equiangular, so it is a regular octagon.

EXAMPLE 3 Find side lengths
ALGEBRA A head of a bolt is shaped like a regular hexagon. The expressions shown represent side lengths of the hexagonal bolt. Find the length of a side.

Solution: First, write and solve an equation to find the value of x. Use the fact that the sides of a regular hexagon are congruent.

\[ \frac{4x + 3}{4} = \frac{5x - 1}{x} \]

Write an equation.

\[ 4x + 3 = 5x - 1 \]

Simplify.

Then evaluate one of the expressions to find a side length when \( x = 4 \).

\[ 4x + 3 = 4(4) + 3 = 19 \]

The length of a side is 19 millimeters.
1.6 Cont.  (Write these on your paper)

**Checkpoint** Tell whether the figure is a polygon and whether it is convex or concave.

1. convex polygon
2. not a polygon

**Checkpoint** Complete the following exercises.

3. Classify the polygon by the number of sides. Tell whether the polygon is *equilateral*, *equiangular*, or *regular*.
   - quadrilateral

4. The expressions $(4x + 8)^\circ$ and $(5x - 5)^\circ$ represent the measures of two of the congruent angles in Example 3. Find the measure of an angle.
   - $60^\circ$
2.1 Use Inductive Reasoning

Obj.: Describe patterns and use inductive reasoning.

Key Vocabulary
- **Conjecture** - A conjecture is an **unproven** statement that is based on **observations**.
- **Inductive reasoning** - You use **inductive reasoning** when you find a **pattern** in specific cases and then write a **conjecture** for the general case.
- **Counterexample** - A **counterexample** is a specific case for which the conjecture is **false**. You can **show** that a conjecture is false, however, by simply finding one counterexample.

EXAMPLE 1 Describe a visual pattern
Describe how to sketch the fourth figure in the pattern. Then sketch the fourth figure.

**Solution:**

Each rectangle is divided into twice as many equal regions as the figure number. Sketch the fourth figure by dividing the rectangle into **eighths**. Shade the section just below the horizontal segment at the left.

EXAMPLE 2 Describe a number pattern
Describe the pattern in the numbers $-1, -4, -16, -64, \ldots$ and write the next three numbers in the pattern.

**Solution:**

Notice that each number in the pattern is **four** times the previous number.

$-1, \quad -4, \quad -16, \quad -64, \ldots$

$x\ 4\ x\ 4\ x\ 4\ x\ 4$

The next three numbers are $-256, -1024, \text{and} -4096$.

EXAMPLE 3 Make a conjecture
Given five noncollinear points, make a conjecture about the number of ways to connect different pairs of the points.

**Solution:**

Make a table and look for a pattern. Notice the pattern in how the number of connections **increases**. You can use the pattern to make a conjecture.

<table>
<thead>
<tr>
<th>Number of points</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Picture</td>
<td>.</td>
<td>-</td>
<td>+</td>
<td>×</td>
<td>*</td>
</tr>
<tr>
<td>Number of connections</td>
<td>0</td>
<td>1</td>
<td>3</td>
<td>6</td>
<td>?</td>
</tr>
</tbody>
</table>

*Conjecture* You can connect five noncollinear points $6 + 4$, or **10** different ways.
EXAMPLE 4 Make and test a conjecture

Numbers such as 1, 3, and 5 are called consecutive odd numbers. Make and test a conjecture about the sum of any three consecutive odd numbers.

Solution:

Step 1 Find a pattern using groups of small numbers.

| 1 + 3 + 5 = 9 | 3 + 5 + 7 = 15 | 5 + 7 + 9 = 21 |
| = 3 · 3 | = 5 · 3 | = 7 · 3 |

Solution: The sum of any three consecutive odd numbers is three times the second number.

Step 2 Test your conjecture using other numbers.

-1 + 1 + 3 = 3 = 1 · 3 ✓
103 + 105 + 107 = 315 = 105 · 3 ✓

EXAMPLE 5 Find a counterexample

A student makes the following conjecture about the difference of two numbers. Find a counterexample to disprove the student’s conjecture.

Conjecture The difference of any two numbers is always smaller than the larger number.

Solution:

To find a counterexample, you need to find a difference that is greater than the larger number.

8 − (−4) = 12

Because 12 ⪯ 8, a counterexample exists. The conjecture is false.

EXAMPLE 6 Real world application

The scatter plot shows the average salary of players in the National Football League (NFL) since 1999. Make a conjecture based on the graph.

Solution:

The scatter plot shows that the values increased each year. So, one possible conjecture is that the average player in the NFL is earning more money today than in 1999.
2.1 Cont.

**Checkpoint** Complete the following exercise.

1. Sketch the fifth figure in the pattern in Example 1.

![Figure 5](image)

**Checkpoint** Complete the following exercises.

2. Describe the pattern in the numbers 1, 2.5, 4, 5.5, . . . and write the next three numbers in the pattern.

The numbers are increasing by 1.5; 7, 8.5, 10.

