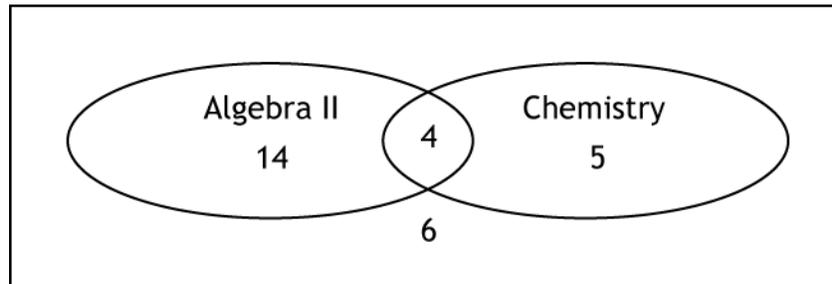


Probability: Determining Probabilities

II.A Student Activity Sheet 1: Using Venn Diagrams

Ms. Snow conducted a survey of her homeroom. She asked students what math course and what science course they were taking this semester. Below are the results.

Students in Ms. Snow's Homeroom



1. Analyze the data in the Venn diagram and list five facts about Ms. Snow's homeroom.

2. If a student is selected at random from Ms. Snow's homeroom, what is the probability that the student is taking Algebra II and Chemistry? Explain your reasoning.

Probability: Determining Probabilities

II.A Student Activity Sheet 1: Using Venn Diagrams

Students survey 758 spectators at a national championship tennis match. The survey results indicate the following:

- 421 are male,
- 256 have a two-handed backhand swing, and
- 176 of the people with a two-handed backhand swing are female.

Draw a Venn diagram and label the data.

6. What is the probability that a person selected at random from the survey group is male? Explain your reasoning.

7. What is the probability that a person selected at random from the survey group is female? Explain your reasoning.

Probability: Determining Probabilities

II.A Student Activity Sheet 1: Using Venn Diagrams

11. **REFLECTION:** Describe the characteristics of a situation that suggest the usefulness of a Venn diagram as a model of the situation.

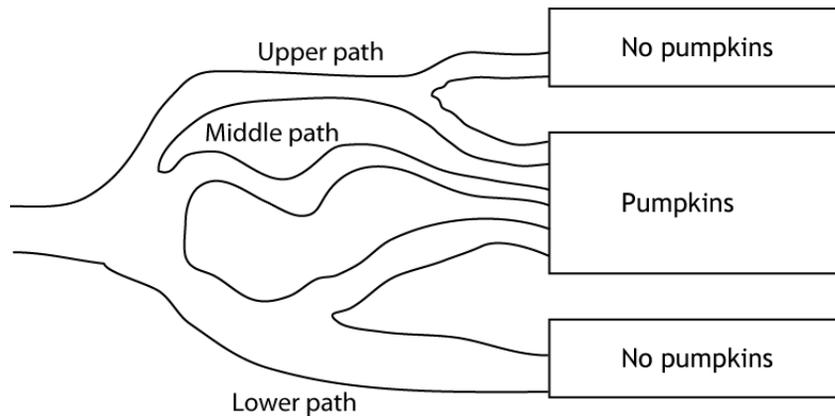
12. **EXTENSION:** Describe a situation that could be modeled with a Venn diagram and create the diagram. Use the diagram to determine the probability of at least two events that are possible in the situation.

Probability: Determining Probabilities

II.A Student Activity Sheet 2: Using Tree Diagrams

A church group in Washington state sells pumpkins every year to raise money for the children of their town. This year's crop, however, produced very small pumpkins. The group decided to construct a corn maze in a field and charge customers to walk through the maze. Customers can only walk forward. If the customers end up at an exit with pumpkins, they win a pumpkin. The church group asked some students to advise it on various possibilities of a customer getting a pumpkin.

Students were shown a simple maze as an example.



1. Make a tree diagram to show the group the possible paths customers might take, entering the maze on the upper, middle, or lower path and proceeding to an exit with or without a pumpkin.

How is this tree diagram different from others you have worked with before?

Probability: Determining Probabilities
II.A Student Activity Sheet 2: Using Tree Diagrams

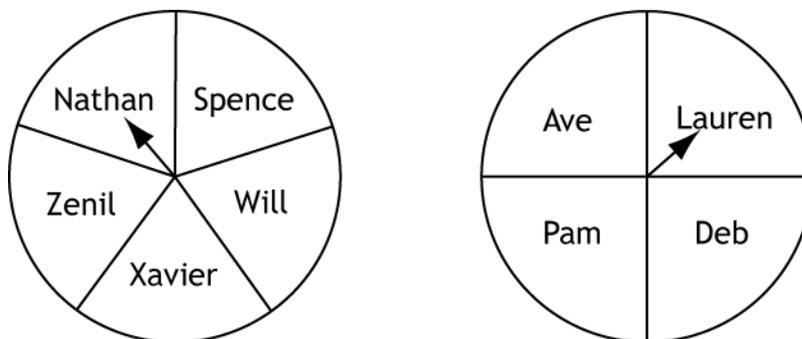
Create-a-Sandwich Menu

Bread	Meat	Cheese
White	Ham	American
Wheat	Turkey	Swiss
	Beef	Provolone
		Muenster

