Probability Models

PROBABILITY MODELS (BINOMIAL AND GEOMETRIC) CHAPTER 16

Essentially, all models are wrong; some are useful.

George E.P. Box (1919 -)

Pop Quiz!!!

This quiz has 10 multiple-choice questions, each with four answer choices (a,b,c,d). Number your paper from 1 to 10.

Each question represents a Bernoulli Trial:

1. Two outcomes (success / fail)
2. Same probability of success $p = 0.25$
3. Trials are independent

Some useful probability models are the result of observing two or more Bernoulli Trials.

Pop Quiz (again!)

Let's take the quiz again (MC – 4 answer choices), only this time you will stop as soon as you answer a question correctly.

If we define X to the number of correct answers out of a pre-determined number (10 questions), we say that X follows a (discrete) **Binomial model**.

If we define X to be the number of questions needed to get one correct, we say that X follows a (discrete) **Geometric model**.

Geometric Model

A **Geometric Model** (discrete random variable) has the following characteristics:

- A series of Bernoulli Trials
- Stop at the first success

For the "Quiz:"

- Each answer is right or wrong
- Each answer has 25% chance of being correct
- Each answer is independent
- You stopped on the first correct answer

Geometric Model

A Geometric Model – Geom(p) – for Bernoulli Trials:

$p =$ probability of success

$q = 1 - p =$ probability of failure

$x =$ number of trials to reach 1 success

$$P(X = x) = q^{x-1} p$$

$$\mu = E(X) = \frac{1}{p}$$

$$\sigma = SD(X) = \sqrt{\frac{q}{p^2}}$$

Geometric Model

Suppose you are collecting LeBron James trading cards from cereal boxes. The manufacturer has put LeBron cards in 20% of the boxes, so you have a probability $p = 0.20$ of getting a LeBron card in the next box you open, and you are willing to keep buying boxes of cereal until you find a LeBron card.

1. Describe a probability model for this situation.

Geometric Model

Suppose you are collecting LeBron James trading cards from cereal boxes. The manufacturer has put LeBron cards in 20% of the boxes, so you have a probability $p = 0.20$ of getting a LeBron card in the next box you open, and you are willing to keep buying boxes of cereal until you find a LeBron card.

2. What is the probability that the 3rd box has the LeBron card?

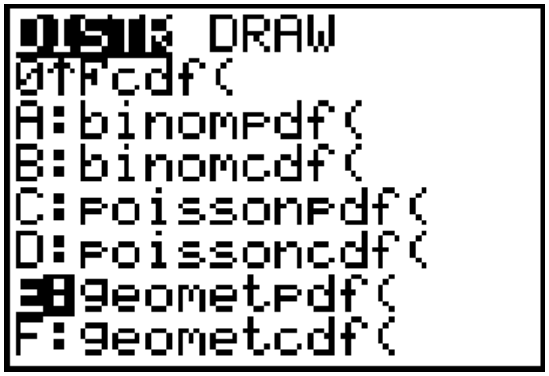
Geometric Model

Suppose you are collecting LeBron James trading cards from cereal boxes. The manufacturer has put LeBron cards in 20% of the boxes, so you have a probability $p = 0.20$ of getting a LeBron card in the next box you open, and you are willing to keep buying boxes of cereal until you find a LeBron card.

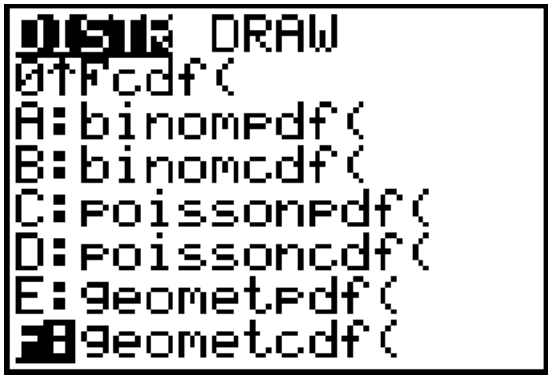
3. What is the probability that a LeBron card appears within the first 3 boxes you open?

Geometric PDF and CDF

On your TI-83/4, the Distribution Menu has two useful commands for geometric probabilities.



