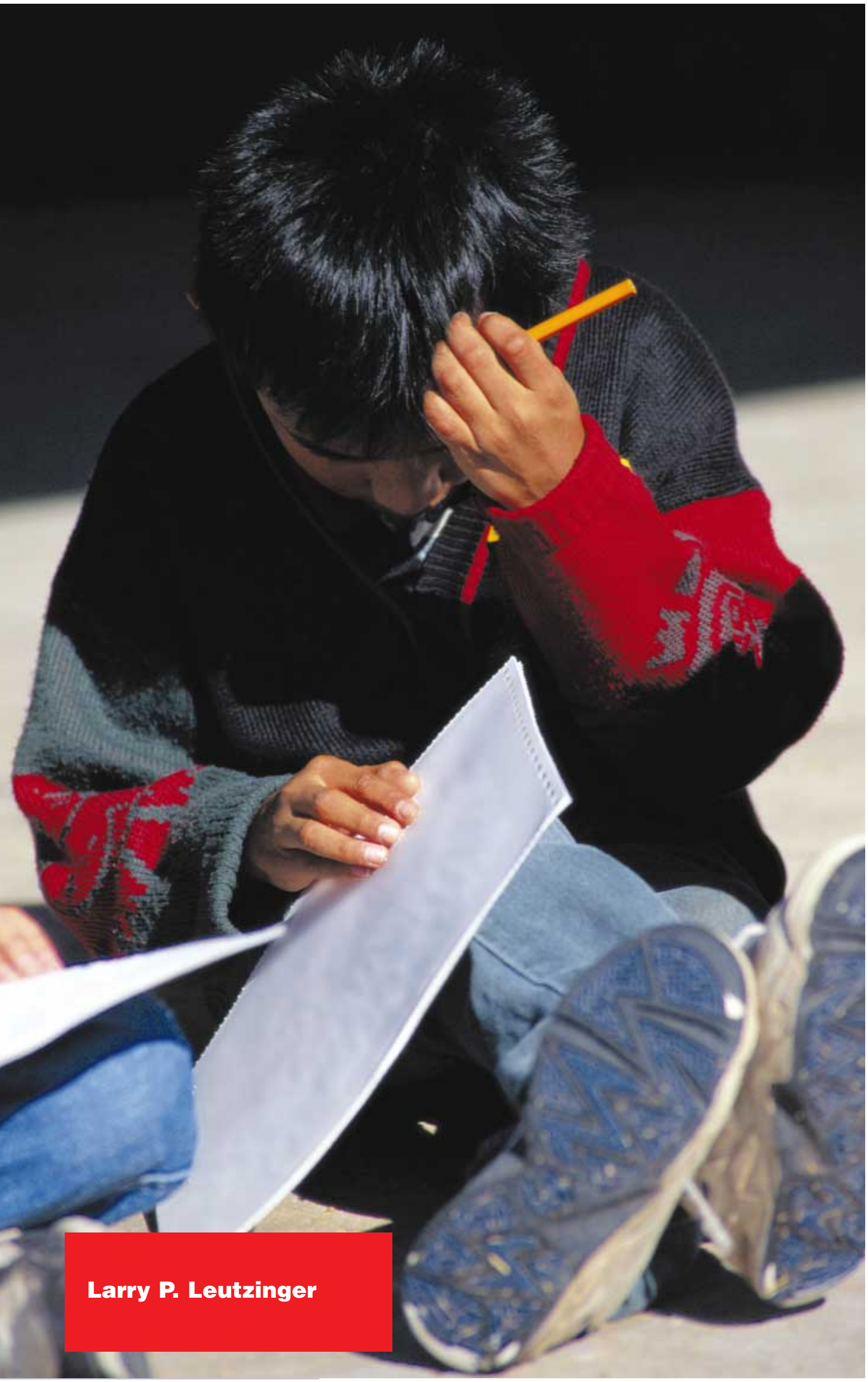
Developing Thinking Strategies for **Addition Facts**



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Each child in a second-grade class was asked to determine the answer to 7 + 9. The following are ways of thinking that certain children used:

Casey: (using counters) 1, 2, 3, 4, 5, 6, 7. [Pause] 1, 2, 3, 4, 5, 6, 7, 8, 9. [Pause] 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16.

