

# Unit 3 Electricity and Magnetism

## 3–14 Electric Charge, Force, and Fields

**Electrostatics:** Involves the dynamics and interactions between charges ( $q$  or  $Q$ ) and electric fields ( $E$ ). This causes force ( $F$ ), electric potential ( $V$ ), and electric potential energy ( $U_E$ ) between charged particles and charged plates. It is considered static since the charges are not transferred between objects.

**Point Charge ( $q$ ):** A quantity of charge that can be reduced to a single point. A charged sphere, where all of the charge acts as though it is located in the center, is similar to gravity where all of the gravity is mathematically located at a planets center. Individual protons and electrons and clusters of these particles are point charges.

**Charged Plates ( $Q$ ):** Two flat conductive surfaces separated by a distance can act as a simple charge storage devise. The large amount of charge and energy they hold is distributed over the plates and cannot be localized to a single point. One way to represent a lot of small  $q$ 's is with a large  $Q$ . While  $q$  and  $Q$  are interchangeable in equations,  $r$  and  $d$  are not. Equations with  $r$  in them pertain to spherical point charges, and equations with  $d$  in them are used for charged plates.

**Charge is Conserved:** Charges cannot be created or destroyed.

**Charge is Quantized:** Comes in set quantities. All charges are a multiple of the charge on an electron.

**Electricity vs. Gravity:** They are both fields and share many characteristics, but have a few critical differences. The charts below compare these two very important field forces to one another.

Gravitational Fields	Electric Fields
<ul style="list-style-type: none"> <li><math>F_g</math>, Force of gravity is caused by gravitational fields.</li> <li><math>m</math>, masses generate gravitational fields.</li> <li><math>g</math> is the gravity field strength.</li> </ul>	<ul style="list-style-type: none"> <li><math>F_E</math>, Force of electricity is caused by electric fields.</li> <li><math>q</math>, charges generate electric fields.</li> <li><math>E</math> is the electric field strength.</li> </ul>
<p><b>Newton's Law of Universal Gravitation</b></p> <p><b>Force of gravity between two masses.</b> Force between masses that attract is equal and opposite, both masses pull on each other with this same value.</p> $F_g = G \frac{m_1 m_2}{r^2}$ $G = 6.67 \times 10^{-11} \frac{N \cdot m^2}{kg^2}$	<p><b>Coulomb's Law</b></p> <p><b>Force of electricity between two charges:</b> Force between charges that attract/repel is equal and opposite, both charges pull/push on each other with this same value. <b>Electric charges attract &amp; repel</b></p> $F_E = k \frac{q_1 q_2}{r^2}$ $F = \frac{1}{4\pi \epsilon_0} \frac{q_1 q_2}{r^2}$ $k = \frac{1}{4\pi \epsilon_0} = 9 \times 10^9 \frac{N \cdot m^2}{C^2}$ $\epsilon_0 = 8.85 \times 10^{-12} C^2 / N \cdot m^2$
<p><b>Force of gravity on a mass in a gravitational field.</b></p> $F_g = mg$	<p><b>Force of electricity on a charge in an electric field.</b></p> $F_E = qE$ $E = \frac{F}{q}$
<p><b>Combine above equations and simplify</b></p> $mg = G \frac{m_1 m_2}{r^2}$ $g = G \frac{m}{r^2} \text{ (not given)}$ <p><b>This is an important often used formula!</b></p>	<p><b>Combine above equations and simplify</b></p> $qE = k \frac{q_1 q_2}{r^2}$ $E = k \frac{q}{r^2} \text{ (not given)}$ <p><b>This is an important often used formula!</b></p>
<p>What if you</p> <ul style="list-style-type: none"> <li>Double one mass?: <math>F_g</math> and <math>g</math> double.</li> <li>Double <math>r</math>?: <math>F_g</math> and <math>g</math> are 1/4.</li> <li>Halve <math>r</math>?: <math>F_g</math> and <math>g</math> are quadrupled</li> </ul> <p>Changes to Mass are directly proportional.</p> <p>Changes in distance involve the <b><u>Inverse Square Law</u></b>.</p>	<p>What if you</p> <ul style="list-style-type: none"> <li>Double one mass?: <math>F_E</math> and <math>E</math> double.</li> <li>Double <math>r</math>?: <math>F_E</math> and <math>E</math> are 1/4.</li> <li>Halve <math>r</math>?: <math>F_E</math> and <math>E</math> are quadrupled</li> </ul> <p>Changes to Charge are directly proportional.</p> <p>Changes in distance involve the <b><u>Inverse Square Law</u></b>.</p>

### Example 14-1: Superposition (Adding electric field vectors)

What is the force of electricity on a +1 C charge located half way between a -3 C charge and a +2 C charge separated by 2 m, as shown in Fig 14.1a?

