Objectives:

Students will be able to:

- Graph functions that are the same in all but one point (by factoring and simplifying)
- Students will be able to determine the location of holes in the function, and correctly determine the domain.
- Determine if x-values will create a hole, a vertical asymptote, or an x-intercept.

Materials: Overhead calculator; homework #8-4 answers overhead; Do Now and answers overhead; pair work; hw #8-5

Time	Activity								
10 min	Review Homework								
	Students take this time to check homework and ask any questions. No time will be used for class								
	review.								
	Problems to grade: 1j, 2c, 3b, 4b, 5 (graph)								
	Return quizzes.								
25 min	Do Now								
	Students practice determining end behavior and vertical asymptotes (both from functions and								
	graphically), stating the domain.								
	Review answers on overhead.								
30 min	Direct Instruction								
	Write the function $f(x) = \frac{x^2 - x - 6}{x + 2}$ on the board. Ask students to predict what it will look like, in terms								
	of end behavior and vertical asymptotes.								
	They will assume there is a vertical asymptote at $x = -2$. Graph the function on the overhead calculator and examine it. They will see there is no asymptote. What is going on here?								
	Use the trace function to see what happens when $x = -2$. The value will be missing. Ask students to find $f(-2)$ by direct substitution, which yields $0/0$. This is an indeterminate form , because $0/0$ could be assigned any value. In Calculus, this will become a very important concept. What does this mean in terms of the function?								
	Hand out the note-taking templates.								
	Lesson 4: Holes in Functions (or, the Hidden Dangers of Simplifying)								
	Examples.								
	Examples: Write down $f(x)$ Ask students to factor yielding $f(x) = 0$								
	$f(x) = \frac{(x+2)(x-5)}{x+2}$								
	From this, it is clear that we can cancel out the $(x + 2)$ factor, to leave $x - 3$, which is the graph that we saw. Question: are the functions $f(x) = \frac{(x+2)(x-3)}{2}$ and $g(x) = x - 3$ the same? Why or why not?								
	x+2 They seem to be but when $x = -2$ we have a problem. There is a hole in the graph at this point. In other								
	words, $f(x) = x - 3$, only in the domain where $x \neq 2$. Graph $g(x) = x - 3$ also, and use TRACE, jumping back and forth between graphs.								
	Write down: $f(x) = \frac{(x+2)(x-3)}{x+2} = x-3, x \neq -2$								
	Concepts:								
	 Simplifying a rational function Factor completely Every canceled factor (x - c) produces a hole at x = c. Write the simplified function with its restricted domain. Graph the simplified function, with an open circle at each hole. 								

	Examples:								
	Graph $f(x)$, and then simplify and graph the following:								
	$g(x) = \frac{3x+12}{x+4} \qquad h(x) = \frac{x^2+8x+15}{x^2-9}$								
	Concepts:								
	Zeros in rational numbers								
	Let k be a real number:								
	• 0/k = 0								
	• $k/0$ is undefined (cannot be defined – cannot be any number)								
	• 0/0 is indeterminate (cannot be determined – could be any number)								
	Zeros in rational functions								
	Let c and k be real numbers:								
	• If $f(c) = 0/k$, (c, 0) is an x-intercept								
	• If $f(c) = k/0$, $x = c$ is a vertical asymptote								
	• If $f(c) = 0/0$, there is a hole in the graph at $x = c$.								
15 min	Pair Work								
	Students practice simplifying functions and graphing them.								

Homework #8-5: The Hidden Dangers of Simplifying

Do Now: Rational Review

Part 1: Graphical Representation

For each graph, determine the equation of each vertical asymptote. Then, determine if the end behavior of the function is approaching a horizontal asymptote, a slant asymptote, or a parabolic asymptote. Finally, write the **domain** of each function.

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End Behavior:





End Behavior:

Domain:













Domain:

VA:

Domain:

3)

Part 2: Algebraic Representation

For each function, write equations of any vertical asymptotes. Then, determine the end behavior function (write it in the form "as $x \rightarrow \pm \infty$, $f(x) \rightarrow ___$ " and describe the type of asymptote that will occur.

Remember, there is only one situation in which you need to actually divide!

That is when: ______

1) $f(x) = \frac{6x-7}{3x^2-5x}$ Vertical asymptotes: End behavior function:

Type of asymptote:

2) $f(x) = \frac{-8x+5}{4x+3}$ Vertical asymptotes:

End behavior function:

Type of asymptote:

3) $f(x) = \frac{4x^2 - 9}{2x + 1}$ Vertical asymptotes: End behavior function:

Type of asymptote:

4)
$$f(x) = \frac{3x^2 - 87x + 149}{9x^2 - 377x + 122}$$
 Vertical asympt
End behavior fu

Vertical asymptotes: (skip this one!) End behavior function:

Type of asymptote:

5) $f(x) = \frac{4x^3 - 2x^2 + 8}{x - 2}$

Vertical asymptotes:

End behavior function:

Type of asymptote:

Simplifying Rational Functions

Fill in the blank and then graph f(x):

$$f(x) = \frac{x^2 + 3x + 2}{x + 2}$$
, so $f(x) = x + 1$, $x \neq$ _____



Simplify each function. Make sure to write the correct domain for the new function. Then, graph the function, correctly indicating any holes.

