**Pre-AP Algebra 2**
**Lesson 2 – Using inverse functions to find the range; Graphing the logarithmic function**

**Objectives:** Students will be able to find the range of $f$ by finding the domain of $f^{-1}$. Students will be able to graph the logarithmic function by graphing the inverse of the exponential function.

**Materials:** hw #10-1 solutions; Do Now and answers overhead; Graphing Inverse Functions worksheet and blank overheads; pair work and answers overhead

| Time | Activity |
|---|---|
| 15 min | **Homework Review**<br>Show solutions to hw #10-1. Give students time to check work and ask questions. Making sure to go over the final 2 questions |
| 15 min | **Do Now**<br>Hand out the **Inverse Functions: Domain and Range** worksheet. Give students about 10 minutes to work on it and then review answers on the overhead.<br><br>Point out the relationship – that the domain of $f$ becomes the range of $f^{-1}$ and vice versa. |
| 25 min | **Direct Instruction**<br>Hand out the **Graphing Inverse Functions** worksheet. Students will practice graphing the inverse of a function by picking key points and inverting their coordinates. *Note that inverse functions are always symmetrical across the line $y = x$.* Students should graphically determine the domain and range of each function and its inverse.<br><br>Then, give students the equation of the original function and ask them to determine the equation of the inverse function.<br><br>Equations:<br>1) $y = (x - 2)^2 - 1$ , $x \geq 2$<br>2) $y = \frac{1}{2}x - 3$, $x \geq -2$<br>3) $y = 2^x$ |
| 25 min | **Pair Work**<br>Hand out the **Graphing Exponential and Logarithmic Functions** worksheet. Students practice finding the inverse of logarithmic functions, graphing them, and using those graphs to pointwise find the graph of the original function.<br><br>Coach as needed and review answers on the overhead in the last few minutes. |

Homework #10-2: Connecting Logs and Exponentials

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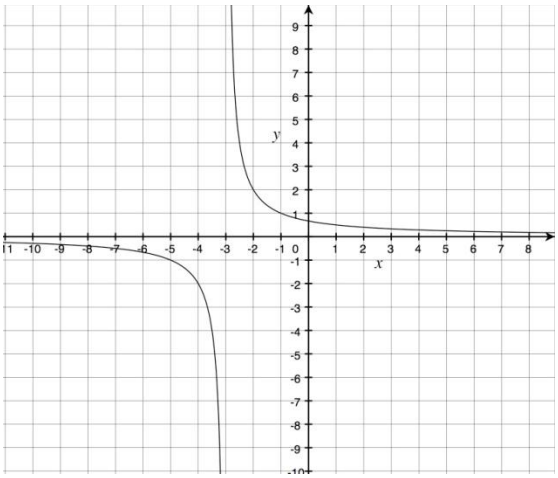
**Do Now**

# Domain and Range of Inverse Functions

**1)** Both graphs and functions are given to you. Use this information to answer the questions.

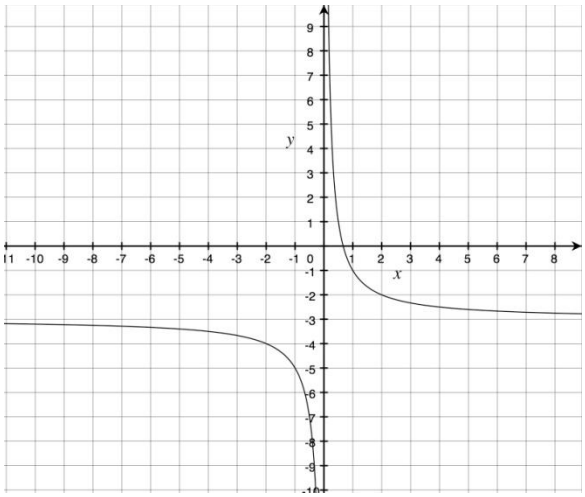
$f(x) = \dfrac{2}{3+x}$



Domain:

Range:

$f^{-1}(x) = \dfrac{-3x+2}{x}$



Domain:

