Pre-AP Algebra 2
Unit 9 - Lesson 2 – Introduction to Logarithms

Objectives:

- Students will be able to convert between exponential and logarithmic forms of an expression, including the use of the common log.
- Students will solve basic equations with logs and exponentials.

Materials: #9-1 exploration answers overhead; Do Now and answers overhead; note-taking templates; practice worksheet; homework #9-2

<table>
<tr><th>Time</th><th>Activity</th></tr>
<tr><td>20 min</td><td>HW Review
Put up answers to lesson #9-1 on the overhead; students check.</td></tr>
<tr><td>10 min</td><td>Do Now
Students work on check for understanding problems and an exploration for logarithms</td></tr>
<tr><td>30 min</td><td>Direct Instruction

Background Information:
Solve:
1) $100x^5 = 9600$ 2) $100(5)^x = 9600$

We have all the tools we need to solve the first equation. The second one is not yet solvable because we don't have an operation that allows us to <u>isolate the exponents</u>. A logarithm is an operation that allows us to do just that.

Concepts:
• A logarithm is an operation that isolates an exponent.
• Logs are used to solve equations with a variable in the exponent.
• Definition: If $b^y = x$, then $y = \log_b x$.
  o This is pronounced “y equals log base b of x”.
  o Notice that the exponent y has been isolated.
• A log's base b can be any positive number (except 1). There are two special cases:
  1. $\log_{10}x$ is the same as log x. This is called the “common log”.
  2. (we will come back to this in a couple of weeks)

Examples:

Convert between logs and exponentials:
(do #1 – 2 together, students do #3 – 8 on their own)
1) if $x = 3^y$, then $y = \log_3 x$
2) if $y = \log_5 x$, then $5^y = x$ In the log, identify the isolated y as the exponent and the 5 as the base.
3) if $1.2^3 = m$, then… $3 = \log_{1.2} m$
4) if $a^4 = 24$, then… $4 = \log_a 24$
5) if $10^x = 17$, then… $\log_{10} 17 = x$, so $x = \log 17$
6) if $\log_a 4 = 5$, then… $a^5 = 4$
7) if $\log_{25} c = ½$, then… $25^{1/2} = c$
8) if $\log x = 7$, then… $10^7 = x$

Find the exact value of a logarithmic expression:

1) $\log_2 16$
  $\log_2 16 = y \rightarrow 2^y = 16$ $y = 4$, therefore $\log_2 16 = 4$
2) $\log_3(1/27)$
  $\log_3(1/27) = y \rightarrow 3^y = (1/27)$ $y = -3$, therefore $\log_3(1/27) = -3$</td></tr>
</table>

	Solve equations: 1) $\log_3(4x - 7) = 2$ $3^2 = 4x - 7$, etc. 2) $\log_x 64 = 2$ $x^2 = 64$, etc. 3) $10^{2x} = 5$ $\log_{10}5 = 2x$ log 5 = 2x x = (log 5)/2 4) Solve $5\left(3^x\right) + 20 = 435$ $5\left(3^x\right) = 415$. $3^x = 83$ *Estimate the value of x. It must be between 4 and 5, closer to 4, so about 4.1 or 4.2.* *To find the exact value of x, we need to get it by itself. How can we isolate x? Convert to logarithm form.* **Convert to log form:** $x = \log_3 83 \approx 4.1$ **Concepts:** Change of Base Theorem - Can be used to convert between any base. - We often convert to base 10 because that's what your calculator can do. - Theorem: $\log_{oldbase} x = \dfrac{\log_{newbase} x}{\log_{newbase} oldbase}$ - Useful example: $\log_b x = \dfrac{\log_{10} x}{\log_{10} b} = \dfrac{\log x}{\log b}$ **Examples:** $x = \log_3 83 = \dfrac{\log 83}{\log 3} \approx 4.022$. Find the exact value and then estimate: $\log_5 89$ (2.789) $\log_7\left(\dfrac{3}{237}\right)$ (-2.245)
20 min	**Pair Work** Practice worksheet

Homework #9-2: Introduction to Logarithms

Pre-AP Algebra 2 **Name: ___________________________**
9-2 Do Now

DO NOW

9-1 Check for Understanding
Rewrite with rational exponents

5 – Exemplary
4 – Proficient
3 – Nearly Proficient
2 – Emerging
1 – Beginning

1) $\sqrt[4]{5}$

2) $\sqrt{3^5}$

Simplify the expressions. Don't leave any negative exponents.