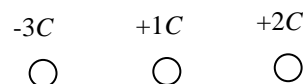


Fig 14.1a

Find  $F_E$  for the -3 & +1 charges, and then the +2 & +1 charges. Then add the two  $F_E$  vectors to find  $\Sigma F_E$ .

$$F_E = k \frac{q_1 q_2}{r^2} \quad F_E = 9 \times 10^9 \frac{N \cdot m^2}{C^2} \frac{(3C)(1C)}{[1/2(2m)]^2} = 27 \times 10^9 N \quad \text{-3 charge attracts the +1 charge to the left, negative direction.}$$

$$F_E = 9 \times 10^9 \frac{N \cdot m^2}{C^2} \frac{(2C)(1C)}{[1/2(2m)]^2} = 18 \times 10^9 N \quad \text{+2 charge repels the +1 charge to the left, negative direction.}$$

Note that the minus sign on the -3C charge was not included in the equation. The equation solves for the magnitude of the vector, which has an absolute value. Do not include minus signs on charge when solving for force electric. The minus signs are used to determine whether the object will repel or attract. To the right of each equation is an explanation of how the signs on charges are used. Once the sign on each vector is made then you can sum them.

Sum the vectors  $-27 \times 10^9 N + -18 \times 10^9 N = -45 \times 10^9 N$  The negative sign means to the left  $\boxed{45 \times 10^9 N, \text{left}}$ .

What is the electric field strength at point P located half way between a -3 C charge and a +2 C charge separated by 2 m, as shown in Fig 14.1b?

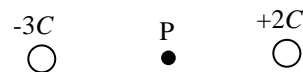


Fig 14.1b

Find  $E$  at point P for the -3 and then the +2 charge. Then add the two  $E$  vectors to find  $\Sigma E$ .

$$E = k \frac{q}{r^2} \quad E_{\text{from -3}} = 9 \times 10^9 \frac{N \cdot m^2}{C^2} \frac{(3C)}{[1/2(2m)]^2} = 27 \times 10^9 N/C$$

The -3 charge attracts a positive test charge to the left, negative direction.

$$E_{\text{from +2}} = 9 \times 10^9 \frac{N \cdot m^2}{C^2} \frac{(2C)}{[1/2(2m)]^2} = 18 \times 10^9 N/C$$

The +2 charge repels a positive test charge to the left, negative direction.

Again the minus signs are not included in the magnitude calculation. They are used to determine repulsion and attraction.

Sum the vectors  $-27 \times 10^9 + -18 \times 10^9 = -45 \times 10^9 N/C$  The negative sign means to the left  $\boxed{45 \times 10^9 N/C, \text{left}}$ .

**Alternate way to find the force on the +1C charge placed at point P.** We solved for the electric field strength at point P independent of a charge existing there. This is actually preferred, as once we have this value for empty space, we can easily put any charge there and find the force. If we insert a +1C charge at point P in Fig 14.1b, then we have the same scenario as Fig 14.1a. But, now that we have the electric field at point P, let's see how easy it is to find force.

$$F_E = qE \quad F_E = (1C)(45 \times 10^9 N/C) = \boxed{45 \times 10^9 N} \quad \text{This is the same answer as in the first part above.}$$

The advantage to this second method is that you can do superposition for the electric field once, and then use it for any charge placed at that point. If you use the force superposition method, you must do a new force superposition problem for every charge placed at point P.

### Example 14-2: Electric Force

A negatively charged wall repels a charged mass, attached to the wall by a string, shown in Fig 14.2a. Fig 14.2b shows the FBD for this scenario, and a diagram of the three vectors added tip to tail. Note that the three vectors sum to zero. The object is not moving and therefore the sum of force should be zero.

What is the force tension in the string?

$$T^2 = F_g^2 + F_E^2$$

$$T = \sqrt{F_g^2 + F_E^2}$$

$$T = \sqrt{mg^2 + qE^2}$$

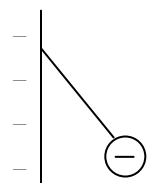


Fig 14.2a

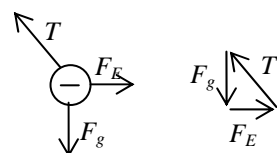


Fig 14.2b

**Electric Field Lines:** An imaginary way to view the electric field, like the way imaginary contour lines are used on maps. In mapping, steep slopes are diagrammed by drawing the contour lines closer together. In a similar fashion a concentration of any field lines (gravitational, electric, and magnetic) indicates higher field strength. Gravitational field lines are based on the direction of movement of a test mass. **Electric field lines are based on the direction of motion of a imaginary positive test charge in an electric field.** As a result electric field lines leave positive charges and enter negative charges. Every charged object generates an electric field. Field lines **leave and enter surfaces perpendicular to the surface.** The field lines around a proton, an electron, between a proton and an electron, and between charged plates as shown in Fig 1.2. The field direction at any point in the diagrams below is easy to identify. Simply imagine a positive charge at the point in question and ask what direction it will go. (+x, -x, +y, -y, etc.). When dealing with a curved portion the field is tangent to a curved surface. The field is solved for a point **P** in each case.