Alex: (using fingers) 10, 11, 12, 13, 14, 15, 16. The answer is 16.

Lindsey: 9 + 9 = 18, and this is 2 less, so the answer is 16.

Marcus: 7 + 7 = 14, 15, 16. The answer is 16.

Jamie: 7 + 9 is the same as 8 + 8, so the answer is 16.

Latesia: 7 + 9 is the same as 10 + 6, so the answer is 16.

Jonathon: 7 + 10 is 17, so this is 1 less, or 16.

In any first- or second-grade class, children will use different ways to figure out answers to basic facts. All the ways stated here yield the correct answer for 7 + 9, but they vary in efficiency and the amount of reasoning used by the children. Casey's way was simply to count both amounts and then the total, which is very time-consuming. Alex used the counting-on strategy, which is faster than just counting the entire total. The other children used thinking strategies in which they figured out a fact that they did not know from one that they already knew. They derived or reasoned out harder facts from easier ones.

For children to develop mastery of addition facts, these efficient thinking strategies must be mentally accessible. Research clearly shows that children's facility in basic facts is enhanced by their first developing effective thinking strategies (Cobb and Merkel 1989; Thornton 1990). It should be a goal of each first- and second-grade teacher that every child in her or his classroom have available at least one effective strategy for any addition fact.

Teaching basic facts has always been an important component of any successful mathematics program. Certainly, mental mathematics and estimation are difficult without a mastery of basic facts. Many recommendations made in the *Curriculum and Evaluation Standards for School Mathematics* (NCTM 1989) assume that children have fluency with the basic facts:

> Children should master the basic facts of arithmetic that are essential components of fluency with paper-and-pencil and mental computation and with estimation. At the same time, however, mastery should not be expected too soon. Children will need many exploratory experiences and the time to identify relationships among numbers and efficient thinking strategies to derive answers to unknown facts from known facts. Practice to improve speed and accuracy should be used but only under the right conditions; that is, practice with a cluster of facts should be used only after children have developed an efficient way to derive answers from those facts. (NCTM 1989, 47)

In the past, learning basic facts was often overemphasized, with too much class time spent on repetitive practice and too little time spent on exploratory experiences that gave children opportunities to develop efficient thinking strategies, such as counting on, using doubles, and making ten. The following activities have two features: (1) they are structured to help children develop the use of specific thinking strategies for basic facts, and (2) they involve other topics, such as probability, spatial sense, and money, and offer rich opportunities for problem solving, reasoning, and communication.

Counting-On

The ability to count on is a vital skill for all children. This skill requires children to start from a number other than 1 and count on from there. Alex used this strategy for 7 + 9 in the opening scenario. Children who are successful at learning this strategy can use it to count on from larger numbers, such as 49 + 3 or 2 + 314. The children simply start with the larger number and count on using the smaller. This strategy is most effective when one of the addends is small, say, less than 4, for example, 1 + 7, 8 + 2, 3 + 9, and so on. Alex had a tendency to use it for numbers larger than 4; with such numbers, counting on is not as efficient.

Although counting on is a relatively easy strategy to learn and implement, young children are so used to counting everything from 1, often rotely, that they may at first be reluctant to count on. They need to be presented with many instances for which counting on is appropriate. Children could be asked to start at 6 and count on 2, then model that situation with counters. They could also count out seven counters, cover those with a paper with a "7" written on it, then place three more counters next to the paper and determine how many in all. The following activity gives children additional opportunities to develop the counting-on strategy.

Activity 1—counting on

K–4 NCTM Standards involved: Problem solving, reasoning, number sense, concepts of whole-number operations, statistics and probability, patterns and relationships

Materials: One copy of **figure 1** for each group of three children; two dice for every three children—one die marked with the numerals 4, 5, 6, 7, 8, and 9; the other marked as follows:

If blank dice are not available, use two sets of cards with the numerals 4–9 on them and four sets of cards with

on them. All the 4–9 cards are placed in one pile and all the

cards in another.

FIGURE 1

Counting On

Roll the two dice 30 times.
Add the numbers shown on the dice.
What total will come up the most?
Write your guess here. _______
Record the total for each roll by placing a tally mark by the proper number.

<u>Total</u>		<u>Total</u>	
1		10	
2		11	
3		12	
4		13	
5		14	
6		15	
7		16	
8		17	
9		18	

Put your results on the chalkboard.