2. Create a tree diagram showing all possible sandwiches.

Probability: Determining Probabilities
 II.A Student Activity Sheet 2: Using Tree Diagrams

As president of the high school band, Catrina needs to pick a committee of 2 to accompany her each time she visits middle schools. The director told her that each committee had to consist of 1 boy and 1 girl; 5 boys and 4 girls volunteered to go. To be fair, Catrina makes a spinner with the boys' names and a spinner with the girls' names. Each time she schedules a visit, Catrina spins each spinner once to determine who goes with her. If a spinner lands on a line, she spins again.



9. Draw a tree diagram to show all the possible combinations of volunteers who might go with Catrina. How many outcomes are in the sample space?

Probability: Determining Probabilities

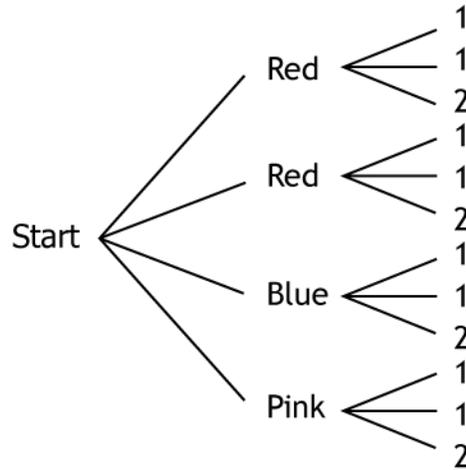
II.A Student Activity Sheet 2: Using Tree Diagrams

10. Are all the outcomes equally likely? What would make the outcomes not equally likely?
11. What is the probability that Nathan will be selected? Explain your reasoning. List the possible outcomes for 2-person committees that include Nathan.
12. If Ave decides she cannot go on a visit she is scheduled for, how does this change the probability for other boys or other girls to be selected? Explain your reasoning.

Probability: Determining Probabilities

II.A Student Activity Sheet 2: Using Tree Diagrams

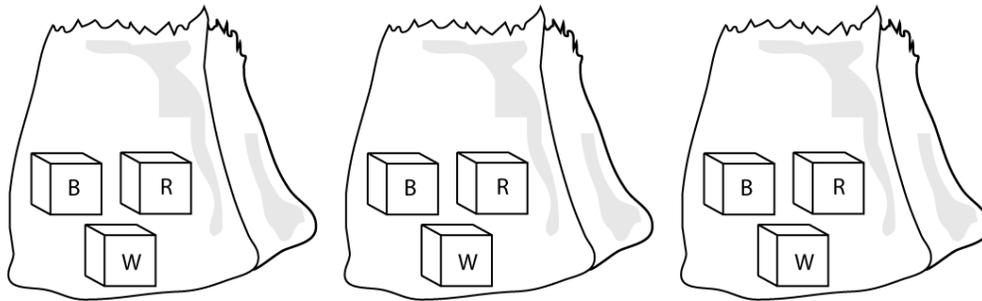
13. **EXTENSION:** Create a scenario for the tree diagram below. Write three probability problems that can be answered using the tree diagram, and then provide the answers.



Probability: Determining Probabilities

II.A Student Activity Sheet 2: Using Tree Diagrams

14. **EXTENSION:** Mr. Silvas surprises his students with a probability challenge that will determine whether they will take a quiz. He puts three cubes in each of three paper bags: a red cube, a white cube, and a blue cube. He divides the class into four groups of students and tells them that each group will draw one cube from each bag. Each group is challenged to come up with a rule to determine whether they will take the quiz based on if their selection of cubes matches or does not match a criteria they identify in advance. Two groups establish a rule for a cube combination that leads to the outcome of taking the test, and the other two groups establish a rule for a cube combination that leads to the outcome of not taking the test.



- a. Draw a tree diagram to show the sample space.

Probability: Determining Probabilities

II.A Student Activity Sheet 2: Using Tree Diagrams

- b. Evaluate each group's decisions. Determine the probability that the outcome chosen by each group will occur.

Group 1: If the group ends up with a red cube, a white cube, and a blue cube (order does not matter), its members take the test.

Group 2: If the group ends up with at least two red cubes, its members take the test.

Group 3: If the second cube selected is white, the group does not take the test.