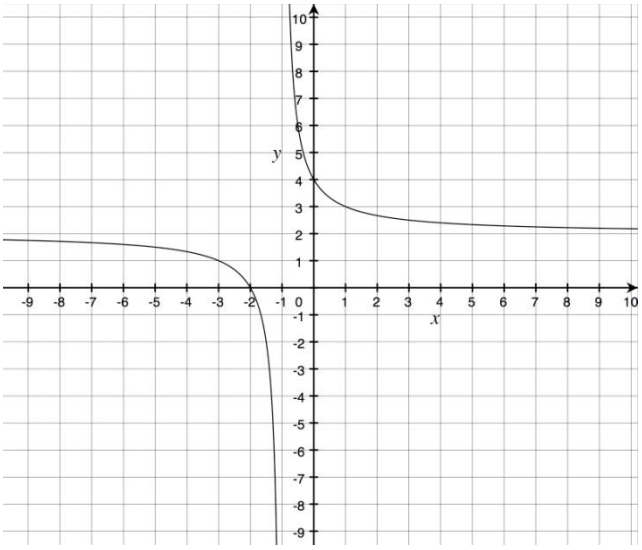
Range:

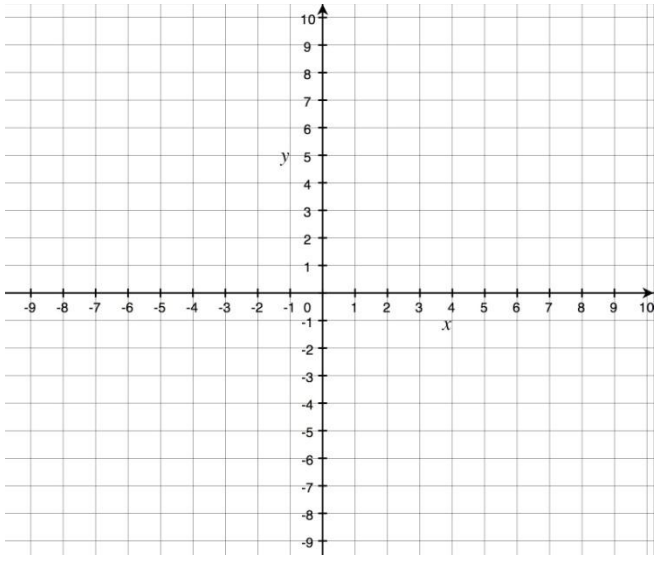
What is the relationship between the domain and range of $f(x)$ and its inverse $f^{-1}(x)$?

**2)** Find the equation of the inverse function $g^{-1}(x)$, and graph it. Then, find the domain and range of both functions.

$$g(x) = \frac{2x+4}{x+1}$$

$$g^{-1}(x) =$$




Domain:

Range:

Domain:

Range:

What is the relationship between the domain and range of $g(x)$ and its inverse $g^{-1}(x)$?

*Workspace to find the inverse function:*

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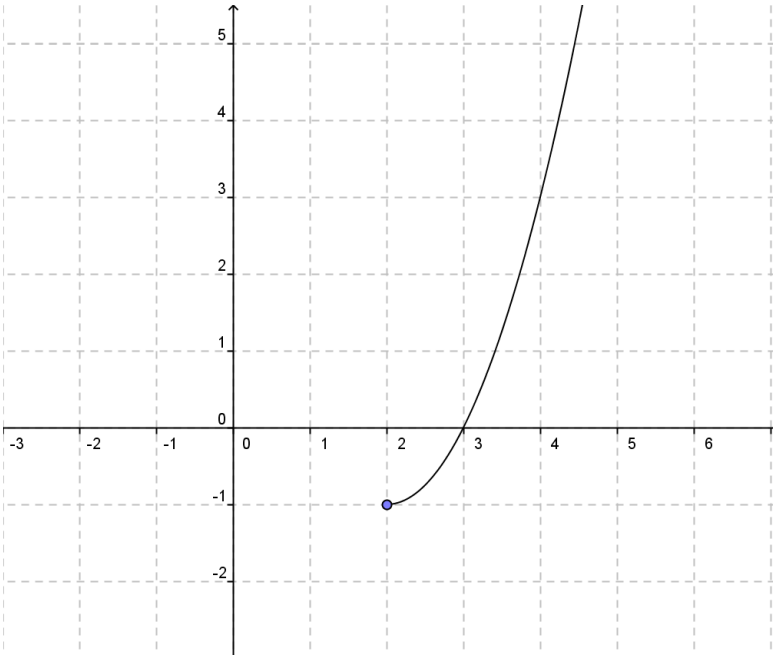
**Pair Work**