Check each group's results.
How do they compare with yours?
Who is right?

Before starting the activity, the teacher should ask the children to decide what possible totals could be rolled or drawn and to predict which of these totals they think will come up the most. The children then share their selections and the reasons they used to determine them.

Children in some classes who have used this activity have stated the following:

"8 because it's in the middle between 5 and 13 [*sic*]." [5 being the lowest number possible and 12 being the highest]

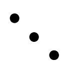
"7 comes up a lot with dice." [Did not realize that the number pairs here are different from those on standard dice.]

"10 is an important number."

"9, since you can get 9 with 6 + 3 or 7 + 2 or 8 + 1."

Although none of these answers is complete, they indicate some level of understanding of the probabilities involved. The teacher can pursue an answer and the reason in a way appropriate to the age and ability of the children in the class, or the guesses can simply be recorded and checked later.

Children should work in groups of three. One child rolls the dice or draws a card from each pile, another determines the total, and the third records the total with a tally mark next to the proper number on the sheet (**fig. 1**). For instance, a roll of 6 and



is recorded as a total of 9. The children should change duties after each ten rolls of the dice. If the children are not counting on to find the total, the teachers should encourage them to do so.

After each group rolls the dice thirty times, one member should record the group's tallies on a large chart drawn on the chalkboard. The teacher should encourage the children to group tally marks by fives when they are recording the results. The children then count the number of tally marks next to each total, compare the numbers, and determine how many more tally marks occurred for certain totals.

The differences in the various groups' results should be discussed. Why are the results for each group different? Which group is right? Opportunities arise to discuss informally the ideas of probability and chance.

The children report what total their group guessed would occur the most. Did any of the groups guess correctly? If the dice were tossed fifteen times, how many totals of 10 would be expected? How many totals of 5? Of 11? Of 12? Each group makes its guess then rolls to check. Continuing to use these dice or cards helps children develop the counting-on skill.

Using Doubles

A powerful strategy for children to possess is using facts that they know to assist them in determining the answer to facts that they do not know. Easy facts for children are the doubles, such as 4 + 4 and 6 + 6, in which both addends are the same. Children can use the doubles facts to help them find the answers to related facts. For instance, for the fact 6 + 7, a child could think that 6 + 6 is 12, so 6 + 7 is 13. In the opening vignette, Marcus used this type of strategy for 7 + 9. Notice that he counted on 2. Lindsey used the double 9 + 9 and counted back 2. Jamie used a double to find the answer to 7 + 9 by compensating. He thought of taking 1 from the 9 and adding 1 to the 7, making 8. The strategy of using a double works best when the two addends are close together. Some children discover that when the addends are exactly two apart, a good strategy is "double the skipped number."

The doubles strategy can be applied to larger numbers, as well. But it is important to remember that the children must know the answer to a double that is close to the two addends. For instance, 25 + 26 is a combination that many children could solve mentally, since they know that 25 + 25 is 50 and so 25 + 26 is one more, or 51. The strategy might not work as well for 47 + 48, since children might not know that 47 + 47 is 94. They might know, however, that 45 + 45 is 90 and then count 2 and 3 more to arrive at 95, or realize that 50 + 50 is 100 and count back 5. Just making children aware that they can use known facts in this way may stimulate them to use a similar strategy in other instances. If they know that 7 + 3 is 10, it is easy to determine the answer to 7 + 4.

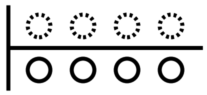
The following activity allows the children to practice using doubles to determine related facts.

Activity 2—using doubles

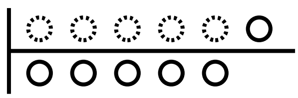
K–4 NCTM Standards involved: Problem solving, reasoning, number sense, concepts of whole-number operations, geometry, and patterns and relationships

Materials: One Reflecta, Mira, or mirror; counters; and a copy of **figure 2** for each pair of children

Before starting the activity, the teacher should allow the children to place counters in front of the Reflecta or Mira and see that both the objects and their reflections are visible. (See the diagram.) The Reflecta or Mira doubles the total number of objects in front. Children tell how many objects they see in all and explain their thinking. The teacher should encourage responses like 4 and 4 is 8 or double 4 is 8.



