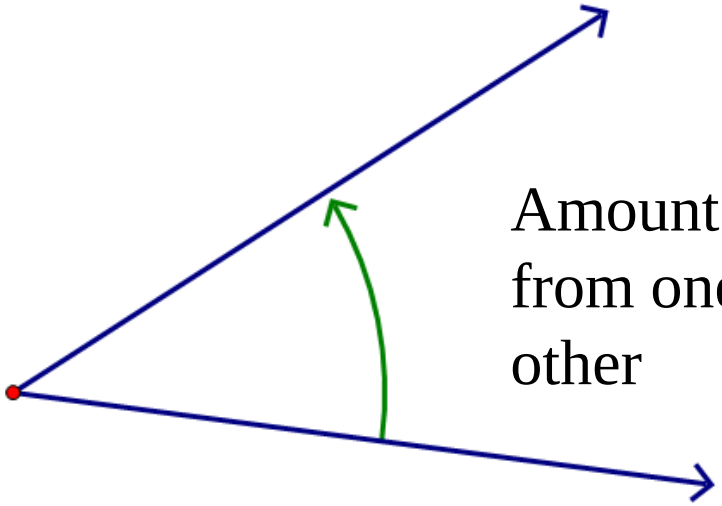
The Measure of an Angle

In geometry, we **measure an angle** by the smallest amount of rotation about the vertex from one side to the other.

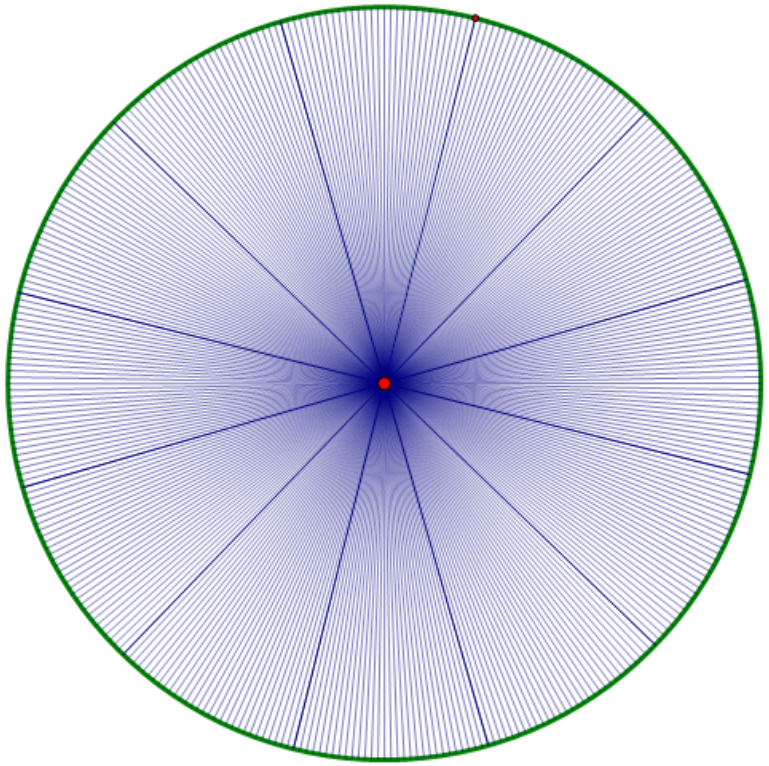


Amount of rotation from one side to the other

In geometry, we usually measure angles in degrees, from 0° to 180°.

How Big is a Degree?

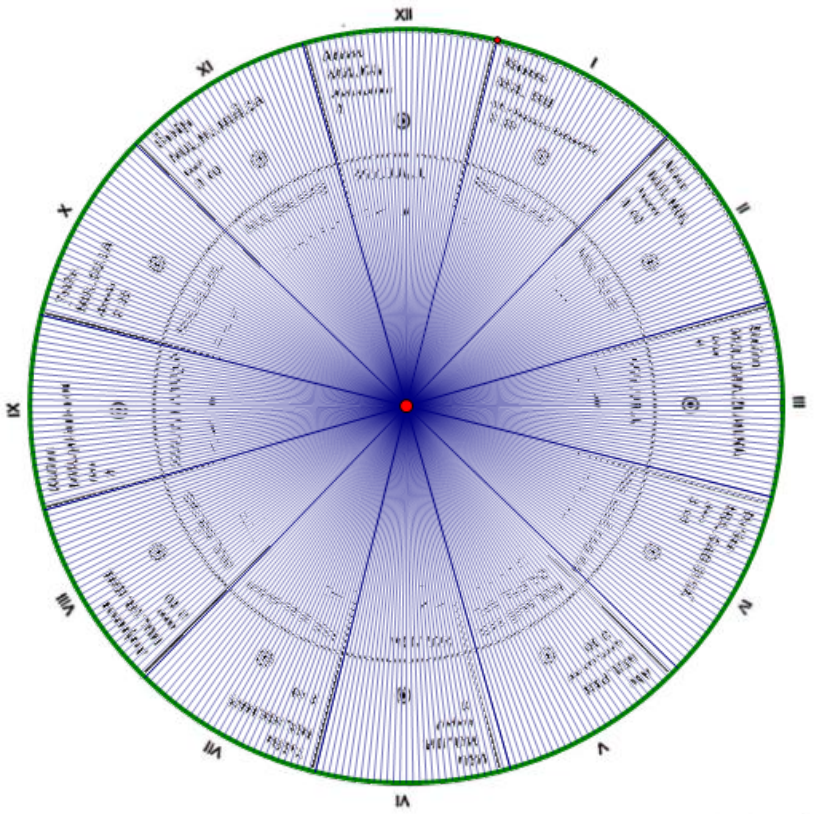
Just how big is a degree? Well, imagine taking a circle, and with a pizza cutter, dividing that circle into 360 congruent parts. Each one of those would be a degree.