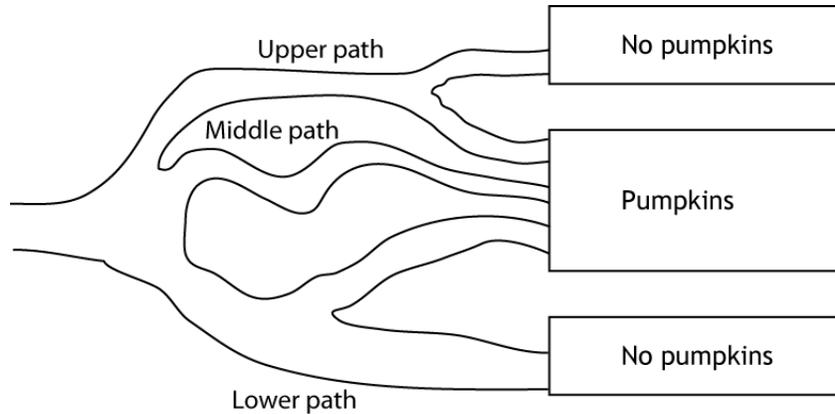
Group 4: If the group ends up with two cubes of the same color, its members do not take the test.

- c. Suggest another outcome. What is the probability that your outcome will occur?

- d. Which group would you join if you got to choose? Why?

Probability: Determining Probabilities
 II.A Student Activity Sheet 3: Using Area Models

Recall the rules for the pumpkin problem you looked at in Student Activity Sheet 2 with a tree diagram: A customer walks forward through the maze with the possibility of winning a pumpkin; this depends on whether there is a pumpkin at the exit where they come out of the maze. One student, Kyra, draws an area model that demonstrates the probability of getting a pumpkin using this maze.



Kyra explains, “As customers enter the maze, what are the path possibilities? They can take the upper path, middle path, or lower path. These three options lead you to divide the area model into three sections. Next, look at each path and decide how to divide each section. The upper path divides into two paths, the middle path stays one path, and the lower path divides into two paths. Next, decide which part of the model of the maze gets a pumpkin and which part does not.”

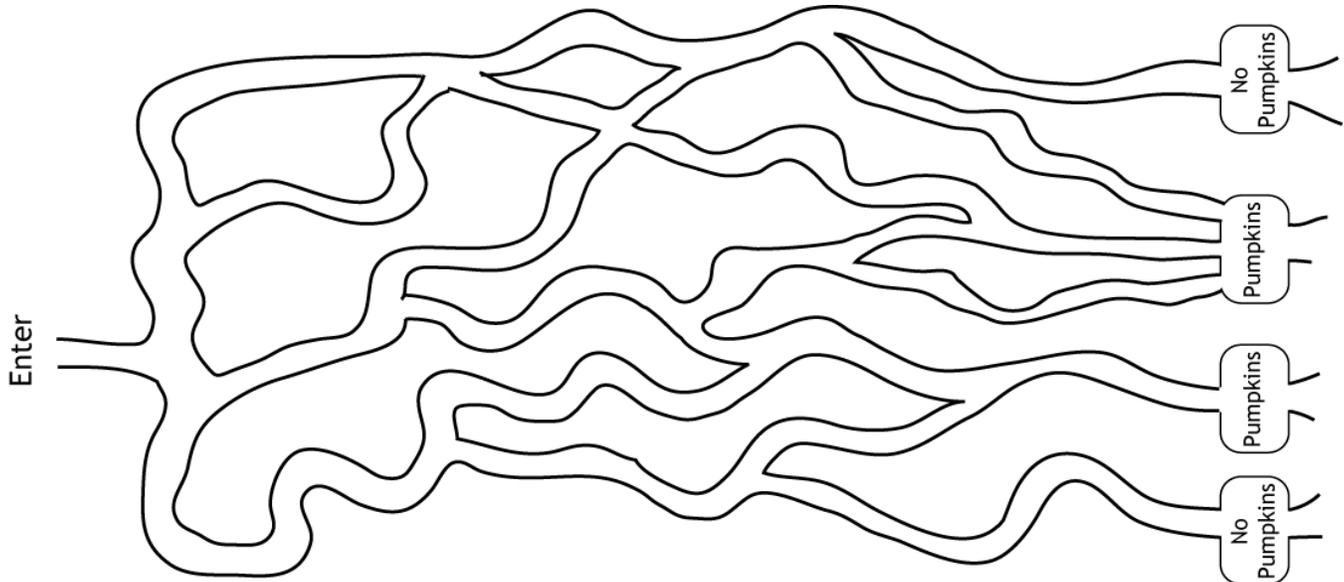
Upper path	No pumpkins	Pumpkins
Middle path	Pumpkins	
Lower path	Pumpkins	No pumpkins

The probability of getting a pumpkin is $\frac{2}{3}$, and the probability of not getting a pumpkin is $\frac{1}{3}$.

Probability: Determining Probabilities

II.A Student Activity Sheet 3: Using Area Models

Below is a drawing of a second maze the church decided to construct.