3) $125^{2/3}$

4) $9^{3/4} \cdot 9^{5/4}$

5) $27^{-1/3}$

6) $36^{1/5} \times 36^{3/10}$

Explore: (Put these in your calculator)

log(10) = ______

log(100) = ______

log(1000) = ______

log(100000) = ______

log(.1) = ______

log(.01) = ______

log(.001) = ______

log(.0001) = ______

log(.00000001) = ______

log(1) = ______

What do you think the log function of your calculator gives you?

What do you think you will get for $\log_2 8$ **?**

Pre-AP Algebra 2 **Name: ___________________________**
9-2 Pair Work

Introduction to Logarithms

Change each <u>exponential</u> expression to an equivalent expression using <u>logarithm</u> form

1) $9 = 3^2$ 2) $a^2 = 1.6$ 3) $1.1^2 = M$

4) $10^x = 7.2$ 5) $x^{\sqrt{2}} = \pi$ 6) $e^x = 8$

Change each <u>logarithmic</u> expression to an equivalent expression using <u>exponential</u> form

1) $\log_4 1024 = 5$ 2) $\log_3 \frac{1}{9} = -2$ 3) $\log_b 4 = 2$

4) $\log 2 = x$ 5) $\log_3 N = 2.1$ 6) $\log x = 4$

Find the exact value of the logarithm <u>without using a calculator</u>

1) $\log_5 125 =$ 2) $\log_{17} 1 =$ 3) $\log_8 8 =$

4) $\log 1000 =$ 5) $\log_3 \frac{1}{27} =$ 6) $\log 0.001 =$

7) $\log_4 2 =$ 8) $\log_{16} 2 =$ 9) $\log_{1/2} 16$

10) $\log_{\sqrt{2}} 4 =$ 11) $\log_{1.6} 1.6^5 =$ 12) $\log_\pi 1 =$

What do the following equal?

$\log_a 1 =$

$\log_a a =$

These are important properties of logarithms to remember.

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Solve each equation. Check your answers. Remember that the base of a logarithm is always a positive number.

Your first step for each problem should be to convert it from one form to the other.

1) $\log_3 x = 2$

2) $\log_2(2x + 1) = 3$

3) $\log_x 4 = 2$

4) $\log_x(1/8) = 3$

5) $\log_5 625 = x$

6) $\log_6 36 = 5x + 3$

Write the exact value of *x*. Then, use your calculator to estimate *x* to the thousandths place.

7) $10^x = 128$

8) $5(10^x) = 30$

Pre-AP Algebra 2 **Name: ________________________**
Homework #9-2

Homework #9-2: Introduction to Logarithms

Part 1:
Change each <u>exponential</u> expression to an equivalent expression using <u>logarithm</u> form.

1) $1000 = 10^3$ 2) $y^4 = -2.9$ 3) $(¼)^2 = A$

4) $e^x = 31$ 5) $x^{\sqrt{5}} = e$ 6) $10^x = 5$

Change each <u>logarithmic</u> expression to an equivalent expression using <u>exponential</u> form.

7) $\log_5 \frac{1}{5} = -1$ 8) $\log_a 3 = 6$ 9) $\log_3 2 = x$

10) $\log 6 = x$ 11) $\log_7 H = 7.9$ 12) $\log x = -2$

Find the exact value of the logarithm <u>without using a calculator</u>.

13) $\log_2 1 =$ 14) $\log_7 7 =$ 15) $\log_x x^5 =$

16) $\log \frac{1}{10} =$ 17) $\log_{1/2} 32 =$ 18) $\log_{125} 5 =$

Part 2: Solve for x in each equation.