Five Days Short…

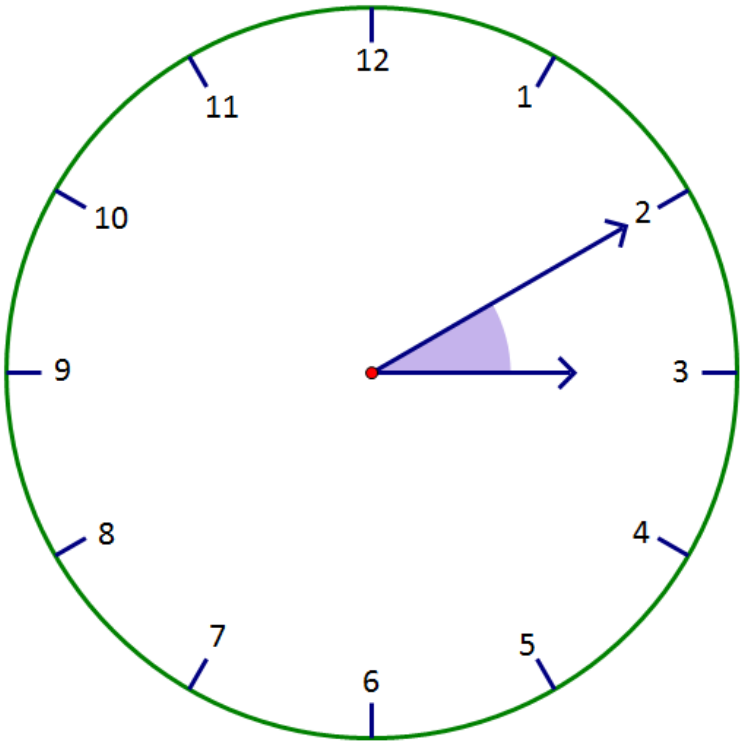
The curious among you may be wondering why there are 360 degrees in a circle, while others may not even care. The answer is actually pretty simple: It's because there are 360 days in the year. At least that's what the Babylonians thought, and they are the ones who came up with the crazy idea called a degree.

Five Days Short…



Each year, of course, is made up of 12 “months.” Further, each of those “months” is made up of 30 “days.” 12 times 30 equals 360 degrees, I mean days.