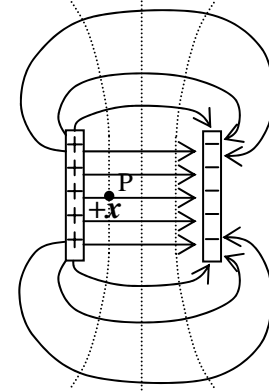
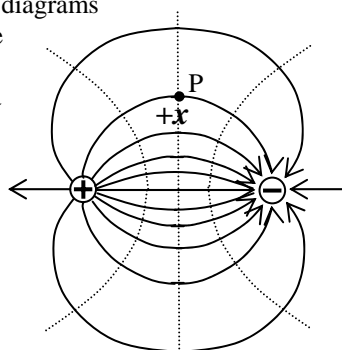
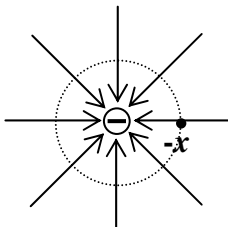
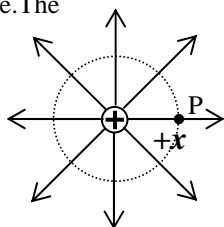


Fig 14.2

## Potential

**Voltage:**  $V = Ed$  In gravity it would be  $gh$ . We never worked with this term since  $g$  is weak, and does not change significantly over short distances. We normally concern ourselves only with changes in  $h$ , unless we leave a planet for some point in space, or we visit a large gravitational source such as a black hole. Electricity on the other hand is billions and billions of times stronger than gravity. A change in distance  $d$  (since no clear  $h$  exists in electricity) is accompanied by a significant change in electric field  $E$  as well. The electrical equivalent of  $gh$  is potential  $Ed$  and is commonly called voltage where  $V = Ed$ .

**Equipotential Lines:** Lines of equal electric potential (a component of potential energy). If a charge moves along an equipotential line it does not change its potential or voltage. This is like hiking in the mountains along the contour lines where your height and gravity above sea level don't change. Equipotential lines are always perpendicular to field lines. The dotted lines in the diagrams above are some examples of the many possible equipotential lines. The electric field  $E$ , is a component of potential ( $E = V/d$ ) and will not change if potential  $V$  and distance  $d$  (or  $r$ ) do not change. If you find the electric field strength at **P** in the two left diagrams it will be the same at any equal distance from the center of the point charge. At this distance an equipotential sphere, with radius  $r$  exists where  $Ed$  stays the same. This is why  $r$  is used for distance around point charges. The potential  $V$  and electric field  $E$  are constant everywhere in a sphere of radius  $r$ . In the right most diagram, we assume that the electric field between the plates is uniform. So if potential  $V$  and distance  $d$  are not changing along the straight dotted line between and parallel to the plates, then the electric field  $E = V/d$  on this line is equal to that at point **P**. So potential and electric field depend on the distance  $d$  between the plates. Use formulas with  $d$  for charged plates. The curving electric field in the combined proton / electron diagram and outside the plates in the diagram to the right is not equal in strength. Remember equipotential lines mean that potential is the same, not necessarily that the electric field strength is equal. The electric field is only equal along equipotential lines where distance ( $d$  or  $r$ ) from the charge/charges is unchanging.

**Faraday Cage:** Any enclosed metal structure, even one made of chicken wire, acts as a Faraday Cage. Charges pile up on the outer surface of a metal enclosure. Due to a combination of  $q$  and  $r$  at any point within a box, cylinder, sphere, etc. the electric field is zero inside. This is why you are not electrocuted in a car or airplane if it is struck by lightning. Shielding in electronic means Faraday Cage.

**Conductors:** Substances in which charges can move freely. When a conductor is charged the individual charges pile up over the outside surface area of a conductor. **The electric field inside a charged conductor is zero.** Outside the surface the electric field drops according to the inverse square law. The relationship of electric field to radius, in a conductor, is shown in Fig 14.3.

**Insulator:** Substance that does not allow charges to move freely. Insulators can become charged. A plastic comb run through your hair is one example. But, the charges do not distribute over the surface area. Only the area affected becomes charged.

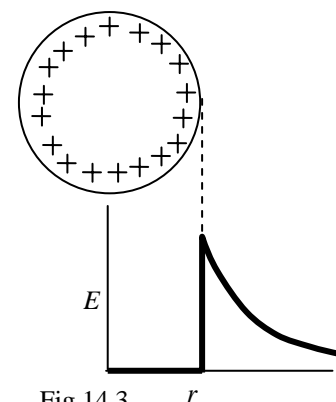


Fig 14.3