3. Rework Example 3 if you are given six noncollinear points.

15 different waves

**Checkpoint** Complete the following exercise.

4. Make and test a conjecture about the sign of the product of any four negative numbers.

The result of the product of four negative numbers is a positive number;

\[(-1)(-2)(-5)(-1) = 10.\]

**Checkpoint** Complete the following exercises.

5. Find a counterexample to show that the following conjecture is false.

Conjecture The quotient of two numbers is always smaller than the dividend.

\[\frac{4}{1} = 8\]

\[\frac{1}{2}\]

6. Use the graph in Example 6 to make a conjecture that could be true. Give an explanation that supports your reasoning.

The average salary of an NFL player in future years will be higher than the previous year; the average salary of an NFL player increased for the 5 years from 1999 to 2003.
2.3 Apply Deductive Reasoning

Obj.: **Use deductive reasoning to form a logical argument.**

Key Vocabulary
- **Deductive reasoning** - Deductive reasoning uses facts, definitions, accepted properties, and the laws of logic to form a *logical argument*.

**** This is different from **inductive reasoning**, which uses specific examples and patterns to form a conjecture.****

Laws of Logic

**KEY CONCEPT**

**Law of Detachment**
If the hypothesis of a true conditional statement is true, then the **conclusion** is also **true**.

**Law of Syllogism**
If hypothesis *p*, then conclusion *q*. If these statements are **true**, then hypothesis *q*, then conclusion *r*.

**EXAMPLE 1 Use the Law of Detachment**
Use the Law of Detachment to make a valid conclusion in the true situation.

a. If two angles have the same measure, then they are congruent. You know that \( m\angle A = m\angle B \).

Solution:

a. Because \( m\angle A = m\angle B \) satisfies the hypothesis of a true conditional statement, the conclusion is also true. So, \( \angle A \cong \angle B \).

b. Jesse goes to the gym every weekday. Today is Monday.

Solution:

b. First, identify the hypothesis and the conclusion of the first statement. The hypothesis is “If it is a weekday,” and the conclusion is “then Jesse goes to the gym.”

“Today is Monday” satisfies the hypothesis of the conditional statement, so you can conclude that Jesse will go to the gym today.

**EXAMPLE 2 Use the Law of Syllogism**
(2.3 cont.)

If possible, use the Law of Syllogism to write a new conditional statement that follows from the pair of true statements.

a. If Ron eats lunch today, then he will eat a sandwich. If Ron eats a sandwich, then he will drink a glass of milk.
Solution:  

b. If $x^2 > 36$, then $x^2 > 30$. If $x > 6$, then $x^2 > 36$.

Solution:

Notice that the conclusion of the second statement is the hypothesis of the first statement, so you can write the following.

If $x > 6$, then $x^2 > 30$.

c. If a triangle is equilateral, then all of its sides are congruent. If a triangle is equilateral, then all angles in the interior of the triangle are congruent.

Solution:  

Neither statement’s conclusion is the same as the other statement’s hypothesis. You cannot use the Law of Syllogism to write a new conditional statement.

EXAMPLE 3 Use inductive and deductive reasoning  
What conclusion can you make about the sum of an odd integer and an odd integer?

Solution:

Step 1  
Look for a pattern in several examples. Use inductive reasoning to make a conjecture.

$-3 + 5 = 2$, $-1 + 5 = 4$, $3 + 5 = 8$

$-3 + (-5) = -8$, $1 + (-5) = -4$

$3 + (-5) = -2$

Conjecture: Odd integer + Odd integer = Even integer

Step 2  
Let $n$ and $m$ each be any integer. Use deductive reasoning to show the conjecture is true.

$2n$ and $2m$ are even integers because any integer multiplied by 2 is even.

$2n - 1$ and $2m + 1$ are odd integers because $2n$ and $2m$ are even integers.

$(2n - 1) + (2m + 1)$ represents the sum of an odd integer $2n - 1$ and an odd integer $2m + 1$.

$(2n - 1) + (2m + 1) = 2(n + m)$

The result is the product of 2 and an integer $n + m$. So, $2(n + m)$ is an even integer.

The sum of an odd integer and an odd integer is an even integer.

EXAMPLE 4 Reasoning from a graph

Tell whether the statement is the result of inductive reasoning or deductive reasoning. Explain your choice.

a. The runner’s average speed decreases as time spent running increases.

Solution:  

Inductive reasoning, because it is based on a pattern in the data

b. The runner’s average speed is slower when running for 40 minutes than when running for 10 minutes.

Solution:  

Deductive reasoning, because you are comparing values that are given on the graph
2.3 Cont.

**Checkpoint** Complete the following exercises.

1. If $0^\circ < m\angle A < 90^\circ$, then $A$ is acute. The measure of $\angle A$ is $38^\circ$. Using the Law of Detachment, what statement can you make?

   $\angle A$ is acute.

2. State the law of logic that is illustrated below.

   If you do your homework, then you can watch TV. If you watch TV, then you can watch your favorite show. If you do your homework, then you can watch your favorite show.

   Law of Syllogism

**Checkpoint** Complete the following exercise.

3. Use inductive reasoning to make a conjecture about the sum of a negative integer and itself. Then use deductive reasoning to show the conjecture is true.