# Graphing Inverse Functions

**Find key points on the given function. Use them to sketch the graph of the inverse function on the same axes.**




f(x) =

Domain:

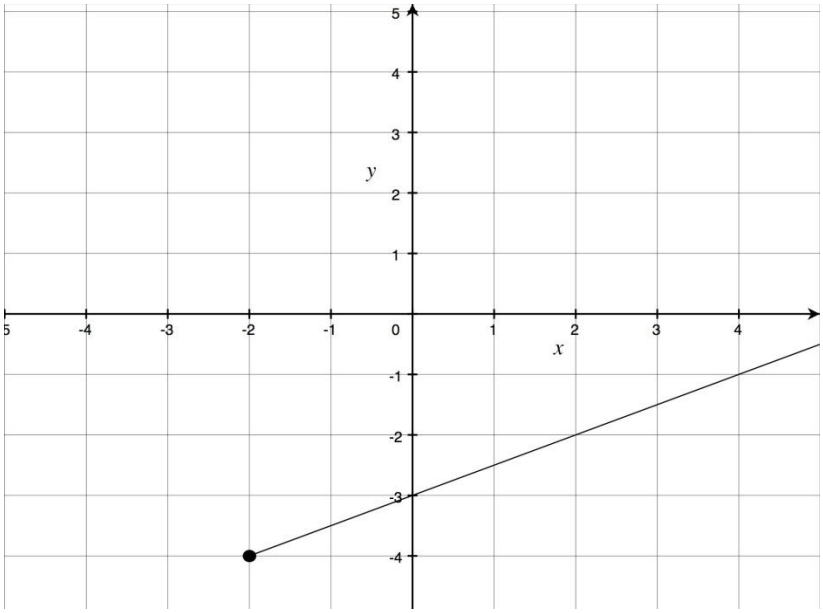
Range:

_____

$f^{-1}(x)$ =

Domain:

Range:




f(x) =

Domain:

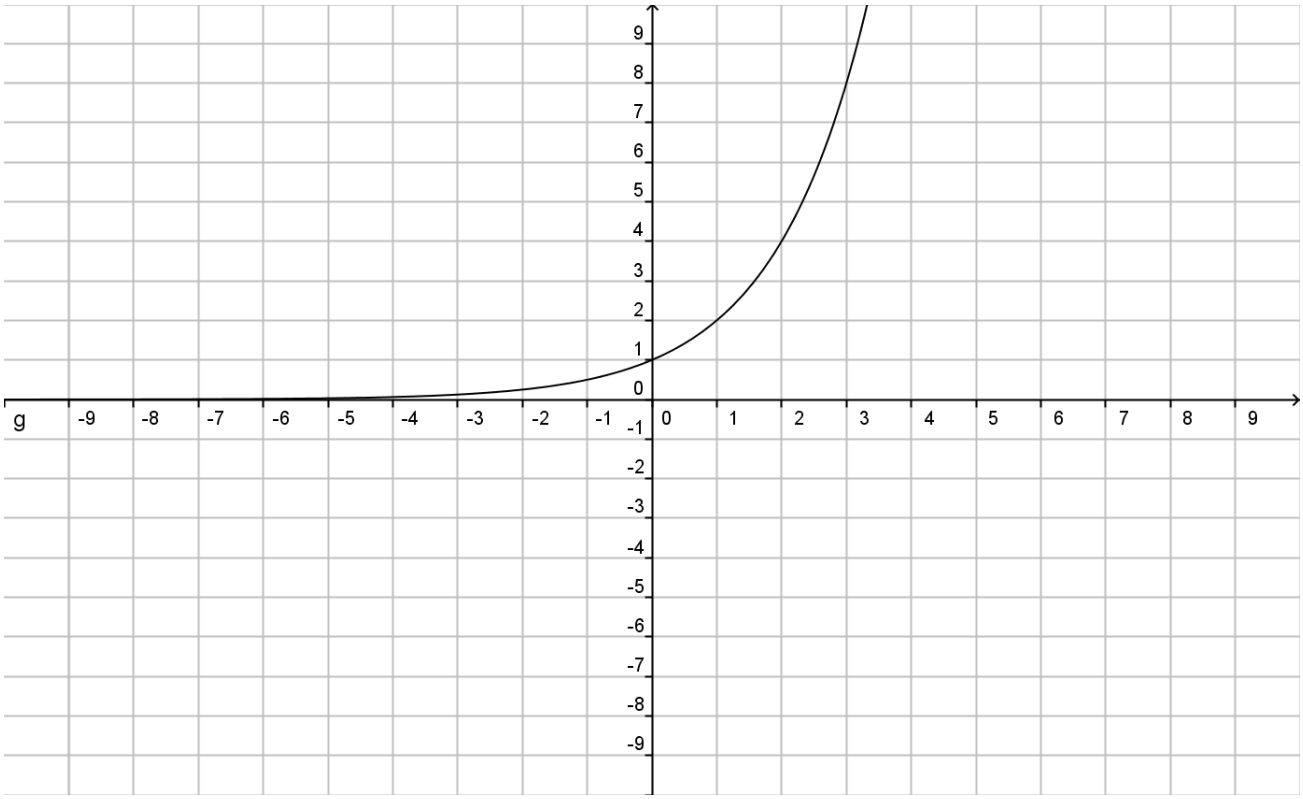
Range:

_____

$f^{-1}(x)$ =

Domain:

Range:




$f(x) =$

$f^{-1}(x) =$

Domain:

Domain:

Range:

Range:

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**Pair Work**

# Graphing Logarithmic and Exponential Functions

**Find the domain of each log function. Remember, the argument of the log must be greater than 0.** ***When finding the domain of a log function, the specific base doesn't matter.***

**1)** $f(x) = \log_4(x - 5)$

**2)** $f(x) = \ln(2x + 3)$

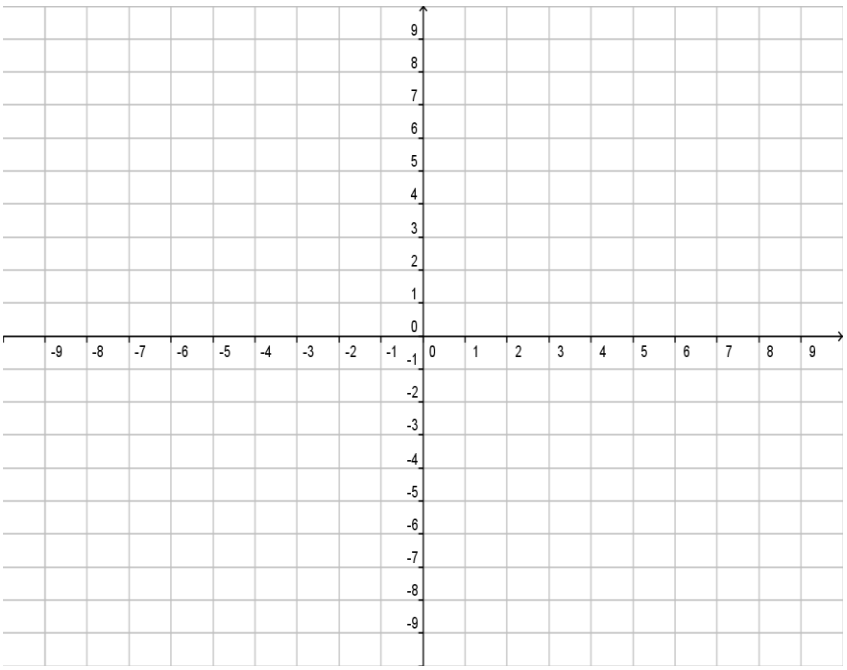
**3)** $f(x) = \log_5(x^2)$

**4)** $f(x) = \log_{0.5}(x^2 - 8x + 15)$
*hint: solve $x^2 - 8x + 15 > 0$ by factoring…*

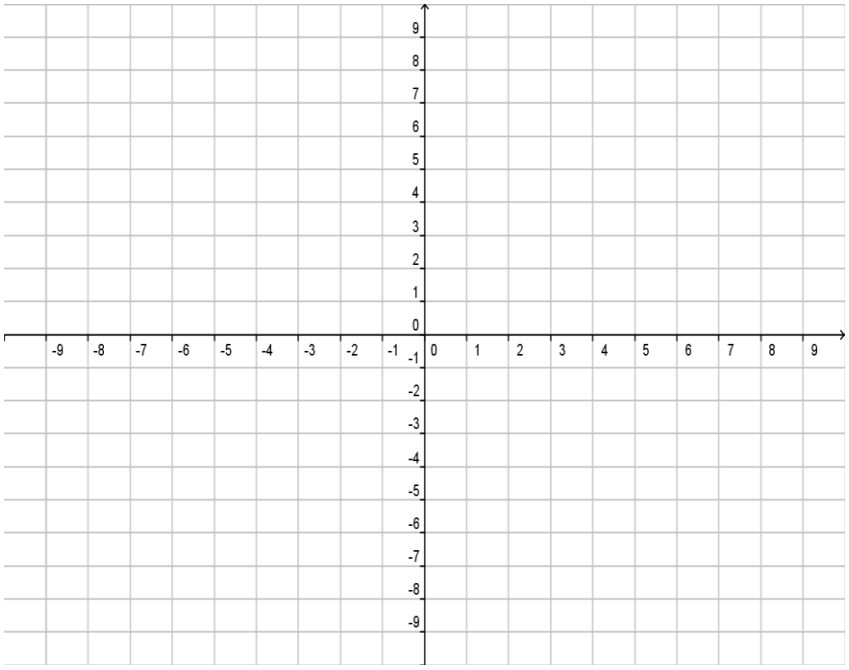
**Make a graph of each logarithmic function. To do this:**

- Determine the inverse function (it should be exponential!)
- Make an x-y table and graph the inverse function (include the horizontal asymptote)
- Pick key points on the graph, switch their coordinates, and plot the new points. Include the vertical asymptote, and think about the relationship between the asymptotes.

**1)** $f(x) = \log_3(x - 1)$ $f^{-1}(x) =$



**2)** f(x) = ln (2x – 5)  $f^{-1}(x)$ =



*Note: to graph a logarithmic function on the calculator, you must change the base. For example, to graph* $f(x) = \log_2(x-4)$, *you could graph* $f(x) = \dfrac{\log(x-4)}{\log 2}$.

Go back to problems 1 and 2, and graph the original function and its inverse. Make sure your window is set to the same dimensions as the graphs on the worksheet. Were your graphs accurate?

On your calculator, add in the graph of **y = x** to the pair of functions. What do you notice about the relationship of this line to the other graphs?

