Probability: Everyday Decisions Based on Probabilities

II.B Student Activity Sheet 6: Driving and Risk

Javier will be a high school senior next year. He wants to get a vehicle to celebrate his graduation. Javier's mother researched vehicle safety and found that 1 of every 6 teenage drivers was involved in some kind of accident. While talking to his math teacher, Javier mentioned that he did not think the risk was high enough to be concerned. Javier decided to survey 500 students, 230 of whom were male, to help him convince his mother to allow him to get a vehicle. No student has both a car and a motorcycle.

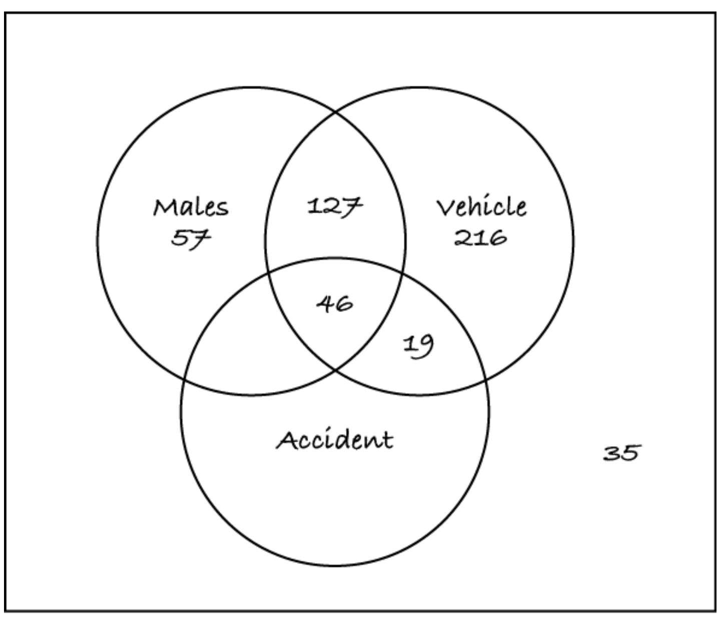
The following are the data from Javier's survey:

	Car	**Motorcycle**
Males with vehicle	150	23
Males involved in accident	40	6
Females with vehicle	225	10
Females involved in accident	15	4

1. Draw a Venn diagram and a tree diagram of the data.

Answers will vary. The following are examples of a possible Venn diagram and tree diagram.

