Chapter 5
Functions and Their Representations

Mathematical Overview

Throughout this chapter, students learn to use more than one representation to model given situations, know the reasons why they might need to do this, and select the representation that is most appropriate for their needs. As they progress through the chapter they learn to graph a linear equation using a table of values, using the slope-intercept form of the equation and using just the intercepts. Likewise, given the slope and y-intercept of a line, or a point and a line either parallel or perpendicular to the line, students write an equation of the line.

Lesson Summaries

Lesson 5.1 ***Investigation: Representing Functions***

In this lesson, students investigate various types of representations that can be used to describe and represent functions. They use many of these representations during the Investigation to describe a function that could help a business determine the price of a product. As students progress through the lesson and practice items, they move from one form of representation to another.

Lesson 5.2 ***Activity: Linear Relationships***

In this Activity, students collect data and use what they know about direct variation, slope, and rate of change to analyze the data. During the Activity students identify independent and dependent variables, linear relationships, and the difference between discrete and continuous variables. This lesson concludes with students using a graph, table, or equation to make a prediction.

Lesson 5.3 ***Investigation: Identifying Linear Functions***

In this lesson, students focus on one particular type of function, the linear function. They examine the characteristics of linear functions and learn to identify them by examining equations, graphs, and tables. The Investigation focuses on depreciation of the value of a computer used for business purposes. Students determine that the depreciation function is a linear function.

Lesson 5.4 ***R.A.P.***

In this lesson, students **R**eview **A**nd **P**ractice solving problems that require the use of skills and concepts taught in previous math levels. The skills reviewed in this lesson are skills that are needed as a basis for solving problems throughout this course.

Lessons 5.5 ***Graphing Linear Equations and***
5.6 ***Investigation: Writing Equations of Lines***

In Lesson 5.5, students use their knowledge of slope and plotting points to graph linear equations to get a visual summary of the relationship that exists between the two variables. They graph equations using the slope and the y-intercept, using the x- and y-intercepts, and using a graphing calculator. Then, in Lesson 5.6, the Investigation provides students with a graph from which they must describe what the slope and intercepts tell them about the relationship between the two variables. Finally, students learn to write an equation of a line when they do not have a graph, but are given two points on the line or one point and the slope of the line.

Lesson 5.7 ***Slopes of Parallel and Perpendicular Lines***

In this lesson, students investigate the slopes of parallel and perpendicular lines and use the information to find the equations of lines parallel (or perpendicular) to a given line.

Chapter 5 ***Extension: Special Functions***

In this Extension, students graph absolute value functions, a subset of piece-wise functions. Students also examine one example of a step function, the greatest integer function, $y = [\![x]\!]$. Given a table of values and a graph of the function, students determine the domain and range, evaluate the function for several given values, and then use the function to model a given situation.

Lesson Guide

Lesson/Objectives	Materials	
Chapter 5 Opener: How Can You Find the Value of an Unknown Quantity? • recognize that equations can be used to predict the value of an unknown quantity from a value of a known quantity.		
5.1 Investigation: Representing Functions • represent functions in multiple ways. • identify the domain and range of a function. • identify the problem domain of a situation.		*Optional:* • grid paper • TRM table shell for Question 3. • TRM table shell for Exercise 14.
5.2 Activity: Linear Relationships • collect and analyze data that have a constant rate of change.	*Per group:* • scale (grams or tenths of an ounce) • 10 objects, uniform in weight, weighing 0.2–0.4 oz (marbles, miniature candy bars, large metal washers, and 10-penny nails) • container weighing between 3 and 6 ounces that is large enough to hold the 10 objects	*Optional:* • grid paper • 100-cm rope • ruler • TRM table shell for Question 2.
5.3 Investigation: Identifying Linear Functions • identify linear functions from tables, graphs, and equations.		*Optional:* • grid paper • Slinky® • yardstick or meter stick • small paper cup • paper clips • marbles • TRM table shell for Question 1.
5.4 R.A.P. • solve problems that require previously learned concepts and skills.		
5.5 Graphing Linear Equations • graph a linear equation using a table. • graph a linear equation in slope-intercept form. • graph a linear equation using the intercepts.		*Optional:* • grid paper
5.6 Investigation: Writing Equations of Lines • write an equation of a line given its slope and y-intercept. • use the definition of slope to write an equation of a line.		*Optional:* • grid paper
5.7 Slopes of Parallel and Perpendicular Lines • write an equation of a line that passes through a given point and is parallel to a given line. • write an equation of a line that passes through a given point and is perpendicular to a given line.		*Optional:* • grid paper
Chapter 5 Extension: Special Functions • graph absolute value functions. • graph piecewise-defined functions. • graph step functions.		*Optional:* • grid paper