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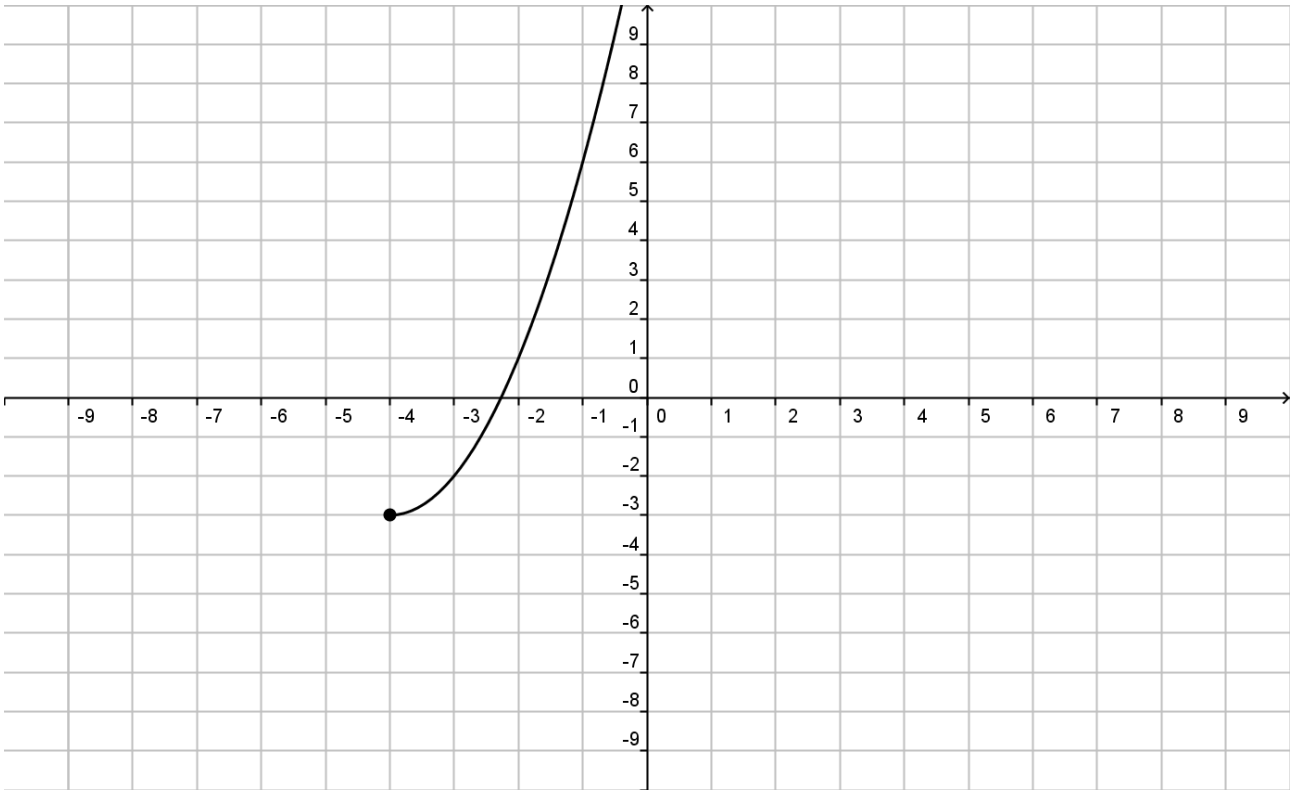
**Homework #10-2**

# HW #10-2: Connecting Logs and Exponentials

1) Find key points on the function graphed below. Use them to sketch the graph of the inverse function on the same axes.




2) a) Determine the inverse function algebraically.

**$f(x) = (x + 4)^2 - 3, \quad x \geq -4$** **$f^{-1}(x) =$**

b) Now verify your answer by evaluating $f(f^{-1}(x))$ and $f^{-1}(f(x))$.

3) Find the domain of each function. Remember that the argument for a logarithm must be greater than 0, and a radicand must be greater than *or equal to* 0.