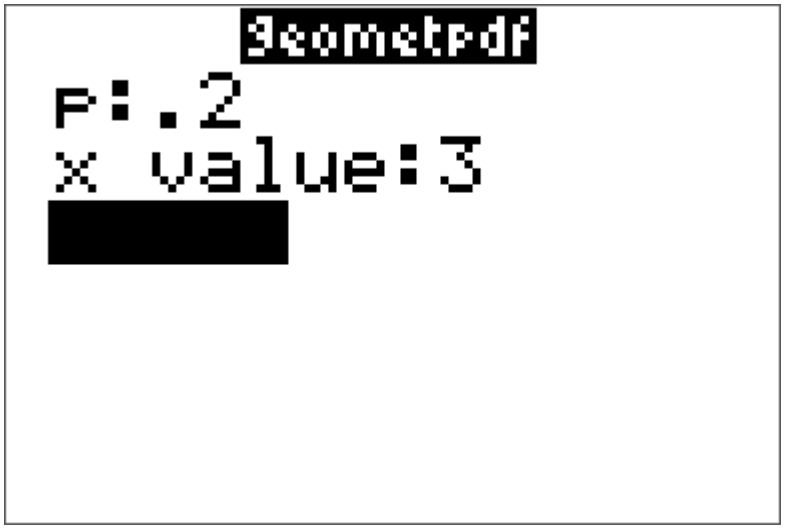
- Geometric Probability Density Function
- Use for $P(X = x)$
- geometpdf(p,x)



- Geometric Cumulative Distribution Function
- Use for $P(X \le x) = P(X = 1) + \cdots + P(X = x)$
- geometcdf(p,x)

Geometric PDF

To compute the geometric probability of the first success occurring on a particular trial:



$$\text{Geom}(p = 0.2)$$

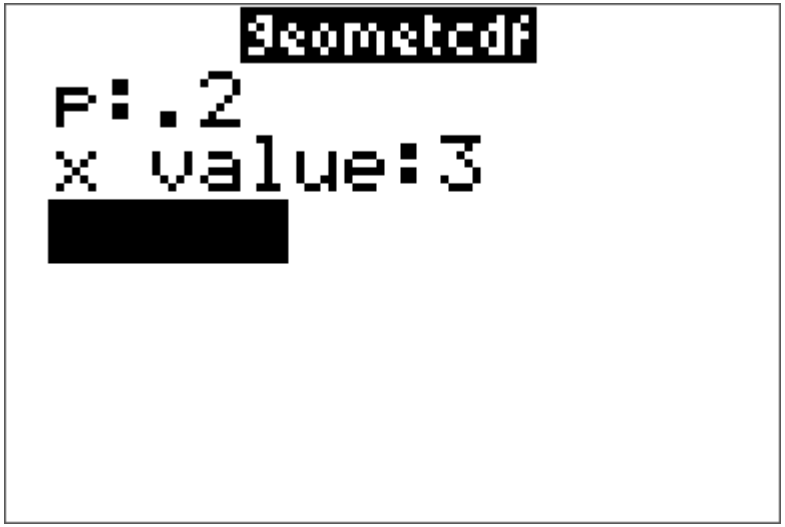
$$P(X = 3) = (0.8)(0.8)(0.2)$$

See TI Tips box on p. 418

Geometric CDF

To compute the geometric probability of the first success occurring on any trial up to a particular trial:

$$\text{Geom}(p = 0.2)$$



$$P(X \le 3) = P(X = 1) + P(X = 2) + P(X = 3)$$

See TI Tips box on p. 418

Geometric Model

Suppose you are collecting LeBron James trading cards from cereal boxes. The manufacturer has put LeBron cards in 20% of the boxes, so you have a probability $p = 0.20$ of getting a LeBron card in the next box you open, and you are willing to keep buying boxes of cereal until you find a LeBron card.

4. How many boxes do you expect to open to find a LeBron card?

Binomial Model

The original "quiz" had 10 questions, with each question a Bernoulli trial. When we know in advance the number of trials, the probability model is called the **Binomial Probability Model** (discrete random variable).

For the "Quiz:"

- Each answer is right or wrong
- Each answer has 25% chance of being correct
- Each answer is independent
- You stopped after a pre-determined number of trials

Binomial Model

A Binomal Model – Binom(n,p) – for Bernoulli Trials:

$p =$ probability of success

$q = 1 - p =$ probability of failure

$x =$ number success in n trials

$$P(X = x) = \binom{n}{x} p^x q^{n-x}$$

$$\mu = E(X) = np \qquad \sigma = SD(X) = \sqrt{npq}$$

Binomial Model

(Back to LeBron …)

Suppose you are willing to buy 8 boxes of cereal (recall that 20% of boxes have LeBron cards) in order to get as many LeBron cards as possible.

1. Describe an appropriate model.

Binomial Model

(Back to LeBron …)

Suppose you are willing to buy 8 boxes of cereal (recall that 20% of boxes have LeBron cards) in order to get as many LeBron cards as possible.