Accident Data



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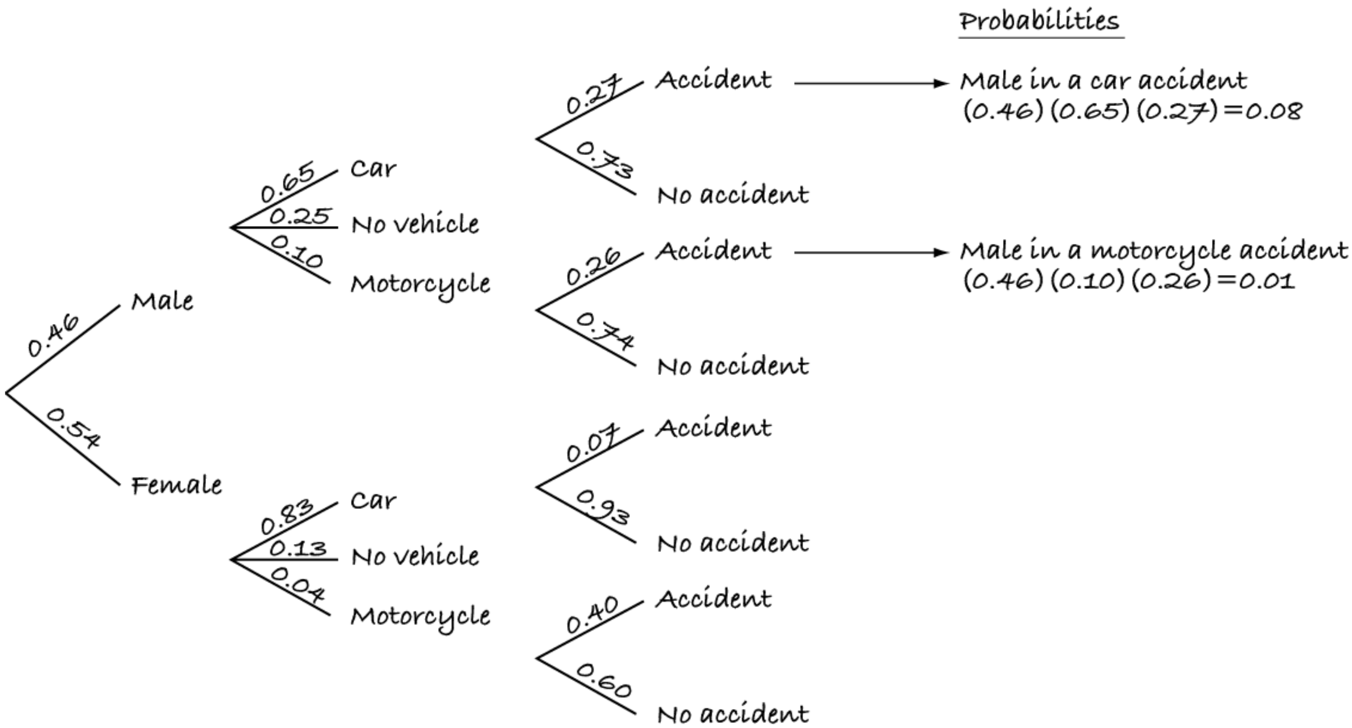
II.B Student Activity Sheet 6: Driving and Risk



2. Using the data, what is the probability that Javier will be involved in an accident if he gets a motorcycle? Explain your reasoning.

Of the 23 males who had a motorcycle, 6 were involved in an accident. Therefore, the probability is $\frac{6}{23}$, or 26%.

3. Based on these survey data, Javier told his mother that he only has a 1% chance of getting in an accident. Is he correct? Why or why not?

Javier is not correct because he is using only the probability if he gets a motorcycle. Since he is also considering a car, the total probability of getting in an accident based on Javier's survey data is 0.08 + 0.01, which equals 0.09, or 9%.

Taking a different approach, Javier could be considerably off in his comment to his mother. It could be argued that 46 of the 230 males surveyed were involved in an accident, or 20%.

4. Use your Venn diagram to write three facts that help Javier convince his mother to let him get a vehicle.

Sample student response:

- Only 46 males were involved in an accident.
- Of the 500 people surveyed, 435 people did not get in an accident.
- 127 males with vehicles did not have an accident.

5. What probability model would you advise Javier to use when he tries to convince his mother?

Answers will vary. Sample student response:

Advise Javier to use the tree diagram. The tree diagram separates the males from the females. The female data helps Javier's cause because females get in fewer accidents per person with a vehicle.

6. **REFLECTION:** List some advantages and disadvantages for each type of model used in this problem.

Advantages

- A tree diagram enables you to see a listing of all the outcomes.
- A tree diagram enables you to isolate events more easily.
- If you have fewer factors to consider in your data, a Venn diagram works nicely.

Disadvantages

- A Venn diagram still needs a large amount of interpretation.
- To determine the probabilities, you must multiply each branch of the tree diagram. Your result(s) is not immediately known.

7. **EXTENSION:** Research your favorite vehicle's safety measures and its likelihood of being involved in an accident. Prepare a short presentation of your findings.

Responses to this Extension will be unique to students' thinking and reasoning skills. Students may communicate their answers in a variety of ways.