a) $f(x) = \log_4(3x - 7)$

b) $f(x) = \log_{1/2}\left(x^2 - 4\right)$

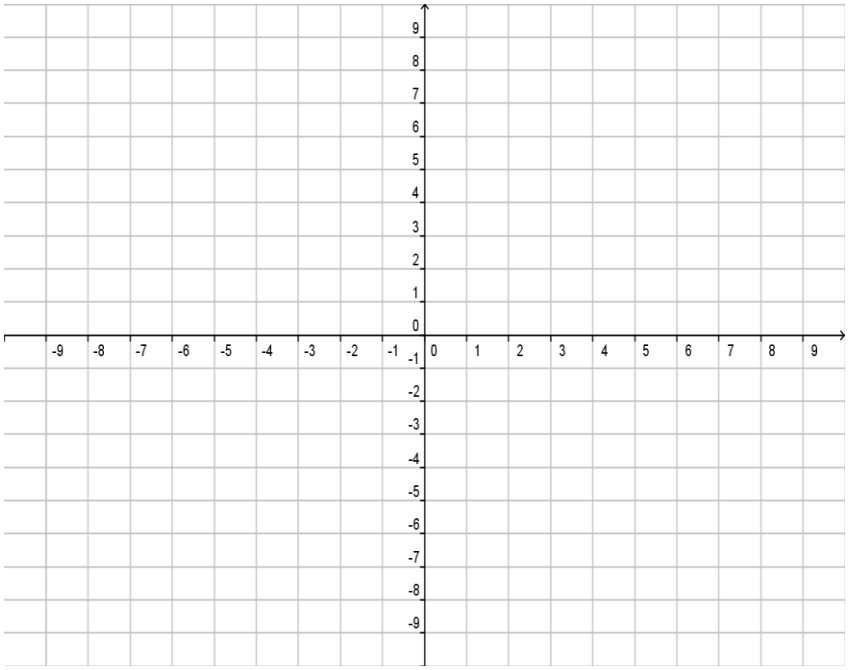
c) $f(x) = \sqrt{2x - 9}$

d) $f(x) = \sqrt{x^2 + 3x - 10}$

4) Make a graph of the logarithmic function f(x) like we did in class. (Find the inverse function, make a table of points, and graph it; then, switch the coordinates to plot the original function. Don't forget to show the asymptotes of each function.)

**$f(x) = \log_2(x + 2)$**

**$f^{-1}(x) =$**




5) Graph the line **y = x** on the axes. What is the relationship between this line and the graphs of f(x) and $f^{-1}(x)$?

6) Find the inverse of each function. Then graph the original function and its inverse on your calculator, along with the line $y = x$. Are the functions mirror images across the line y = x?

a) $f(x) = \log_4(x + 1)$ *(Remember, to graph f(x) on your TI, you need to change the base.)*

b) $g(x) = 8(x - 1)^3 + 5$

c) $h(x) = \sqrt{2x - 6} - 2$

**7)** Solve each inequality. Write your solution in interval notation. *Remember your number line models? Plot the x-intercepts and use a test-value to determine if each interval is + or - .*

a) $4x - 7 > 0$

b) $x^2 - 9 > 0$