Example 1

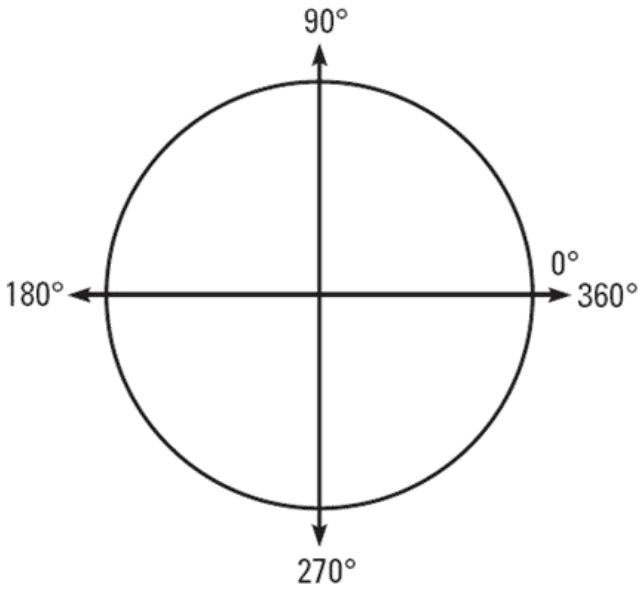


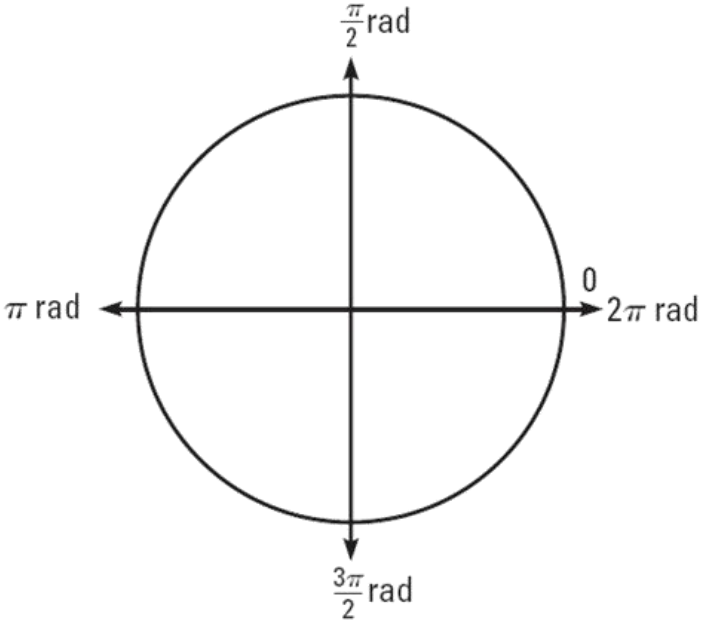
1. On a clock, how many degrees does the hour hand rotate each hour?
2. How many degrees does the minute hand rotate each minute?

Radians

You can also measure an angle in **radians**.

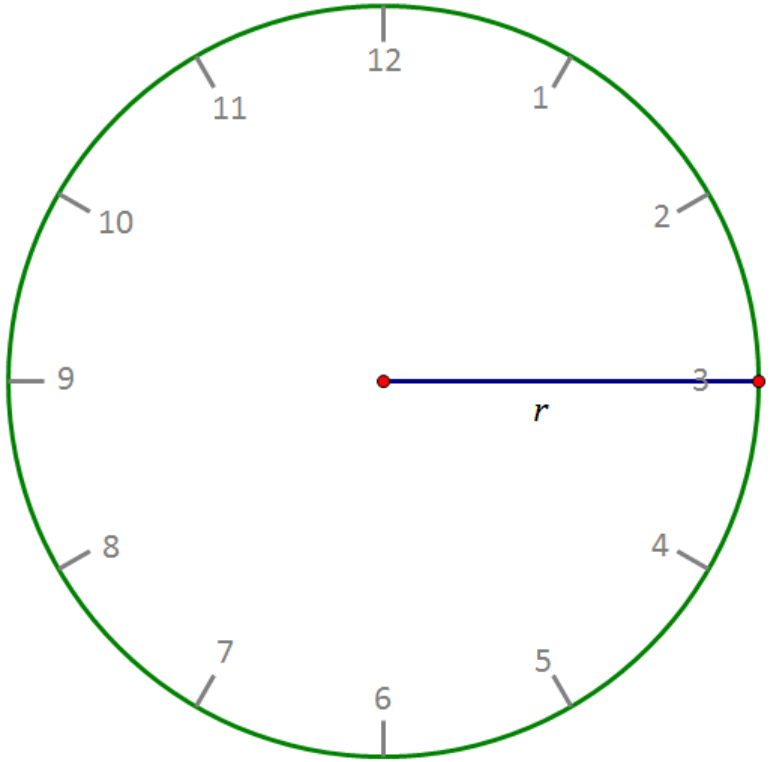
Radians are like the less well-known greasy, nerdy-type who eats lots of pie.





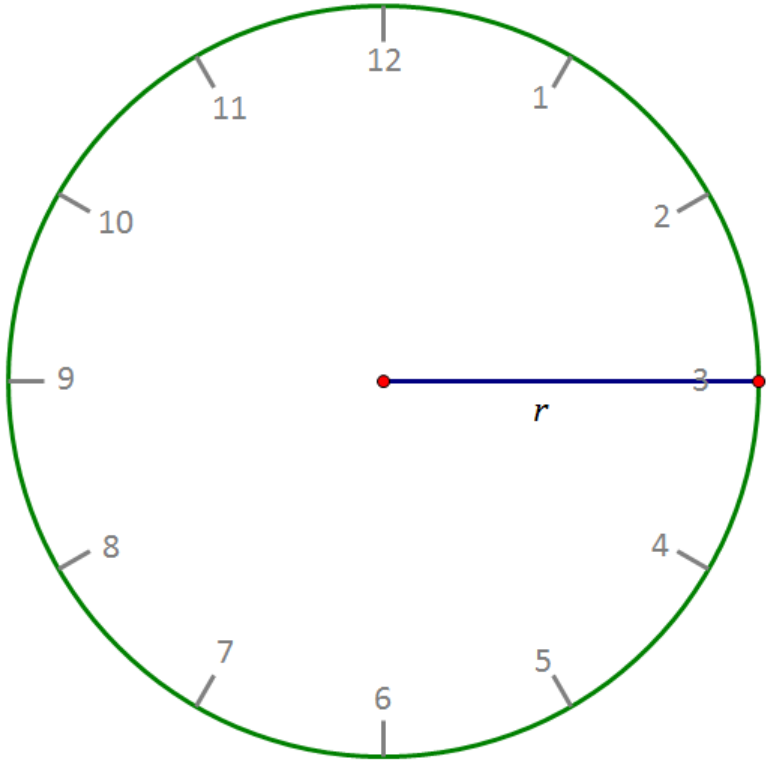
Radians

Here's an interesting question: If you were to take the radius of a circle and wrap it around the circle's circumference, how far would it reach?