1) $\log_5 x = 2$ 2) $\log_4(x - 5) = 3$

3) $\log x = 5$ 4) $\log_x 5 = ½$

5) $\log 10^x = 21$ 6) $\log_x 8 = 3$

7) $\log_2(3x + 5) = 4$ 8) $\log_3 81 = 7x - 10$

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Part 3: Solving Exponential Equations

Solve each equation by isolating the power, then converting to a logarithm. Write the exact answer (as a logarithm), and then use the Change of Base Theorem and your calculator to estimate the solution to the thousandths place.

1) $5^x = 96$ Exact: Estimate:

2) $3(7^x) = 237$ Exact: Estimate:

3) $-2(1.5^x) + 28 = 4$ Exact: Estimate:

4) $45 – 4(2.5^x) = 1$ Exact: Estimate:

Part 4: STAAR Review

1) What is the y-value of the solution to the matrix equation below?

$$\begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

a. -6 b. 14 c. -5 d. 12

2) Which of the following quadratic functions does not have zeros of -15 and 6?

a. $f(x) = \frac{1}{3}x^2 + 3x - 30$ b. $f(x) = -x^2 - 9x + 90$

b. c. $f(x) = -\frac{2}{3}x^2 - 6x + 60$ d. $f(x) = -x^2 - 9x - 90$

3) The base of a triangle is 3 inches less than twice its height. If the area of the triangle is 126 square inches, which of the following equations can be used to find h, the height of the triangle in inches?

a. $2h^2 - 3h + 63 = 0$ b. $2h^2 - 3h - 63 = 0$

c. $2h^2 - 3h + 252 = 0$ d. $2h^2 - 3h - 252 = 0$

4) Find all solutions: $\frac{1}{x} + \frac{2}{x^3} = \frac{3}{x^2}$

a. x = 1 b. x = 1, 2 c. x = 0, 1, 2 d. x = 2, 4

Lesson Name: *Introduction to Logarithms* **Date:** _______________ **Student:** ________________________

Portfolio Section: *Exponents and Logs*

Concepts	Examples	Background
.**Concepts:** • A **logarithm** is an operation that isolates an exponent. • Logs are used to solve equations with a variable in the exponent. • **Definition:** If $b^y = x$, then $y = \log_b x$. This is pronounced "y equals log base b of x". Notice that the **exponent** *y* has been **isolated**. • A log's base *b* can be any positive number (except 1). There are two special cases: 1. $\log_{10}x$ is the same as log x. This is called the "common log". 2. *(we will come back to this in a couple of weeks)*	**Examples:** **Convert between logs and exponentials:** 1) if $x = 3^y$, then 2) if $y = \log_5 x$, then 3) if $1.2^3 = m$, then 4) if $a^4 = 24$, then 5) if $10^x = 17$, then 6) if $\log_a 4 = 5$, then 7) if $\log_{25} c = ½$, then 8) if log x = 7, then **Find the exact value of a logarithmic expression:** 1) $\log_2 16$ 2) $\log_3(1/27)$	**Background Information:** Solve: 1) $100x^5 = 9600$ 2) $100(5)^x = 9600$ *We have all the tools we need to solve the first equation. The second one is not yet solvable because we don't have an operation that allows us to <u>isolate the exponents</u>. A **logarithm** is an operation that allows us to do just that*

Concepts	Examples
Change of Base Theorem - Can be used to convert between any base. - We often convert to base 10 because that's what your calculator can do. - Theorem: $\log_{oldbase} x = \dfrac{\log_{newbase} x}{\log_{newbase} oldbase}$ - Useful example: $\log_b x = \dfrac{\log_{10} x}{\log_{10} b} = \dfrac{\log x}{\log b}$	**Solve equations:** 1) $\log_3(4x - 7) = 2$ rewrite as $3^2 = 4x - 7$ 2) $\log_x 64 = 2$. 3) $10^{2x} = 5$ 4) $5\left(3^x\right) + 20 = 435$. *Estimate the value of x.* *To find the exact value of x, we need to get it by itself. How can we isolate x?* *Convert to logarithm form.* Find the exact value and then estimate: $\log_3 83 =$ $\log_5 89 =$ $\log_7\left(\dfrac{3}{237}\right) =$