   The sum of a negative integer and itself is twice the integer; $-n + (-n) = -2n = 2(-n)$.

**Checkpoint** Complete the following exercises.

4. Use inductive reasoning to write another statement about the graph in Example 4.

   *Sample answer:* The faster the average speed of the runner, the less time he or she is running.

5. Use deductive reasoning to write another statement about the graph in Example 4.

   *Sample answer:* The runner’s average speed is faster when running for 10 minutes than when running for 40 minutes.
2.2 Analyze Conditional Statements

Obj.: Write definitions as conditional statements.

Key Vocabulary
• Conditional statement - A conditional statement is a logical statement that has two parts, a hypothesis and a conclusion.
• Converse - To write the converse of a conditional statement, exchange the hypothesis and conclusion.
• Inverse - To write the inverse of a conditional statement, negate (not) both the hypothesis and the conclusion.
• Contrapositive - To write the contrapositive, first write the converse and then negate both the hypothesis and the conclusion.
• If-then form, Hypothesis, Conclusion - When a conditional statement is written in if-then form, the “if” part contains the hypothesis and the “then” part contains the conclusion. Here is an example:

\[
\text{If it is raining, then there are clouds in the sky.}
\]

\[\text{hypothesis} \hspace{1cm} \text{conclusion}\]

• Negation - The negation of a statement is the opposite of the original statement.
• Equivalent statements - When two statements are both true or both false, they are called equivalent statements.
• Perpendicular lines - If two lines intersect to form a right angle, then they are perpendicular lines. You can write “line } l \text{ is perpendicular to line } m \text{ as } l \perp \hspace{0.1cm} m.

\[\text{KEY CONCEPT} \hspace{1cm} l \perp \hspace{0.1cm} m\]

• Biconditional statement – A biconditional statement is a statement that contains the converse and the phrase “if and only if.”

EXAMPLE 1 Rewrite a statement in if-then form
Rewrite the conditional statement in if-then form. All vertebrates have a backbone.
Solution:

First, identify the hypothesis and the conclusion. When you rewrite the statement in if-then form, you may need to reword the hypothesis or conclusion.

All vertebrates have a backbone.

If an animal is a vertebrate, then it has a backbone.
EXAMPLE 2 Write four related conditional statements

Write the if-then form, the converse, the inverse, and the contrapositive of the conditional statement “Olympians are athletes.” Decide whether each statement is true or false.

Solution:

If-then form

If you are an Olympian, then you are an athlete. True, Olympians are athletes.

Converse

If you are an athlete, then you are an Olympian. False, not all athletes are Olympians.

Inverse

If you are not an Olympian, then you are not an athlete. False, even if you are not an Olympian, you can still be an athlete.

Contrapositive

If you are not an athlete, then you are not an Olympian. True, a person who is not an athlete cannot be an Olympian.

EXAMPLE 3 Use definitions

Decide whether each statement about the diagram is true. Explain your answer using the definitions you have learned.

a. $AC \perp BD$

b. $\angle AED$ and $\angle BEC$ are a linear pair.

Solution:

a. The statement is true. The right angle symbol indicates that the lines intersect to form a right angle. So you can say the lines are perpendicular.

b. The statement is false. Because $\angle AED$ and $\angle BEC$ are not adjacent angles, $\angle AED$ and $\angle BEC$ are not a linear pair.

EXAMPLE 4 Write a biconditional

Write the definition of parallel lines as a biconditional.

Definition: If two lines lie in the same plane and do not intersect, then they are parallel.

Solution:

Converse: If two lines are parallel, then they lie in the same plane and do not intersect.

Biconditional: Two lines are parallel if and only if they lie in the same plane and do not intersect.
2.2 Cont.

**Checkpoint** Write the conditional statement in if-then form.

1. All triangles have 3 sides.
   
   If a figure is a triangle, then it has 3 sides.

2. When \( x = 2 \), \( x^2 = 4 \).
   
   If \( x = 2 \), then \( x^2 = 4 \).

**Checkpoint** Complete the following exercises.

3. Write the if-then form, the converse, the inverse, and the contrapositive of the conditional statement “Squares are rectangles.” Decide whether each statement is true or false.
   
   If-then form: If a figure is a square, then it is a rectangle. True, squares are rectangles.
   
   Converse: If a figure is a rectangle, then it is a square. False, not all rectangles are squares.
   
   Inverse: If a figure is not a square, then it is not a rectangle. False, even if a figure is not a square, it can still be a rectangle.
   
   Contrapositive: If a figure is not a rectangle, then it is not a square. True, a figure that is not a rectangle cannot be a square.

4. Decide whether each statement about the diagram is true. Explain your answer using the definitions you have learned.

   a. \( \angle GLK \) and \( \angle JLK \) are supplementary.
   
   b. \( \overrightarrow{GJ} \perp \overrightarrow{HK} \)
      
      (a) True; linear pairs of angles are supplementary.
      
      (b) False; it is not known that the lines intersect at right angles.

5. Write the statement below as a biconditional.

   **Statement:** If a student is a boy, he will be in group A. If a student is in group A, the student must be a boy.

   A student is in group A if and only if the student is a boy.