3. Use an area model to determine the theoretical probability of a customer taking home a pumpkin.

Probability: Determining Probabilities

II.A Student Activity Sheet 3: Using Area Models

6. If you do not want to give away too many pumpkins, at which three exits do you put the pumpkins? Explain your reasoning.
7. **REFLECTION:** What would a maze look like with equally likely outcomes? What would the corresponding area model look like? What is an advantage of the area model?

Probability: Determining Probabilities
 II.A Student Activity Sheet 3: Using Area Models

8. **EXTENSION:** Because this year’s maze was such a success, Emma draws a plan for next year. Draw a maze that fits her plan.

- Y–The customer gets a pumpkin.
- N–The customer does not get a pumpkin.

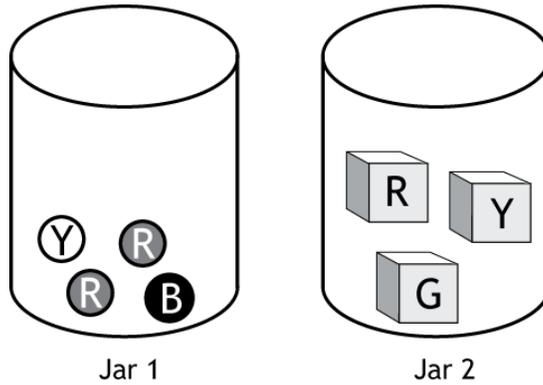
Y	N	Y	Y
N		Y	
N	N	Y	

a. Find $P(Y)$. Explain your reasoning.

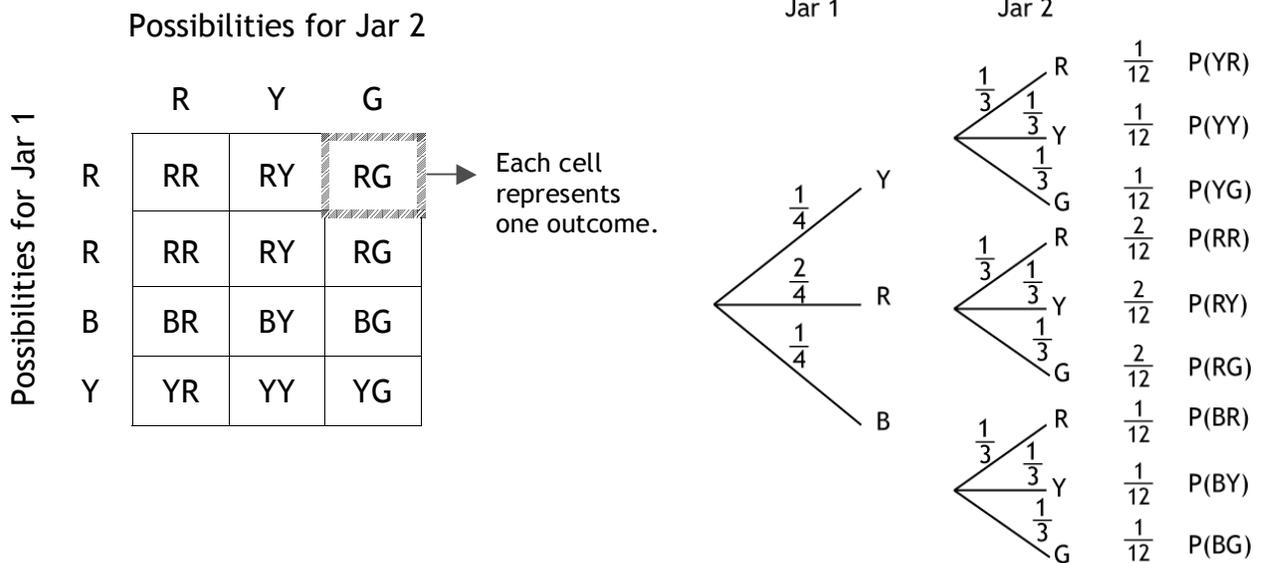
b. Find $P(N)$. Explain your reasoning.

Probability: Determining Probabilities
 II.A Student Activity Sheet 3: Using Area Models

9. **REINFORCEMENT:** You can use an area model to analyze probability situations that involve more than one stage. The following example involves selecting a marble (yellow, red, or blue) from one jar and a cube (yellow, red, or green) from another jar.