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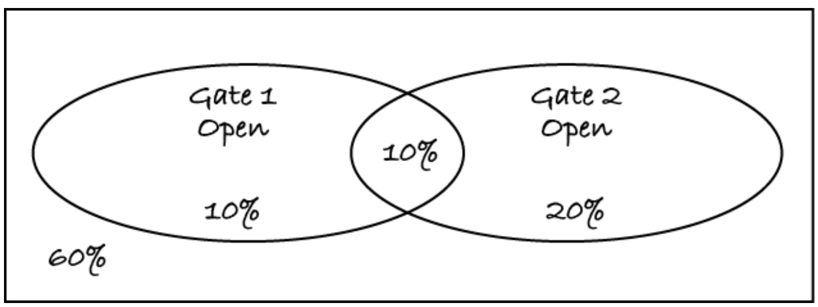
II.B Student Activity Sheet 5: Probability in Games

Victoria is playing a new video game in which the object is to find hidden treasures. To do so, she must travel through several levels, clashing with guards and watchdogs. In one part of the journey, Victoria must pass through two gates (Gate 1, then Gate 2) to get to the next level.

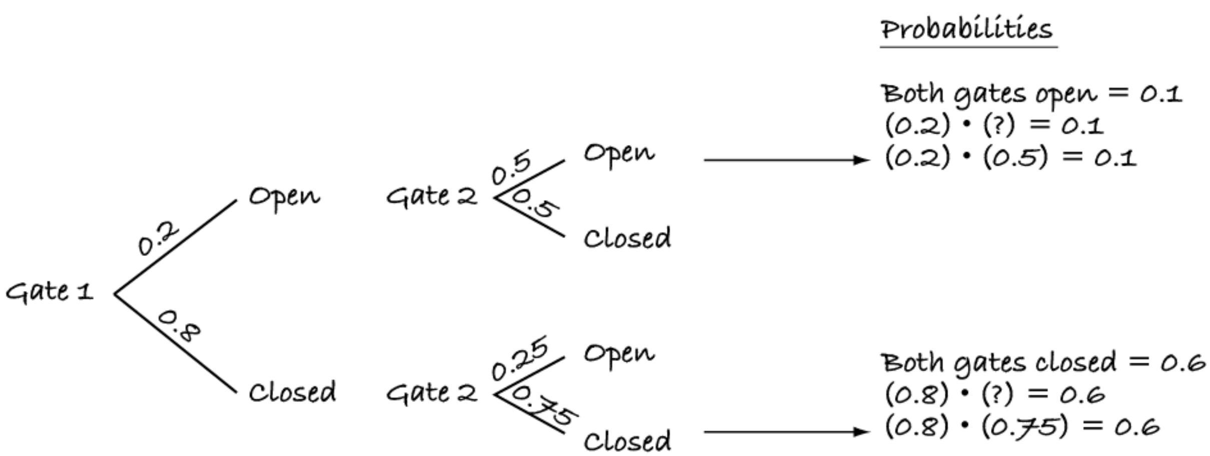
- The chance that Gate 1 is open is 20%.
- The chance that Gate 2 is open is 30%.
- The game designer has programmed the gates so that the probability of both being open at the same time is 0.1.

Draw a model of the situation to help you answer Questions 1–5. Explain why you chose the particular type of model from among the various probability models.

Venn Diagram



Tree Diagram



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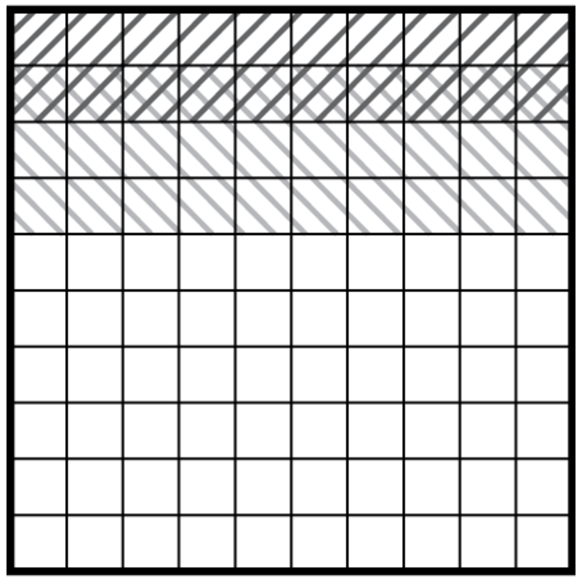
Area model



Gate 1 open –

Gate 2 open –

Neither gate open –

1. What is the probability that both gates are open when Victoria reaches this part of the game? Explain your reasoning.

The game designer has programmed the gates so that both are concurrently open 10% of the time. This probability is shown in the intersection of both the Venn diagram and the area model.