2. What is the probability of finding 2 LeBron cards?

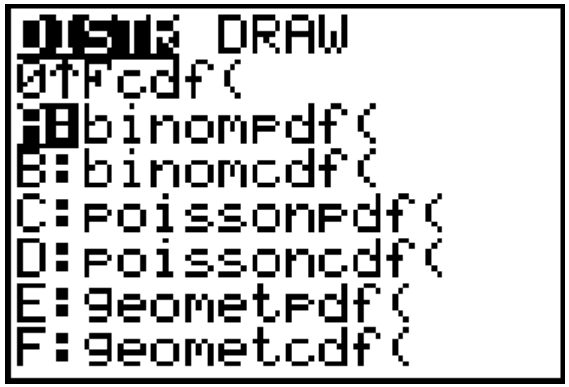
Binomial Model

(Back to LeBron …)

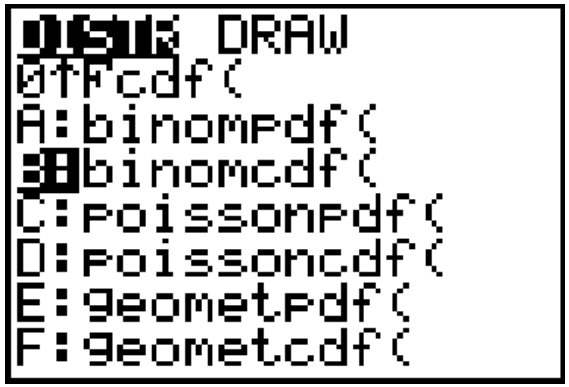
Suppose you are willing to buy 8 boxes of cereal (recall that 20% of boxes have LeBron cards) in order to get as many LeBron cards as possible.

3. What is the probability of finding 2 or 3 LeBron cards?

Binomial Model



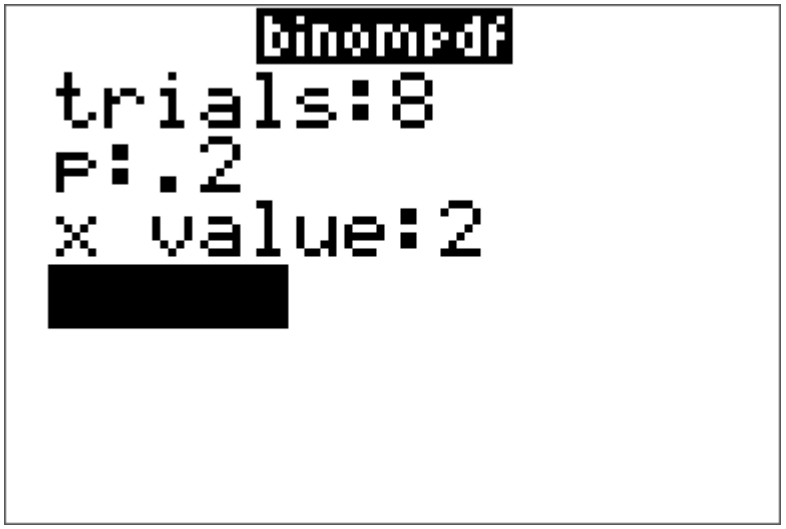
- Binomial Probability Density Function
- Use for $P(X = x)$
- binompdf(n,p,x)



- Binomial Cumulative Distribution Function
- Use for $P(X \le x) = P(X = 0) + \cdots + P(X = x)$
- binomcdf(n,p,x)

Binomial PDF

To compute the binomial probability of a particular number of successes:



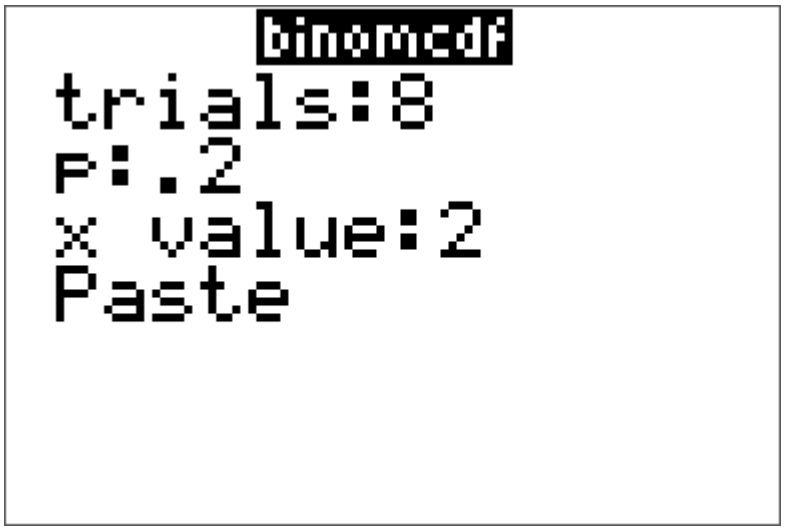
$$\text{Binom}(n = 8,\ p = 0.2)$$

$$P(X = 2) = \binom{8}{2}(0.2)^2(0.8)^6$$

See TI Tips box on p. 422

Binomial CDF

To compute the binomial probability of the number of successes less than or equal to certain number:



$$\text{Binom}\left(n = 8,\ p = 0.2\right)$$

$$P\left(X \le 2\right) = P\left(X = 0\right) + P\left(X = 1\right) + P\left(X = 2\right)$$

See TI Tips box on p. 422

Binomial Model

(Back to LeBron …)

Suppose you are willing to buy 8 boxes of cereal (recall that 20% of boxes have LeBron cards) in order to get as many LeBron cards as possible.

4. What is the probability of finding fewer than 4 LeBron?

Binomial Model

(Back to LeBron …)

Suppose you are willing to buy 8 boxes of cereal (recall that 20% of boxes have LeBron cards) in order to get as many LeBron cards as possible.

5. What is the probability of finding 4 or more LeBrons?

Binomial Model

(Back to LeBron …)

Suppose you are willing to buy 8 boxes of cereal (recall that 20% of boxes have LeBron cards) in order to get as many LeBron cards as possible.

6. What is the mean and standard deviation?

One more example …

An Olympic archer is able to hit the target's bull's-eye with 80% accuracy (assume individual shots are independent of each other).

1. What is the probability she misses her first bull's-eye on her third shot?
2. What is the probability she makes at least 5 out of 6 shots?

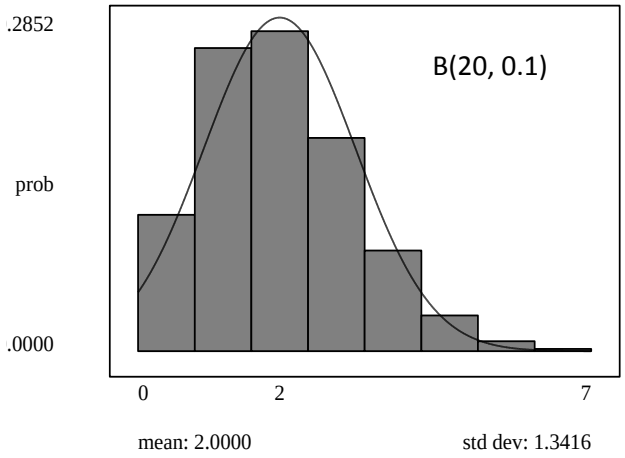
Preview of Coming Attractions …

If the number of trials is large enough that

$$np \geq 10 \quad \text{and} \quad nq \geq 10$$

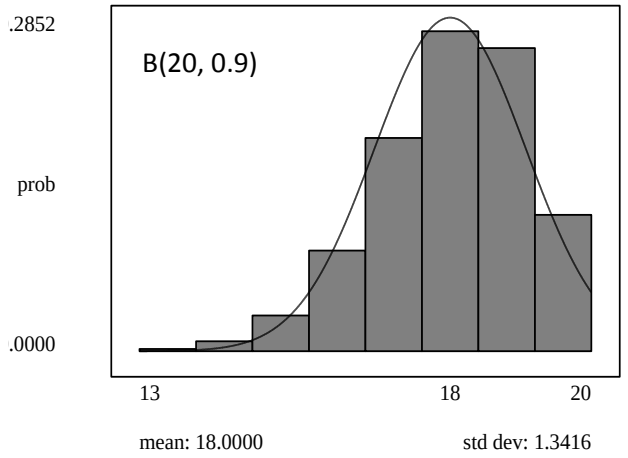
a **Binomial Model** (discrete random variable) can be *approximated* with a **Normal Model** (continuous random variable) with the right parameters.