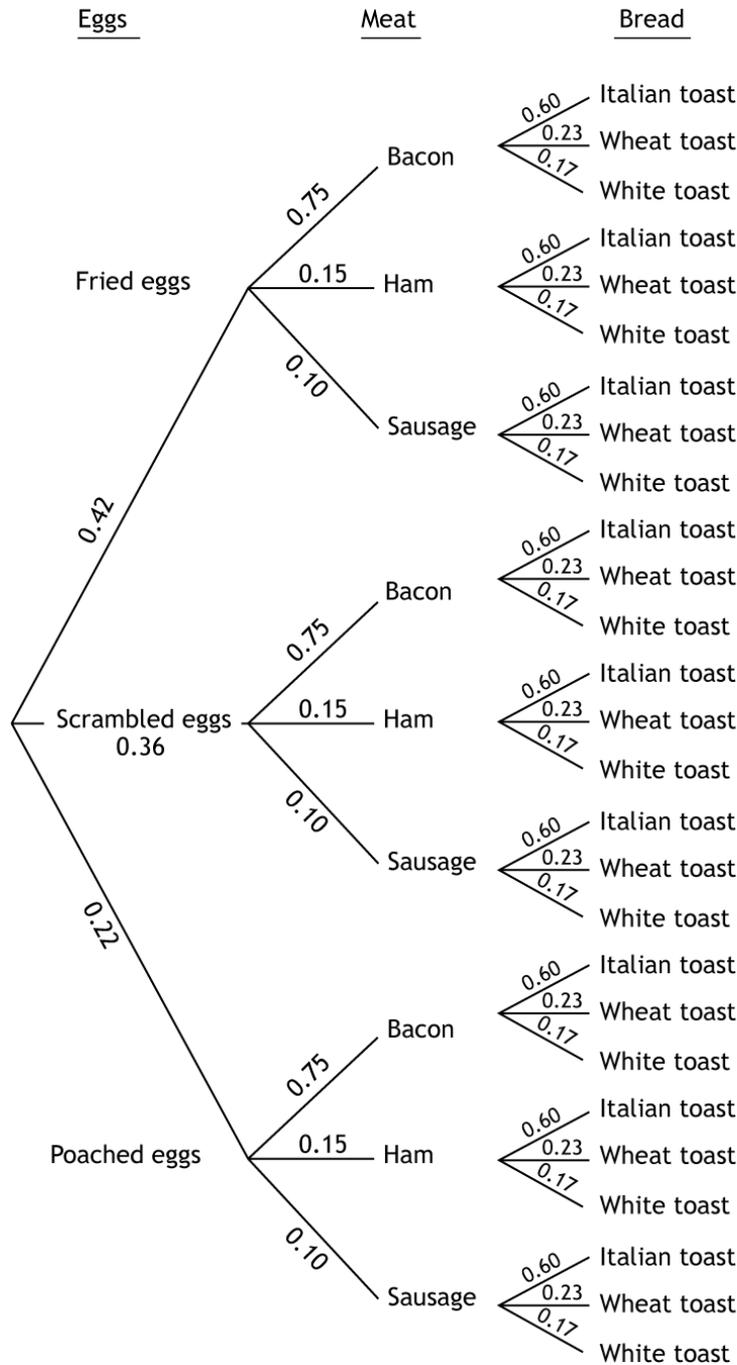


Compare the following Punnett square to a tree diagram representing the same scenario.



Probability: Determining Probabilities
 II.A Student Activity Sheet 4: All-American Breakfast Choices

Fallon's Bistro



**The American Breakfast
Only \$3.99**

Probability: Determining Probabilities

II.A Student Activity Sheet 4: All-American Breakfast Choices

6. What is the probability that the next customer to get the American Breakfast orders either Italian toast or white toast? Explain your reasoning.
7. **REFLECTION:** What would an area model look like that models the choices at Fallon's Bistro?
8. **EXTENSION:** Consider the two options below. Choose one option, and respond to its questions.
- Option 1:* If Fallon's Bistro adds pancakes or waffles as a choice to the American Breakfast, how does this addition affect the sample space? Where did you add the new choice? Why did you add it there? Explain your reasoning.
- Option 2:* What happens if hard-boiled eggs are added to the egg choices of the American Breakfast? How does this addition affect the sample space? Explain your reasoning.

Probability: Everyday Decisions Based on Probabilities

II.B Student Activity Sheet 5: Probability in Games

Victoria is playing a new video game in which the object is to find hidden treasures. To do so, she must travel through several levels, clashing with guards and watchdogs. In one part of the journey, Victoria must pass through two gates (Gate 1, then Gate 2) to get to the next level.

- The chance that Gate 1 is open is 20%.
- The chance that Gate 2 is open is 30%.
- The game designer has programmed the gates so that the probability of both being open at the same time is 0.1.

Draw a model of the situation to help you answer Questions 1-5. Explain why you chose the particular type of model from among the various probability models.

Probability: Everyday Decisions Based on Probabilities

II.B Student Activity Sheet 5: Probability in Games

4. What is the probability that neither gate is open when Victoria reaches this part of the game? Explain your reasoning.