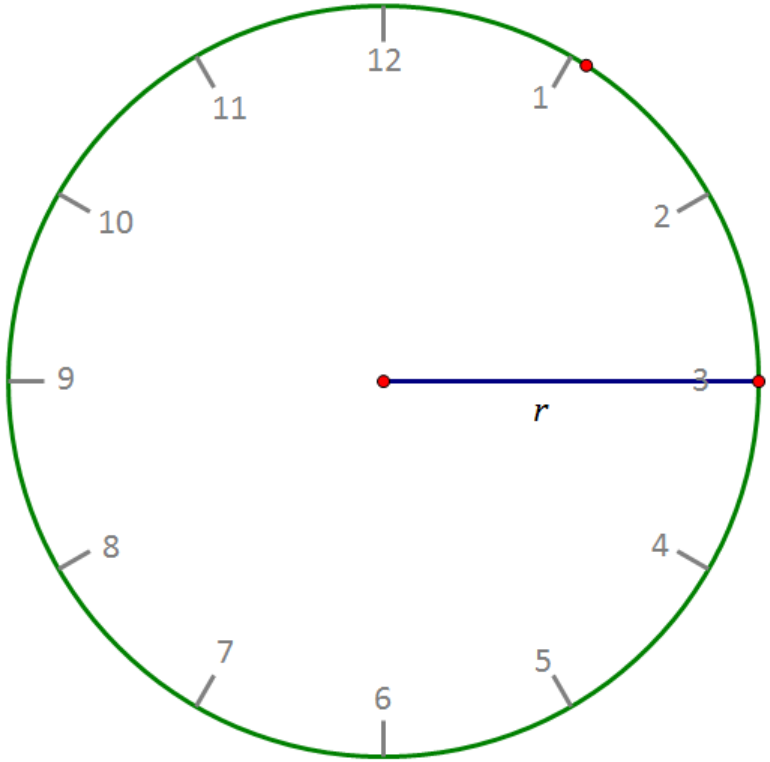
Activity: Radians

Let's draw a radius on this circle that points to the "3." This will be the initial side of an angle.



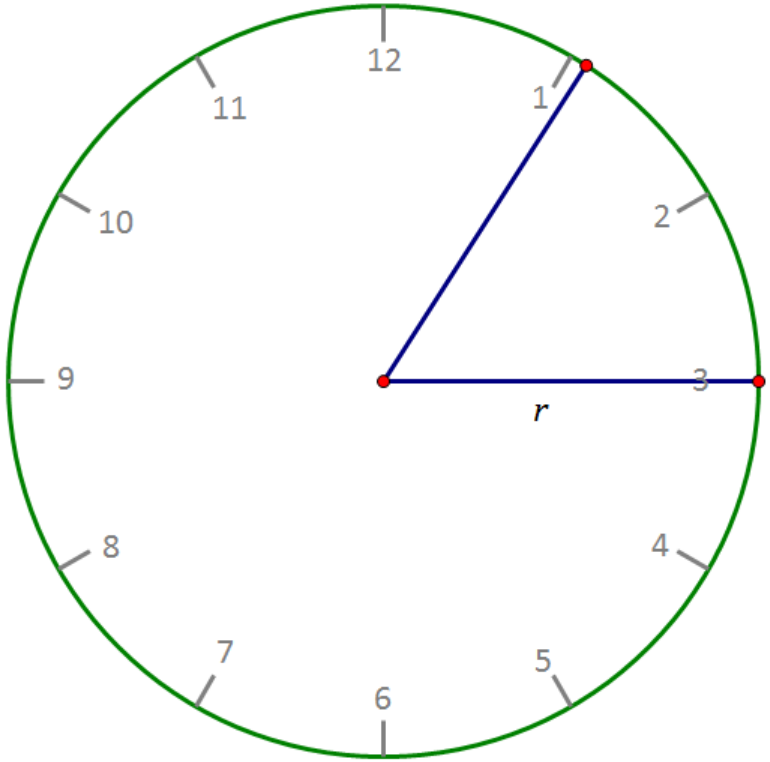
Activity: Radians

Now, let's say we measured the length of that radius, and with a really bendy ruler, marked that same length on the circle starting from the 3.



