Understanding Ratios

All About Pets

Lesson 17-1 Understanding Ratios

Learning Targets:

- Understand the concept of a ratio and use ratio language.
- Represent ratios with concrete models, fractions, and decimals.
- Give examples of ratios as multiplicative comparisons of two quantities describing the same attribute.

SUGGESTED LEARNING STRATEGIES: Interactive Word Wall, Visualization, Create Representations, Look for a Pattern

A **ratio** is a comparison of two quantities. Ratios can represent a comparison of part-to-part, part-to-whole, or whole-to-part. Ratios can be written as fractions, or using the word "to" or a colon.

Example A

Use the tags below. Find each of these ratios:

a. stars to bones

b. stars to total number of tags

c. total number of tags to bones

Write each ratio three different ways. State whether the ratio is a part-to-part, part-to-whole, or whole-to-part.

Solution:

a. stars to bones

part-to-part $\frac{\text{number of stars}}{\text{number of bones}} = \frac{4}{8}$; 4 to 8, 4:8

b. stars to total number of tags

part-to-whole $\frac{\text{number of stars}}{\text{number of tags}} = \frac{4}{12}$; 4 to 12; 4:12

c. total number of tags to bones

whole-to-part $\frac{\text{number of tags}}{\text{number of bones}} = \frac{12}{8}$; 12 to 8; 12:8

Try These A

Use ratios to compare the pet toys shown. Write each ratio three different ways. State whether the ratio is a part-to-part, part-to-whole, or whole-to-part.

a. balls of yarn to mice

b. white balls of yarn to total number of toys

c. gray mice to white mice

My Notes

MATH TERMS

Each part of a **ratio** is called a term. Terms can be:

- Numbers, such as 4 and 8: $\frac{4}{8}$, 4 to 8, 4:8
- Variables, such as x and y: $\frac{x}{y}$, x to y, x:y
- The product of a number and a variable, such as $3x$ and $9y$: $\frac{3x}{9y}$, $3x$ to $9y$, $3x$:$9y$

MATH TIP

Like fractions, ratios can sometimes be rewritten in lowest terms.

$\frac{4}{8} = \frac{1}{2}$, 1 to 2, or 1:2

$\frac{4}{12} = \frac{1}{3}$, 1 to 3, or 1:3

$\frac{12}{8} = \frac{3}{2}$, 3 to 2, or 3:2

My Notes

A ratio is also a multiplicative comparison of two quantities.

The ratio of circles to the total number of shapes below is $\frac{2}{5}$.

This means $\frac{2}{5}$ of all the shapes are circles and that for every 2 circles added, a total of 5 shapes will be added.

Suppose a set of shapes with the pattern above includes 8 circles. You know that $2 \times 4 = 8$, so multiply the number of shapes in the repeating part of the set (2 circles + 3 squares = 5 shapes) by 4 to find the total number of shapes when there are 8 circles: $5 \times 4 = 20$ total shapes.

MATH TIP

Like fractions, ratios can be written as decimals. The ratio $\frac{3}{4}$ is the quotient of $3 \div 4$ or 0.75.

Example B

Make sense of problems. In January, for every 3 cats adopted, 4 dogs were adopted. A total of 16 dogs were adopted. How many cats were adopted?

Step 1: Write a ratio comparing the number of cats to the number of dogs adopted.

$$\frac{\text{number of cats}}{\text{number of dogs}} = \frac{3}{4}$$

The number of cats adopted is $\frac{3}{4}$ times the number of dogs adopted.

Step 2: Multiply the ratio times the number needed to create an equivalent ratio showing 16 dogs.

$$\frac{3}{4} \times \frac{4}{4} = \frac{12 \text{ cats}}{16 \text{ dogs}}$$

Solution: 12 cats were adopted.

Check: Does the ratio of 12 cats to 16 dogs equal $\frac{3}{4}$?

$$\frac{12}{16} = \frac{12 \div 4}{16 \div 4} = \frac{3}{4}$$

Try These B

At the dog park on Monday, 2 dogs out of every 5 were terriers. A total of 20 dogs were at the park.

a. How many terriers were there? Explain how you got your answer.

b. The ratio of Irish terriers to the total number of terriers was 1:4. How many of the terriers were Irish terriers? Explain how you got your answer.

My Notes

Check Your Understanding

1. For a given ratio, how many equivalent ratios can be written? Explain your reasoning.
2. How can you check to see if the ratio 1:2 is equivalent to another ratio?
3. Find as many whole-number ratios equal to 50:100 as you can, using division.

LESSON 17-1 PRACTICE

4. Use ratios to compare the dog bowls shown. Write each ratio three different ways. State whether the ratio is a part-to-part, part-to-whole, or whole-to-part.

   a. white bowls to total number of bowls
   b. black bowls to gray bowls
   c. all bowls to bowls that are not gray

5. At the veterinarian's office, 4 animals out of every 5 seen were cats. A total of 35 animals were seen.
   a. How many cats were seen?
   b. The ratio of male cats to all cats seen was 6:7. How many of the cats seen were males?

6. There are twelve rabbits in a cage. The ratio of white rabbits to all rabbits is 3:4. How many white rabbits are in the cage?

7. **Make sense of problems.** Each veterinarian has seen 40 animals today. Two out of every 5 animals Vet A has seen have been dogs. Three out of every 8 animals Vet B has seen have been dogs. Which vet saw more dogs today? Explain your reasoning.