5. What is the probability that Victoria finds exactly one gate open?

Probability: Everyday Decisions Based on Probabilities

II.B Student Activity Sheet 5: Probability in Games

Victoria encounters another challenge in the game. If she zaps a target in one try, Victoria gets a chance to capture a bonus shield. To capture the bonus shield, she must hit a second target in one try. Victoria can hit a target in one try an average of 60% of the time.

Draw a model of the situations to help you answer Questions 6-8.

6. What is the probability that Victoria hits the first target? Explain your reasoning.

7. What is the probability that Victoria captures the bonus shield? Explain your reasoning.

Probability: Everyday Decisions Based on Probabilities

II.B Student Activity Sheet 5: Probability in Games

8. What is the probability that Victoria hits the first target and does not hit the second target to capture the shield? Explain your reasoning.
9. **REFLECTION:** How would you advise your friends who might be interested in playing a new video game?
10. **EXTENSION:** Create two probability situations that use conditional probability. Describe the outcomes for these situations.

Probability: Everyday Decisions Based on Probabilities

II.B Student Activity Sheet 5: Probability in Games

11. **EXTENSION:** Create a probability situation that uses compound events. Describe the outcomes for the situation. What type of model do you think will help you answer questions about the situation?

Probability: Everyday Decisions Based on Probabilities**II.B Student Activity Sheet 6: Driving and Risk**

Javier will be a high school senior next year. He wants to get a vehicle to celebrate his graduation. Javier's mother researched vehicle safety and found that 1 of every 6 teenage drivers was involved in some kind of accident. While talking to his math teacher, Javier mentioned that he did not think the risk was high enough to be concerned. Javier decided to survey 500 students, 230 of whom were male, to help him convince his mother to allow him to get a vehicle. No student has both a car and a motorcycle.

The following are the data from Javier's survey:

	Car	Motorcycle
Males with vehicle	150	23
Males involved in accident	40	6
Females with vehicle	225	10
Females involved in accident	15	4

1. Draw a Venn diagram and a tree diagram of the data.

Probability: Everyday Decisions Based on Probabilities

II.B Student Activity Sheet 6: Driving and Risk

5. What probability model would you advise Javier to use when he tries to convince his mother?
6. **REFLECTION:** List some advantages and disadvantages for each type of model used in this problem.
7. **EXTENSION:** Research your favorite vehicle's safety measures and its likelihood of being involved in an accident. Prepare a short presentation of your findings.
8. **EXTENSION:** Investigate other applications of decision making in situations involving risk—both situations where the risk is known (that is, you have some sort of data to determine mathematically how likely it is to occur) and situations where risk information is not known ahead of time. Examples include purchasing insurance, increasing or decreasing premiums on insurance, and not being eligible for insurance because of high risks. Prepare a short presentation of your findings to share with the class.

Probability: Everyday Decisions Based on Probabilities

Student Activity Sheet 7: Stocks and Risk

Javier will be a senior next year in high school. To celebrate his graduation, his grandmother gave him a sizeable amount of money. Since he has scholarships for college, Javier decided to investigate investing the money in stocks. He talked to a consultant, who explained some of her investment types and their returns as compared to the market average. Javier's choices were to invest in high- or low-capital stock and domestic or international.

The following is a summary of the data for the last quarter:

	Domestic	International
High-capital stock	250	40
Low-capital stock	200	10
High-capital stock with below-average showing	40	6
Low-capital stock with below-average showing	15	4

1. Draw a Venn diagram and a tree diagram of the data.

Probability: Everyday Decisions Based on Probabilities

Student Activity Sheet 7: Stocks and Risk

- Using the data, estimate the probability that a high-cap domestic stock performs at or above market average.
- What type of stock would you suggest Javier invest in if he wants to limit his risk of the stock performing below market average. Why?
- Contrast the performance of international and domestic stocks as compared to the market average.

Probability: Everyday Decisions Based on Probabilities

Student Activity Sheet 7: Stocks and Risk

5. Write three facts that help Javier decide in which types of stocks to invest.

Did you use the Venn diagram or the tree diagram to find the facts? Which was most helpful? Why?

6. **REFLECTION:** List the advantages and disadvantages of the types of models used in this problem.

Probability: Everyday Decisions Based on Probabilities

Student Activity Sheet 7: Stocks and Risk

7. **EXTENSION:** Javier decides to invest \$100 in high-cap domestic stocks. He narrows his choice to stock in Txtel or stock in TxMart, both of which have the same price today. Txtel is a growth stock in the technology sector. It tends to be more volatile. Based on financial analysis, this stock may go down \$10 with a probability of 0.4 or up \$8 with a probability of 0.6 in the next quarter. TxMart is a value stock in the domestic goods sector. It is projected to drop \$4 with a probability of 0.2 or go up \$2 with a probability of 0.8. Which stock do you recommend? Why?
8. **EXTENSION:** Investigate other applications of decision making in situations involving risk—both situations where the risk is known (that is, you have some sort of data to determine mathematically how likely it is to occur) and situations where risk information is not known ahead of time. Examples include purchasing insurance, increasing or decreasing premiums on insurance, and not being eligible for insurance because of high risks. Prepare a short presentation of your findings to share with the class.