The children should place objects behind the Reflecta or Mira, as well. (See the diagram.) These objects are not reflected but are added onto the total. For the objects in this example, the children might respond, "5 + 5 is 10, and 1 more is 11."



If a mirror is used, the objects behind the mirror must be placed at such a distance that they are visible when viewed from in front of the mirror.

In groups of two, children are next challenged to find different ways to see thirteen objects. Both children determine the total, and one child records the results on the page. See **figure 2.**

The children next share their ways of arranging the counters to see 13, reporting whether they see any patterns in their results. Interestingly, in one class a child placed 6 counters in front of the Reflecta and then placed the Reflecta across the middle of another counter. When he looked, he saw 6 1/2 counters and their reflections, 13 in all.

Next the children are asked in how many different ways they could arrange the objects to see 12, 15, and so on. They might show all the ways for totals from 10 to 20 and record their results in a table. Continuing to use the Reflectas or Miras develops the skills of doubling and adding onto a double.

FIGURE 2

Using Doubles

How many different ways can you see 13 objects? Place some objects in front of the Reflecta and some behind. (An example is given.)

Number in Front	Number Behind
6	1

Make 10

Another strategy that uses facts that children know to help them determine the answer to facts that they do not know is the make-10 strategy. Latesia and Jonathon used this strategy in the opening example. The strategy is useful when one addend is close to 10 or a multiple of 10. For 9 + 5, the child would make a 10 by "borrowing" a 1 from the 5, leaving 4. Therefore, the combination would be changed to 10 + 4. which is 14. A different method would be to think of 10 + 5, which is 15, so 9 + 5 is 1 less, or 14. This strategy can also be used with larger numbers. For 49 + 7, the children can make a 10 and consider 50 + 6, or consider 50 + 7 and subtract 1. The make-10 strategy is appealing for two reasons: *(a)* 10 is an important component of our number system and *(b)* this strategy encourages children to think constantly about the tens place of numbers.

The make-10 strategy uses facts that children know to determine ones that they do not know

The following activity gives children practice making 10.

Activity 3—make 10

K–4 NCTM Standards involved: Problem solving, reasoning, number sense, concepts of whole-number operations, measurement, patterns, and relationships

Materials: Two dice, one marked with 8, 9, 10, 8, 9, 10 and the other with 4, 4, 5, 5, 6, 7; twenty pennies; two dimes; and one copy of **figure 3** for each group of two or three children.

One child rolls the dice. Another child records the numbers on the dice in the first two columns on the chart (see **fig. 3**). A child counts out two piles of pennies equal to the amount on each die. A child then moves pennies from the smaller pile to the larger one to make a total of ten pennies. That child then trades in the pennies for a dime and writes on the chart 1 dime and so many pennies and the total amount. If a ten is rolled, the child simply takes a dime and does not trade, writing 1 dime 0 pennies.

After the children finish filling out the chart,

FIGURE 3

Make 10

Roll the two dice.

Show with pennies the amount on each dice. (Count out pennies for each die.)

Move pennies to make a group of ten. Trade ten pennies for a dime.

How much money do you have? Record. One example is done for you.

First Die	Second Die	Dimes	Pennies	Total
8	5	1	3	13¢

they determine the total amount of money for all their rolls. Two children could play a game by alternating rolls and, when finished, determine their respective totals.

If enough dimes are available, the children could alternate tosses and accumulate the money, trading for a dime each time they got ten pennies. They could race to collect ten dimes. Continuing to use dimes and pennies develops the skill of making 10.

Parents can be encouraged to use these activities at home with their children. Such use not only affords opportunities for practice but also allows the parents to work with their children, observing firsthand the types of rich mathematical activities being used in the classroom to develop thinking strategies for mastering basic addition facts. The children may even have opportunities to teach their parents about the thinking strategies that they have learned.

Conclusion

Presenting activities that encourage children to think allows them to develop strategies at their own rates. These activities help children develop relationships among facts. They begin to view facts as related closely to other facts rather than as separate entities that must be learned independently. Some children need many such opportunities to incorporate strategies into their mental tool kit. By repeating these activities often for short periods of time, most children will become fluent in the use of the strategies. Once that fluency exists, the mastery of basic addition facts is made much easier.

References

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