2. What is the probability that only Gate 1 is open when Victoria reaches this part of the game? Explain your reasoning.

The probability of Gate 1 being open is 20%. Part of that time, however, Gate 2 is also open, so P(only Gate 1 open) $= \frac{10}{100}$, or 10%.

3. What is the probability that only Gate 2 is open when Victoria reaches this part of the game? Explain your reasoning.

The probability of Gate 2 being open is 30%. Part of that time, however, Gate 1 is also open, so P(only Gate 2 open) $= \frac{20}{100}$, or 20%.

4. What is the probability that neither gate is open when Victoria reaches this part of the game? Explain your reasoning.

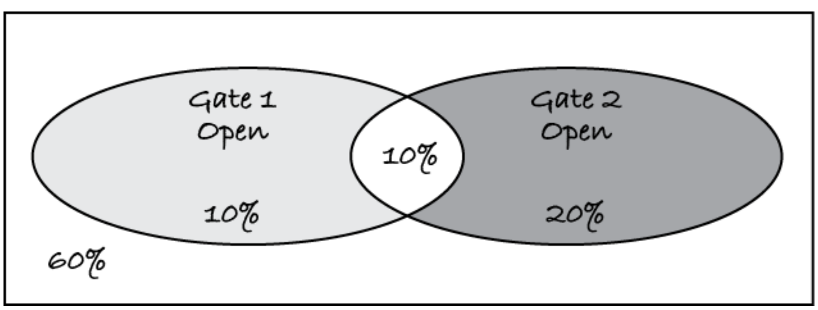
Gate 1 (only) is open 10% of the time, Gate 2 (only) is open 20% of the time, and both are concurrently open 10% of the time. This means one or both gates are open 40% of the time. If students are using a Venn diagram, the probability that Gate 1 *and* Gate 2 are not open is the area outside the two circles.

P(Gate 1 not open or Gate 2 not open) $= \frac{60}{100}$, or 60%

5. What is the probability that Victoria finds exactly one gate open?

To *find exactly one gate open* means one of the gates is open and the other is not. The following events justify this result:

- Gate 1 open and Gate 2 closed or
- Gate 1 closed and Gate 2 open.



The lighter shaded area shows Gate 1 (only) open and the darker shaded area shows Gate 2 (only) open. These are mutually exclusive events; therefore, the probability is $0.10 + 0.20 = 0.30$, or 30%.