1) $f(x) = \frac{5x+15}{2x+6}$	f(x) =
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2)
$$f(x) = \frac{x^2 - 2x - 8}{x + 2}$$
 $f(x) =$

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Hw #8-5: The Hidden Dangers of Simplifying!

- 1) Sometimes, when you think a function will have a vertical asymptote, it doesn't! When does this happen? Instead of a vertical asymptote, what do you see on the function when this happens?
- 2) Write a rational function f(x) that seems like it should have a vertical asymptote, but actually doesn't.
- 3) When you simplify a rational function, what do you have to include as part of your answer? Why?
- 4) Write the correct simplified form of your function from question 2.
- 5) There are three possibilities for dealing with zeros in fractions: 0/k, k/0, and 0/0. Describe what each one results in, and explain why. Describe the result in the graph of a function that each of these situations produce.

- 6) Write a rational function f(x) that has an x-intercept at x = 2 and a vertical asymptote at x = -3.
- 7) Write a rational function f(x) that has an x-intercept at x = 2 and a vertical asymptote at x = -3, and a hole at x = 4.

- 8) Write a rational function f(x) that has x-intercepts at $x = \pm 4$, and vertical asymptotes at x = 2 and x = -5.
- 9) Write a rational function f(x) that has x-intercepts at $x = \pm 4$, vertical asymptotes at x = 2 and x = -5, and a horizontal asymptote at y = 3.
- 10) Write a rational function f(x) that has x-intercepts at $x = \pm 4$, vertical asymptotes at x = 2 and x = -5, a horizontal asymptote at y = 3, and a hole at x = 1.
- 11) Simplify each function and write the domain of the simplified function. Graph f(x).

$$f(x) = \frac{x^3 + 2x^2 - x - 2}{x + 1} \qquad \qquad f(x) =$$

12) Use substitution to determine what happens to each function when x = 2. Is there a hole, a vertical asymptote, an x-intercept, or just a regular point on the graph?

a)
$$f(x) = \frac{x^2 - x - 2}{x^2 - 2x - 24}$$

b) $f(x) = \frac{x^2 + 2x - 8}{3x^2 + 24x - 60}$
c) $f(x) = \frac{x^2 + 4}{x^2 - 4}$
d) $f(x) = \frac{x^3 - 8}{x - 2}$

Bonus: +3 points (Hint – a dictionary will help with some of these. Also, phone numbers in England start a little differently than ours do.)

Text:

Violet and I often have a quiet drink after work in our favorite bar on the Mobius Strip. It's not unusual for some bloke to try to chat her up and she usually brushes them off with a cutting one-liner. One night, a particularly persistent guy just wouldn't give up asking for her phone number, insisting that he recognized her from somewhere.

Eventually, she arched one eyebrow and said:

"My phone number has 11 digits and they sum to 44. Vowels in syzygy, corners on a cube, deadly sins, null in German, love in tennis, strings on a violin, degrees of separation, atomic number of lithium, horsemen of the apocalypse, colors of the rainbow, dots in a quincunx."

He asked her to repeat the problem but, smiling, she refused.

HW 8-4 Answer Sheet
1)a)
$$(x + 5)(x - 5)$$
 b) $(7x + 10)(7x - 10)$ c) $3(1 + 4y)(1 - 4y)$
d) $(x + 8)(x - 3)$ e) $(x - 9)(x - 1)$ f) $(x + 7)(x + 7)$
g) $(x - 2)(x^2 + 2x + 4)$ h) $(3x + 1)(9x^2 - 3x + 1)$
i) $2x(x - 5)(x^2 + 5x + 25)$ j) $(3x - 5)(2x + 3)$
k) $(5x + 3)(5x - 2)$ l) $(x + 3)(x - 3)(x^2 - 5)$
m) $(x + 4)(x - 4)(3x + 2)$
2) a)as $x \to \pm \infty$, $f(x) \to -\infty$ b) as $x \to \pm \infty$, $f(x) \to 3x - 8$
as $x \to -\infty$, $f(x) \to +\infty$
c) as $x \to \pm \infty$, $f(x) \to 0$ d) as $x \to \pm \infty$, $f(x) \to 2$
3) Vertical: $x = -1$ Horizontal: $y = 2$
x-int $(3, 0)$ y-int $(0, -6)$
domain $x \neq -1$ range $y \neq 2$
graph

HW 6-4 Tally Sheet 1)a) c) b) f) d) e) g) j) h) i) k) 1) m) 2)a) b) c) d) 3)Vertical: Horizontal: x-int y-int domain range graph