Activity: Radians

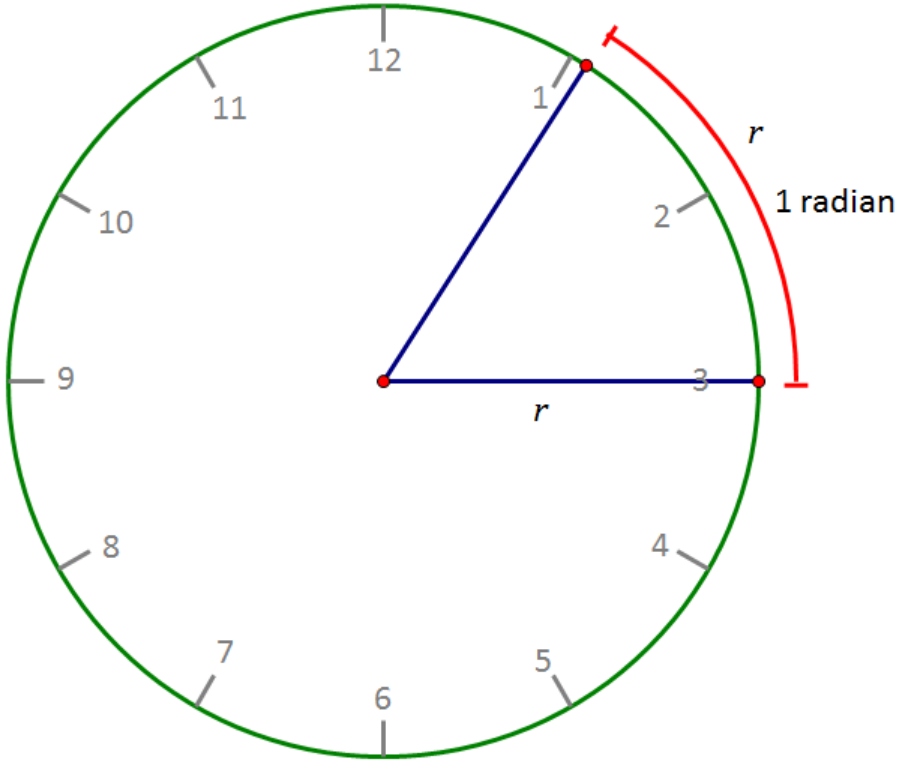
Next, we'll connect this mark to the center of the circle with another radius. This is the terminal side of an angle we'll call θ.



Activity: Radians

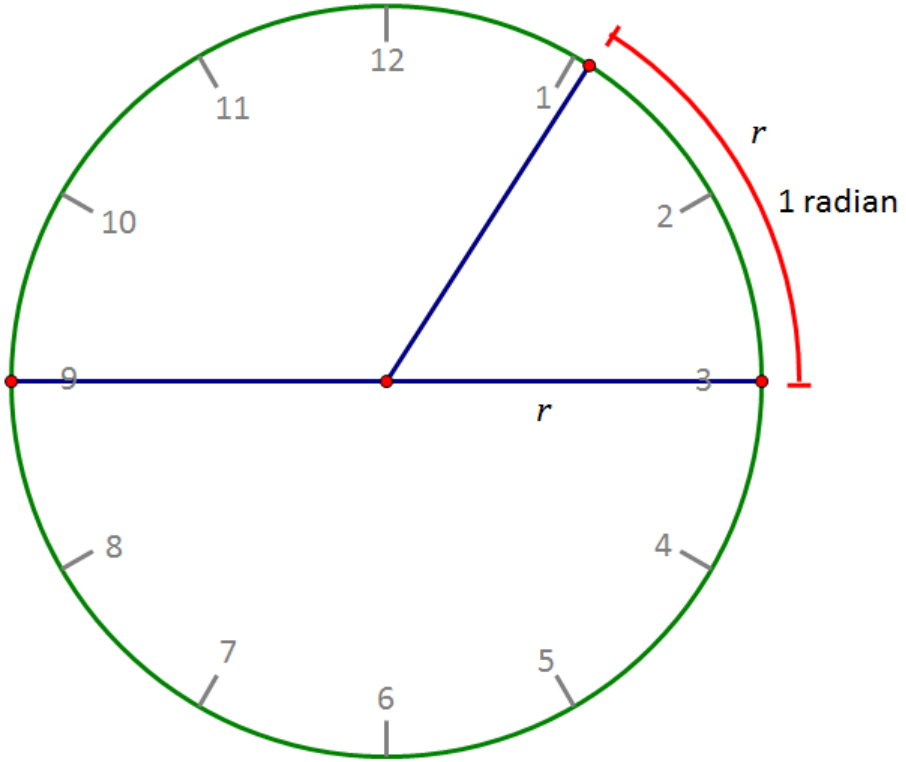
The arc that intercepts θ has length 1 **radian**, so we say the measure of $\theta = 1$ radian.

Approximately how many degrees is 1 radian?