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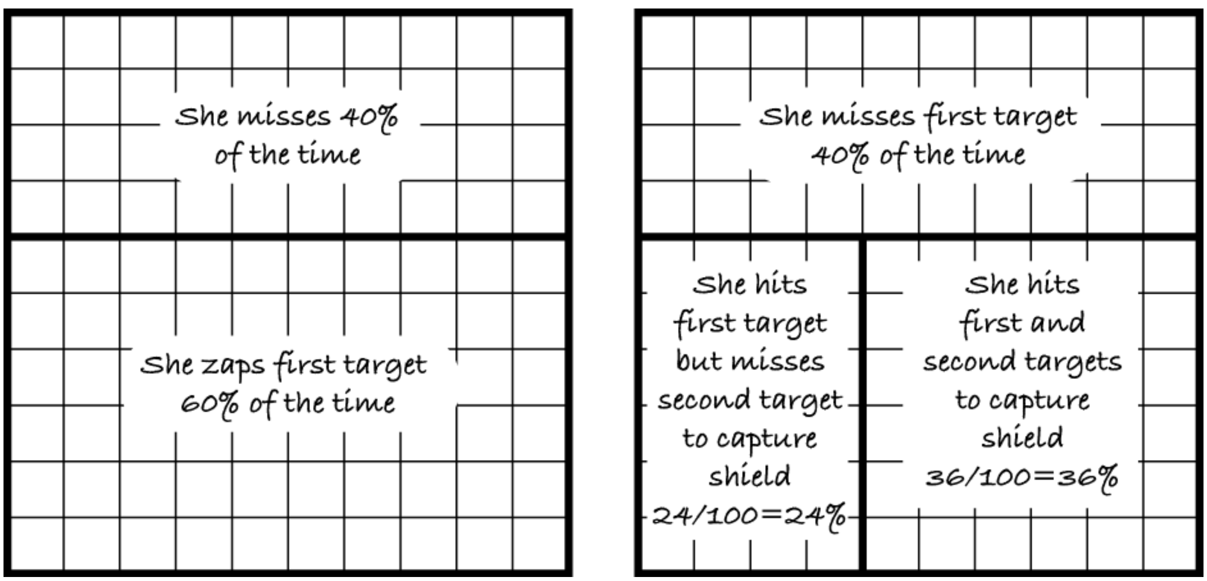
Victoria encounters another challenge in the game. If she zaps a target in one try, Victoria gets a chance to capture a bonus shield. To capture the bonus shield, she must hit a second target in one try. Victoria can hit a target in one try an average of 60% of the time.

Draw a model of the situations to help you answer Questions 6–8.

Answers will vary. This is an example of an area model for the two situations. Students could also use a Venn diagram or tree diagram.



6. What is the probability that Victoria hits the first target? Explain your reasoning.

P(hit first target) = 60%, or 3 out of 5 times $\left(\frac{3}{5}\right)$ *Victoria hits the target because she can hit a target 60% of the time.*

7. What is the probability that Victoria captures the bonus shield? Explain your reasoning.

To capture the bonus shield, Victoria must hit the second target. Since she can hit a target 60% of the time, find 60% of 60%. This means the probability of hitting the second target and capturing the bonus shield is 36%.

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8. What is the probability that Victoria hits the first target and does not hit the second target to capture the shield? Explain your reasoning.

To hit the first target but not capture the shield, Victoria has to hit the first target and miss the second target. The probability of hitting the first target is 60%, and the probability of missing a target is 40%. Therefore, the probability of hitting the first target and not capturing the shield is 24%.

9. REFLECTION: How would you advise your friends who might be interested in playing a new video game?

Responses to this Reflection will show students' different approaches to the question.

10. EXTENSION: Create two probability situations that use conditional probability. Describe the outcomes for these situations.

Answers will vary. Sample student response:

When the probability of Event B depends on Event A, the probability is conditional. For example, a teacher gave her class two quizzes. Only 40% of her class passed both quizzes, and 60% of her class passed the first quiz. What is the probability that those who passed the first quiz also passed the second? You know P(passed first quiz) = 60%, P(passed second quiz) = ?, and P(passed first and second) = 40%.

$$P(\text{passed second/first}) = \frac{P(\text{passed first and second})}{P(\text{passed first})} = \frac{0.40}{0.60} \approx 0.67 \approx 67\%$$

In another example, students are asked to roll a number cube. What is the probability of rolling an even number? What is the probability that the roll is a 2?

$P(\text{even}) = \frac{3}{6}$ or $\frac{1}{2}$, $P(2) = \frac{1}{3}$, $P(2/\text{even})$?

$$P(2/\text{even}) = \frac{P(2 \text{ and even})}{P(\text{even})} = \frac{\frac{1}{3} \bullet \frac{1}{2}}{\frac{1}{2}} = \frac{\frac{1}{6}}{\frac{1}{2}} = \frac{1}{3}$$