Probability: Everyday Decisions Based on Probability

II.B. Student Activity Sheet 8: Choosing Classes

Four teachers offer Zane’s favorite computer class at different times during the day. The school counselor asks Zane if he prefers a morning or afternoon class. Below is a list of teachers and the periods they teach this class. The morning classes are 1st, 2nd, and 3rd periods, and the afternoon classes are 4th, 5th, and 6th periods.

- Mr. Nelson—2nd, 4th, 5th, 6th
 - Ms. Trevino—1st, 2nd, 3rd, 4th, 5th
 - Mr. Garza—1st, 3rd, 4th, 6th
 - Ms. Jones—1st, 2nd, 3rd, 6th
1. Before answering the counselor’s question, Zane wants to list all the possibilities so he can make a choice that gives him the highest probability of getting a teacher he prefers. Create a table and a tree diagram that illustrate the possibilities.

Probability: Everyday Decisions Based on Probability

II.B. Student Activity Sheet 8: Choosing Classes

5. Because this is a required class for all students and Mr. Garza's classes are filled, the school adds another teacher, Ms. Lopez. She will teach 1st and 6th periods. Does this fact affect the probability of getting Mr. Nelson in the morning?
6. While Zane is calculating probabilities so he can make his decision, the class offerings change. (Mr. Garza's classes fill, and Ms. Lopez is added.) If Zane requests an afternoon class, what is his probability of getting Ms. Trevino for 4th period?

Probability: Everyday Decisions Based on Probability

II.B. Student Activity Sheet 8: Choosing Classes

7. **REFLECTION:** What other factors might influence Zane's class selection? Would these factors be reflected in the diagram or data? If so, describe the new diagram or set of data.
8. **EXTENSION:** Meet with the person on your campus who is responsible for creating the master schedule. Prepare a set of questions similar to the ones that Zane faced and record the responses. Share your findings with the class, including at least one decision you can make based on those findings.

Probability: Expected Value

II.C Student Activity Sheet 9: Binomial Probability and Expected Value

At the National Baseball Batting Contest, the organizers have set up game booths for the contestants. Richard wants to win a large stuffed animal. The rules of the game are as follows:

- You are pitched 5 fastballs, and you must hit them into a fair zone to count.
 - If you successfully hit all 5 pitches, you win a large stuffed animal.
 - If you successfully hit 3 or 4 pitches, you win a small stuffed animal.
 - If you successfully hit 1 or 2 pitches, you win a bat-shaped pencil.
 - If you miss all the pitches, you do not win a prize.
 - The game costs \$3 to play (each set of 5 fastballs).
1. What are the possible outcomes that Richard can have on his 5 swings? Explain or show how you arrived at your answer?

Probability: Expected Value

II.C Student Activity Sheet 9: Binomial Probability and Expected Value

2. What is the probability of Richard successfully hitting the following? Justify your answers.

a. All 5 pitches?

b. 4 pitches?

c. 3 pitches?

d. 2 pitches?

e. 1 pitch?

f. 0 pitches?

Probability: Expected Value

II.C Student Activity Sheet 9: Binomial Probability and Expected Value

3. Suppose Richard tries the game 10 times. How many times do you expect him to successfully hit the given amounts of times below? Justify your answers.

a. 5

b. 4

c. 3

d. 2

e. 1

f. 0

Probability: Expected Value

II.C Student Activity Sheet 9: Binomial Probability and Expected Value

9. **EXTENSION:** Research the use of Pascal's triangle. Prepare a short class presentation regarding your findings. Be sure to include the connection to combinatorics.

10. **EXTENSION:** Investigate the probabilities and associated expected values related to winning a state or local lottery game. Prepare a short class presentation about the probable outcomes of event of winning the lottery.

Probability: Expected Value

II.C Student Activity Sheet 10: Expected Allowance

Yvonne gets \$15 a week for allowance. She also loves to play basketball. Yvonne wants to convince her father to try something new with her allowance based on her basket-shooting talent, hoping that it will increase what she receives. Yvonne suggests that instead of getting \$15, she attempt shooting baskets each week for her allowance.

- If she misses the first basket, she gets only \$5.
- If she makes (succeeds with) the first basket, she gets \$15 and a chance to make another basket for an additional \$10.

Yvonne can make a basket 40% of the time

1. To help Yvonne's father decide whether to use the new allowance plan, find the probability of Yvonne making 0 baskets, 1 basket, and 2 baskets. Justify your reasoning with an appropriate model.