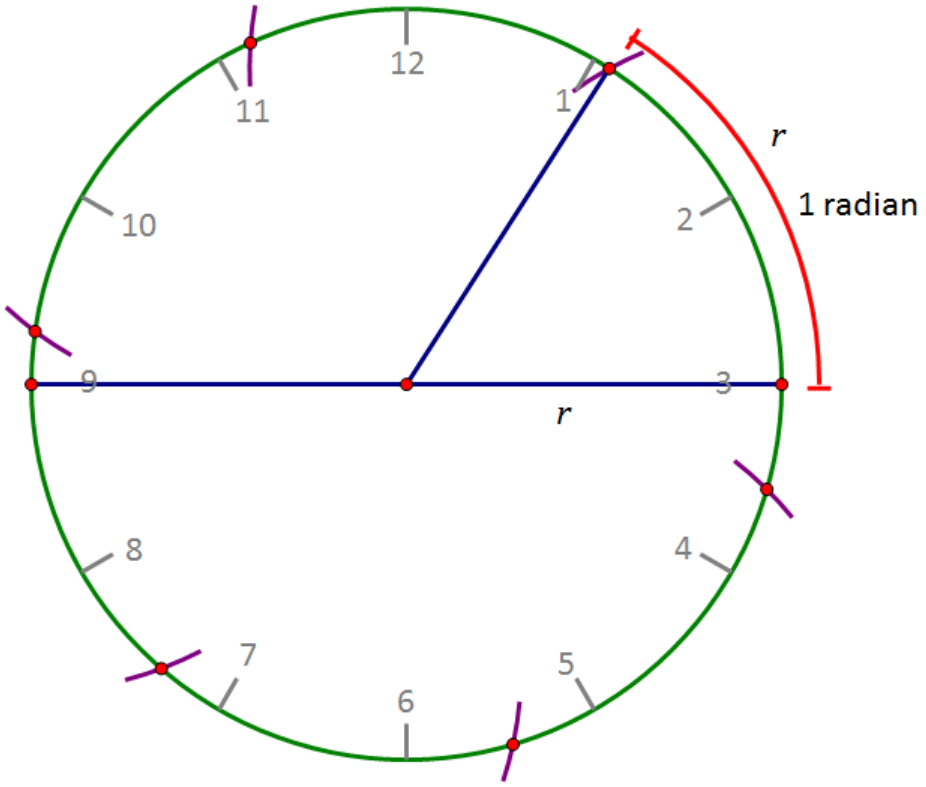
Activity: Radians

Now let's see how many radians it takes to span the circle. First, I'll connect the 3 to the 9 to make a diameter.



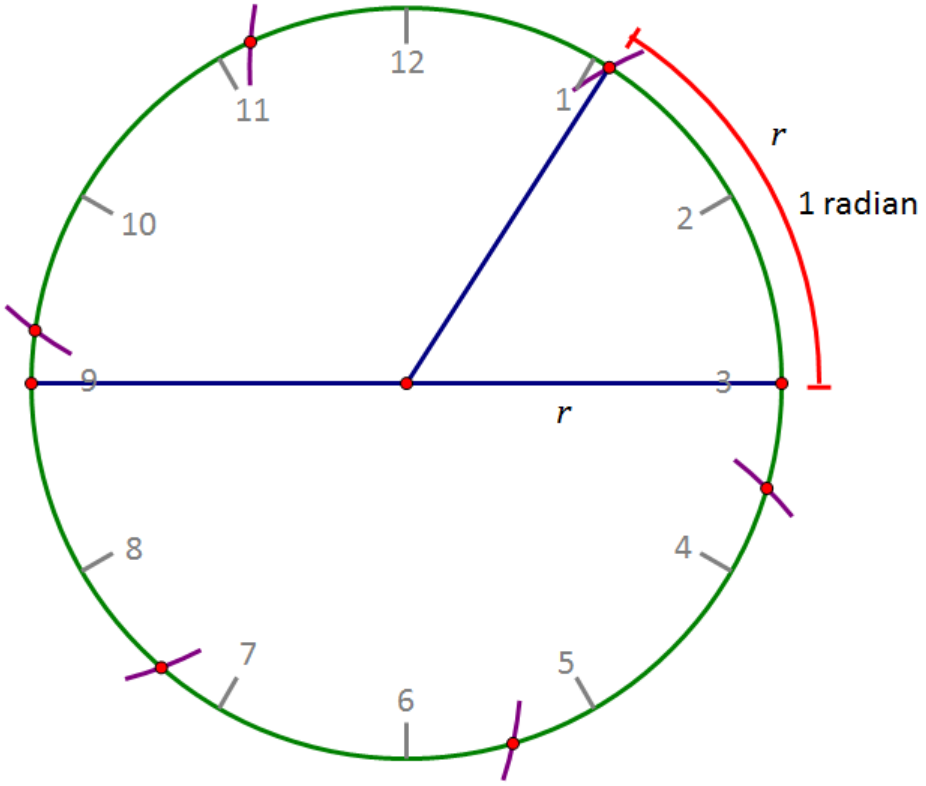
Activity: Radians

You could use a compass to measure the radian arc length. Then you could copy this length around your circle multiple times until you have gone (nearly) all the way around.