8. **Reason abstractly.** The ratio of red collars to black collars sold at one store is 9 to 10. In one month 30 black collars are sold. Is 57 a reasonable number for the total number of red and black collars sold that month? Explain your reasoning.

9. There are 15 black mice in a cage. The ratio of all mice to black mice is 5:1. How many mice are in the cage?

My Notes

Learning Targets:

- Make tables of equivalent ratios relating quantities.
- Use tables to compare ratios.
- Plot the pairs of values on the coordinate plane and describe the relationship.

SUGGESTED LEARNING STRATEGIES: Interactive Word Wall, Visualization, Create Representations, Identify a Subtask

Relationships that have **equivalent ratios** are called **proportional relationships**. All the columns in a **ratio table** show equivalent ratios.

MATH TERMS

Equivalent ratios are ratios that name the same number, just as equivalent fractions do.

Equivalent ratios are found by multiplying or dividing both terms of a ratio by the same number.

Example A

Reason quantitatively. A recipe for a homemade dog treat calls for a mixture of 8 ounces of oats to 12 ounces of finely chopped liver. Complete the ratio table.

	8 ÷ 4	8 ÷ 2		8 × 2	8 × 10
Oats (oz)	2	4	8	16	80
Liver (oz)	3	6	12	24	120
	12 ÷ 4	12 ÷ 2		12 × 2	12 × 10

a. How many ounces of liver are needed with 16 oz of oats?

Solution: 24 oz of liver are needed with 16 oz of oats.

b. How many ounces of oats are needed with 120 oz of liver?

Solution: 80 oz of oats are needed with 120 oz of liver.

c. Use the table to name four ratios equivalent to $\frac{8}{12}$.

Solution: The ratios $\frac{2}{3}$, $\frac{4}{6}$, $\frac{16}{24}$, and $\frac{80}{120}$ are equivalent to $\frac{8}{12}$.

Try These A

a. In one recipe for dog biscuits, the ratio of cups of water to cups of flour used is 3:9. Complete the ratio table

	3 ÷ 3		3 × 2	3 × 4	3 × 6	3 × 9
Water (c)		3	6	12		
Flour (c)	3	9			54	81
	9 ÷ 3		9 × 2	9 × 4	9 × 6	9 × 9

b. How many cups of water are needed with 81 cups of flour?

c. How many cups of flour are needed with 12 cups of water?

d. Use the table to name five ratios equivalent to 3:9.

My Notes

A relationship is proportional if the graph of the relationship is a set of points through which a straight line can be drawn and the straight line passes through the point (0, 0).

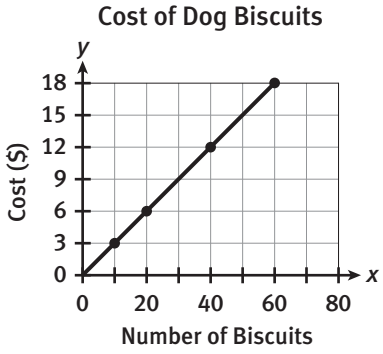
Example B

At the animal food store, 20 dog biscuits cost $6. Is the relationship between the number of biscuits and the cost proportional?

Step 1: Make a ratio table.

Number of Biscuits, x	10	**20**	40	60
Total Cost ($), y	3	**6**	12	18

Step 2: Graph the relationship between the number of biscuits x and the cost y.
Plot the ordered pairs (x, y) from the table: (10, 3), (20, 6), (40, 12), and (60, 18).



Solution: A line passes through all the points and through (0, 0). This means that the relationship is proportional.

Try These B

Graph each relationship in the My Notes section to the right. Determine if the relationship is proportional or not proportional. Explain your reasoning

a.

Number of Hours, x	2	4	6	8	9
Total Cost ($), y	15	25	35	45	50

b.

Number of Hours, x	2	4	6	8	9
Total Cost ($), y	6	12	18	24	27

My Notes

Check Your Understanding

1. How can you use a ratio table to find the value of x in the ratio x:20 if the ratio is equivalent to 5:2? Explain your reasoning.
2. Name two ways to determine if the x- and y-values in a table have a proportional relationship.

LESSON 17-2 PRACTICE

3. **Reason quantitatively.** The recipe for a homemade dog treat calls for a mixture of 2 eggs for every 8 cups of flour.
   a. Complete the ratio table.

Number of Eggs	1	2	6		
Cups of Flour		8		40	64

   b. How many eggs are needed with 40 cups of flour?
   c. How many cups of flour are needed with 6 eggs?
   d. Use the table to name four ratios equivalent to $\frac{2}{8}$.
   e. Which ratio is equivalent to 2:8 in lowest terms?
4. **Model with mathematics.** For every 4 days of dog sitting Julie charges $20.
   a. Complete the table to find the amount Julie should charge for 1, 2, and 8 days of dog sitting.

Number of Days, x	1	2	4	8
Total Cost ($), y			20	

   b. Graph the relationship between the number of days x and the cost y.
   c. Is the relationship between the number of days and the cost proportional? Justify your answer.
   d. Use your graph to determine how much Julie should charge for 6 days of dog sitting.
   e. Is 4:20 equivalent to 10:60? Explain using the graph.
5. Are $\frac{2}{3}$ and $\frac{5}{6}$ equivalent ratios? Justify your answer.
6. Are $\frac{2}{7}$ and $\frac{6}{21}$ equivalent ratios? Justify your answer.
7. Are $\frac{2}{4}$ and $\frac{3}{6}$ equivalent ratios? Justify your answer.