Probability: Expected Value

II.C Student Activity Sheet 10: Expected Allowance

2. How many times in a year do you expect Yvonne to get \$5? \$15? \$25?

3. How much allowance should Yvonne expect to receive in a year?

4. Should Yvonne's father accept the deal? Justify your reasoning.

Probability: Expected Value

II.C Student Activity Sheet 10: Expected Allowance

Yvonne practiced shooting baskets all year. She can now make a basket 60% of the time. Yvonne offers her father the same deal.

5. What is the probability of Yvonne making 0 baskets? 1 basket? 2 baskets?

6. How much money do you expect Yvonne to receive in a year?

Probability: Expected Value

II.C Student Activity Sheet 10: Expected Allowance

Reinforcement

Yvonne's younger sister, Lisa, wants her father to offer her the same deal. Lisa can make a basket 20% of the time.

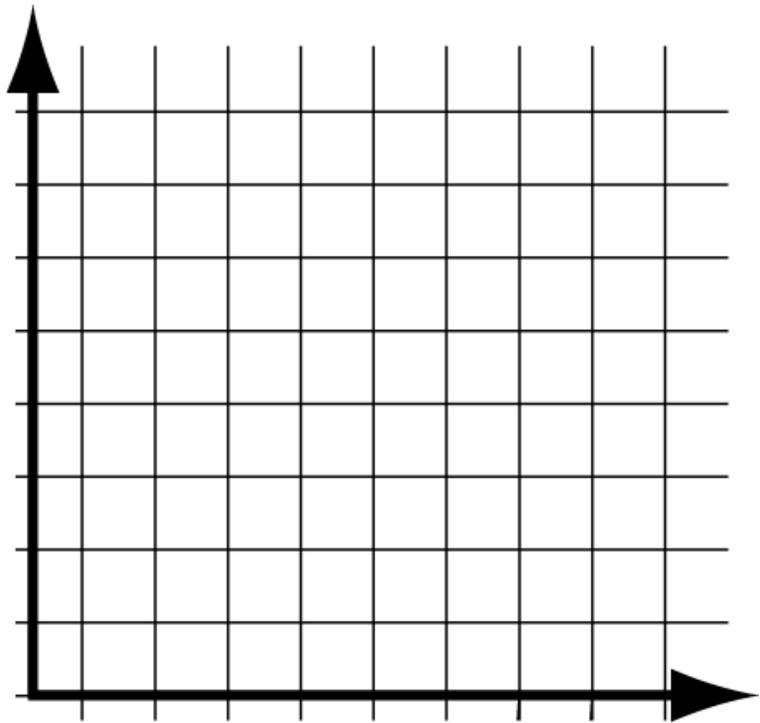
7. How many times do you expect Lisa to receive \$5? \$15? 25? Justify your reasoning using an appropriate model.

Probability: Expected Value

II.C Student Activity Sheet 10: Expected Allowance

8. You have now looked at Yvonne’s situation (with a 40% shooting percentage) and Lisa’s (with a 20% shooting percentage) to determine what percent of the time they are each likely to receive \$5, \$15, and \$25 in allowance. Make a table and a graph that show the relationship between these shooting percentages and the amount of allowance earned, as well as other possible shooting percentages related to the amount of allowance earned. Describe how this relationship changes based on a person’s shooting percentage.

Percentage of Making a Basket	Amount of Allowance			
	\$5	\$15	\$25	Average Allowance Per Week



Probability: Expected Value

II.C Student Activity Sheet 10: Expected Allowance

9. Lisa's ability to make a basket improved to 30%. Using your graph, what average weekly allowance should she expect? What is the actual average amount Lisa should expect?
10. If Yvonne wants to earn an average weekly allowance of \$15, what percentage of baskets must she make?
11. Yvonne's father figured out that he had been giving Yvonne \$20 per week for her allowance. What percentage of the baskets is Yvonne making? What information did you use to answer this question?

Probability: Expected Value

II.C Student Activity Sheet 10: Expected Allowance

12. **EXTENSION:** Research popular basketball players or teams to find current percentages associated with making free throws, 2-point shots, and 3-point shots. Create a geometric probability model of a scenario using the player or team to make the given shot in a particular situation. Provide explanations as needed for sharing with others in the class.

(Note: Basketball statistics do not break out the 2-point percentage in its own category in the same way that free throws and 3-pointers are broken out. Instead 2- and 3-point goals are combined in the Field Goals Made [FGM] category. Determine the number of 2-point shots made by subtracting the 3-point shots made from the FGM.)

13. **MINI-PROJECT:** Design a carnival game that is of interest to your age group.

- Make a scale model of the game.
- Include a report with your model that details the following:
 - a. Experimental and theoretical probability of winning the game
 - b. Expenses
 - c. Expected payoff
 - d. Profit
 - e. Rules of the game
 - f. Prizes and how to earn the prizes
 - g. Why the game should be selected for the carnival