c) $x^2 + 3x - 18 > 0$

d) $x^3 + 2x^2 - 9x - 18 > 0$

If you were stuck on problem 3, go back and try again now…

# HW 10-1 Key

1) a) Not a Function b) Function (one-to-one) c) Function (***not*** one-to-one)
d) Function (***not*** one-to-one) e) Function (one-to-one) f) Function (one-to-one)

2) a) $f(g(x)) = \left(\sqrt[5]{x}\right)^5 = x$
$g(f(x)) = \sqrt[5]{x^5} = x$

b) $f(g(x)) = \left(\sqrt{x-2}\right)^2 + 2 = x - 2 + 2 = x$
$g(f(x)) = \sqrt{(x^2+2)-2} = \sqrt{x^2} = x$

c) $f(g(x)) = 4^{\log_4 x} = x$
$g(f(x)) = \log_4 4^x = x$

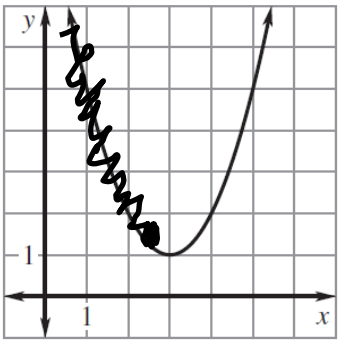
3) a) $f^{-1}(x) = \frac{x-5}{2}$ *or* $\frac{1}{2}x - \frac{5}{2}$

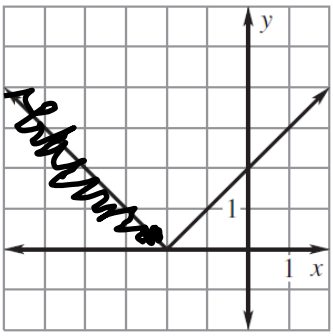
b) $g^{-1}(x) = \sqrt{x-2}$

c) $h^{-1}(x) = \frac{-3x+1}{x-2}$

4) Domain: $x \neq -3$
Range: $y \neq 2$

5) Domain: $x \neq -3$
Range: $y \neq 2$
Inverses are created by reversing the domain and range of the function.

6) a) 


b) 

**HW 10-1 Tally Sheet**

1) a) b) c)
d) e) f)

2) a) b) c)

3) a) b) c)

4) Domain: Range:

5) Domain: Range:

6) a) b)

**HW 10-1 Tally Sheet**

1) a) b) c)
d) e) f)

2) a) b) c)

3) a) b) c)

4) Domain: Range:

5) Domain: Range:

6) a) b)