Binomial ~ Normal



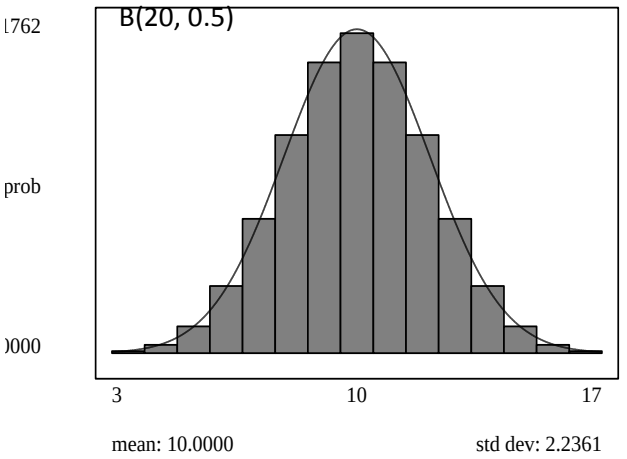
$$np = (20)(0.1) = 2$$

$$nq = (20)(0.9) = 18$$



$$np = (20)(0.9) = 18$$

$$nq = (20)(0.1) = 2$$



$$np = (20)(0.5) = 10$$

$$nq = (20)(0.5) = 10$$

Binomial ~Normal

A recent survey asked a nationwide random sample of 2500 adults if they agreed or disagreed that "I like buying new clothes, but shopping is often frustrating and time-consuming." Suppose that 60% of all adults would agree if asked this question. What is the probability that the sample proportion who agree is from 58% to 62%?

1. What model can be used?

Binomial ~Normal

A recent survey asked a nationwide random sample of 2500 adults if they agreed or disagreed that "I like buying new clothes, but shopping is often frustrating and time-consuming." Suppose that 60% of all adults would agree if asked this question. What is the probability that the sample proportion who agree is from 58% to 62%?

2. What are the mean and standard deviation?

Binomial ~Normal

A recent survey asked a nationwide random sample of 2500 adults if they agreed or disagreed that "I like buying new clothes, but shopping is often frustrating and time-consuming." Suppose that 60% of all adults would agree if asked this question. What is the probability that the sample proportion who agree is from 58% to 62%?

3. What is the probability that the sample proportion is between 58% and 62%?

Assignment

<u>Assignment 1</u>

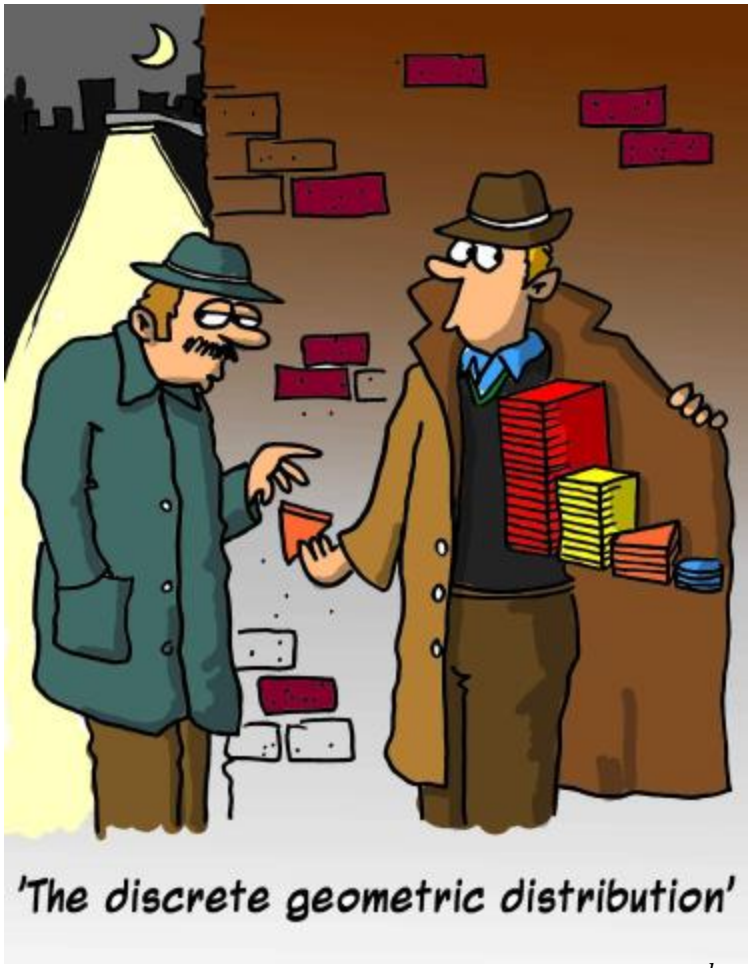
Read Chap 17 pp. 413-418

Exercises #1, 3, 7, 9, 15

<u>Assignment 2</u>

Read Chap 17 pp.418-429

Exercises #17, 21, 25-29 odd, 31, 39



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