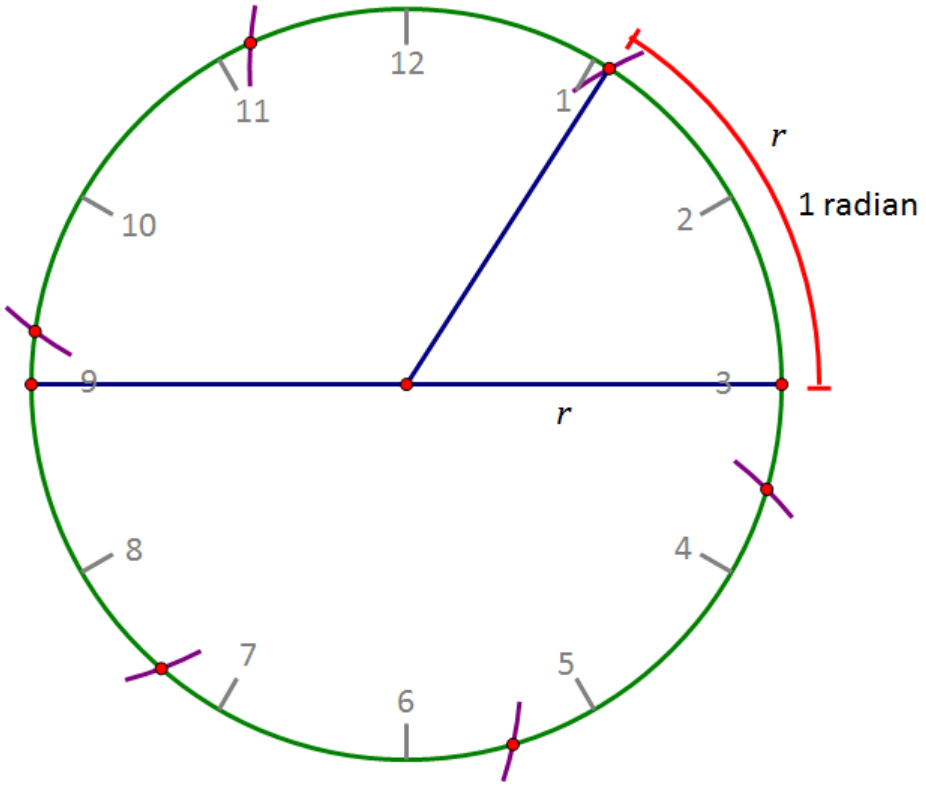
Activity: Radians

You should notice that it takes a little bit more than 3 radians to span a semicircle. In fact, it takes exactly π (≈3.14) radians.



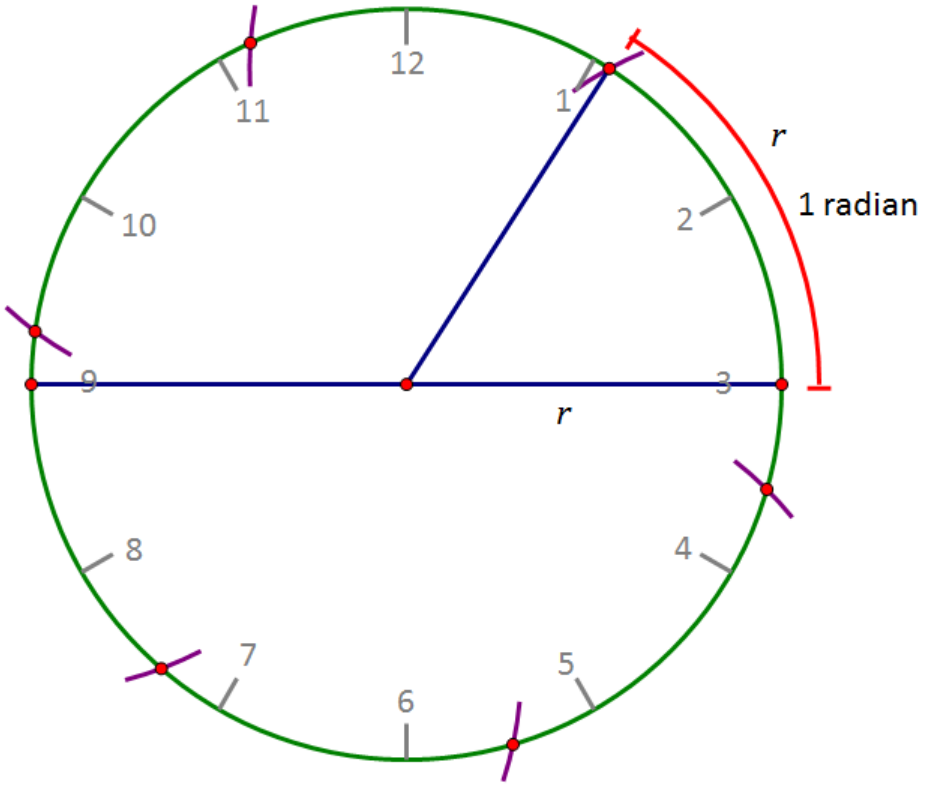
Activity: Radians

Also notice that it takes a bit more than 6 radians to span the full circle, which is exactly 2π (≈6.28) radians.



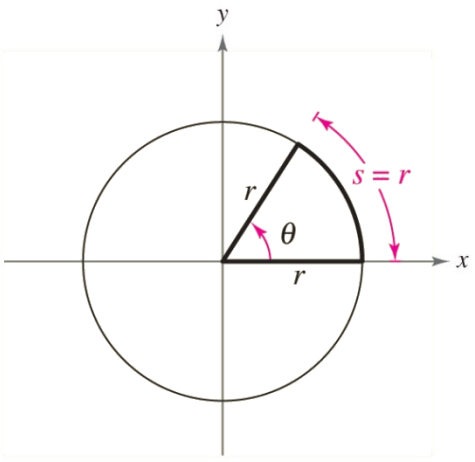
Activity: Radians

This should make sense since the circumference of a circle is $2\pi r$, where r is the radius of the circle.

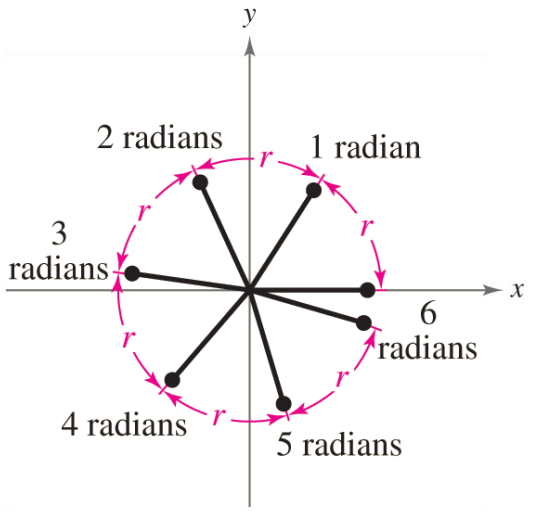


Radians

One radian is the measure of the angle formed by stretching the radius of a circle around its circumference.



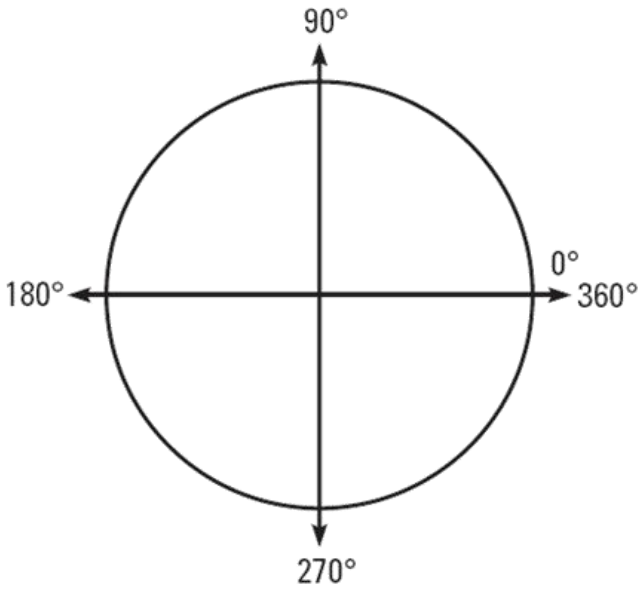
Arc length = radius when θ = 1 radian

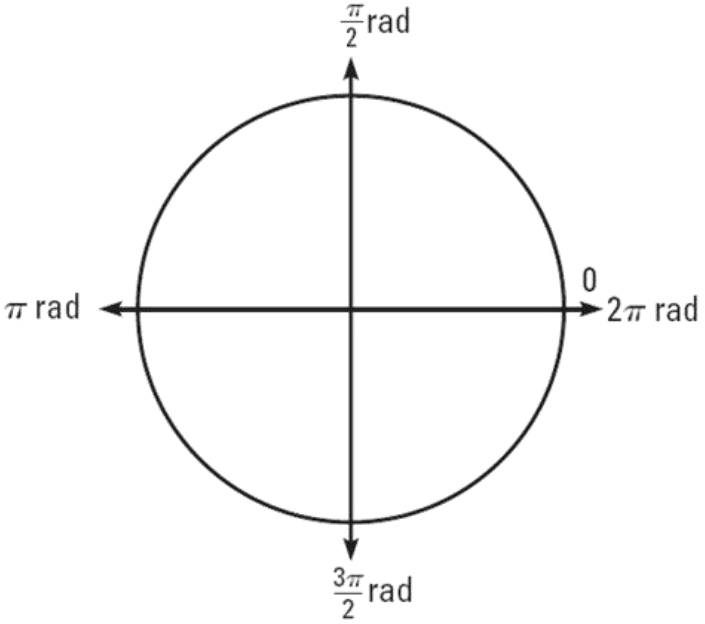


Example 2

How many radians would be the equivalent to one full revolution around the unit circle?

How many radians would equal 180°?





Example 3a

Use the conversion factor 180° = π radians to convert the following angle measures.

1. Convert 27° into radians.

Example 3b

Use the conversion factor 180° = π radians to convert the following angle measures.

2. Convert 150° into radians.

Example 4a

Use the conversion factor 180° = π radians to convert the following angle measures.

1. Convert $\frac{\pi}{6}$ rad into degrees.

Example 4b

Use the conversion factor 180° = π radians to convert the following angle measures.

2. Convert $\frac{3\pi}{4}$ rad into degrees.