See page 135 for the *Pacing Guide* and *Supplement Support.*

CHAPTER 5

Functions and Their Representations

CHAPTER 5

Functions and Their Representations

CONTENTS

Pacing Guide

	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7	Day 8	Day 9	Day 10
Basic	p. 136, 5.1	5.1	5.2	5.3	5.4	5.5	5.6	5.7	project	review
Standard	p. 136, 5.1	5.1, 5.2	5.2, 5.3	5.3, 5.4	5.4, 5.5	5.5, 5.6	5.6, 5.7	5.7, project	project, review	extension
Block	p. 136, 5.1	5.2, 5.3	5.3, 5.4	5.5, 5.6	5.7, project	review, extension				

Supplement Support

See the Book Companion Website at www.highschool.bfwpub.com/ModelingwithMathematics and the Teacher's Resource Materials (TRM) for additional resources.

CHAPTER 5 OPENER

5e Engage

Lesson Objective

- recognize that equations can be used to predict the value of an unknown quantity from the value of a known quantity.

Vocabulary

none

Description

This reading helps students see that mathematics is used in their everyday lives.

TEACHING TIP

After students have read the Chapter Opener, lead a whole-class discussion asking students to name some pairs of measurable quantities that are related to each other in such a way that if you know the value of one quantity, you can determine the value of the other.

Sample answers might include *geometric relationships,* such as finding the area of a circle given its radius, *monetary relationships,* such as calculating cost given time and resources used, and *forecasting,* such as predicting future demand for a product based on past trends.

How Can You Find the Value of an Unknown Quantity?

You may be familiar with Chuck E. Cheese's®, a chain of family entertainment centers headquartered in Irving, Texas. These centers provide arcade games, rides, prizes, food, and entertainment. As patrons win games, they receive tickets that can then be redeemed for prizes. During a single visit, a child might receive hundreds of these tickets.

In the early years of the company, there were no ticket-counting machines as there are today. Rather than count the hundreds of tickets one by one, an employee placed them on a scale. The scale was used to indirectly determine the number of tickets. How did a scale tell the employee how many tickets there were?

It was the connection between the weight of the tickets and the number of tickets that allowed the coupons to be "counted" indirectly. Once the weight of the tickets was found, the employee used a table, looked up the weight in a table, and found the number of tickets associated with that weight.

In this chapter, you explore many different input/output relationships and see how they can be used to predict the value of an *unknown* quantity from the value of a *known* quantity.

Lesson 5.1

INVESTIGATION: Representing Functions

In Chapter 2, you explored quantities that varied directly. You found that two quantities are directly proportional if the ratio of the two quantities is constant. You used tables, graphs, and simple equations to describe these proportional relationships. In this lesson, you will investigate various representations that can be used to describe functions.

Recall

When two variables are related in such a way that for each *input* value there is exactly one *output* value, the relationship is called a *function*.

TYPES OF REPRESENTATIONS

As you have seen in previous chapters, functions can be represented in several ways.

- by verbal descriptions
- by symbolic rules
- by diagrams, such as arrow diagrams
- by tables
- by graphs
- by manipulatives

In this Investigation, you will use many of these representations to describe a function that could help a business determine the price of a product.

Many businesses involve selling products. The total dollar amount received from product sales is called *revenue*. The revenue from a single product depends on its selling price. So, it is sometimes possible to express revenue as a function of price.

1. Suppose you decide to start a small business selling T-shirts in a resort town. On the first day, you decide to charge $10 for each shirt. If no one buys a shirt for $10, what is your total revenue?
2. Now suppose that on the next day, you reduce the selling price to $9. Ten people buy shirts at this price. What is your total revenue for that day?
3. To decide on the best price, you might use a table to see how different prices affect your total revenue. Suppose that reducing the price to $8 results in 20 sales and reducing the price to $7 results in 30 sales. Assume this pattern continues. Complete the table on the next page to show how the total revenue R depends on the selling price p.

Lesson 5.1 Investigation Answers

1. $0
2. $90

3.

Selling Price ($)	Number of Sales	Total Revenue ($)
10	0	0
9	10	90
8	20	160
7	30	210
6	40	240
5	50	250
4	60	240
3	70	210
2	80	160
1	90	90
0	100	0

LESSON 5.1

5e Explore

Lesson Objectives

- represent functions in multiple ways.
- identify the domain and range of a function.
- identify the problem domain of a situation.

Vocabulary

- domain
- intercept
- problem domain
- range

Materials List

none

Description

This Investigation works best if students work in pairs. Before beginning, review the definition of *function* in the **Recall** on the first page.

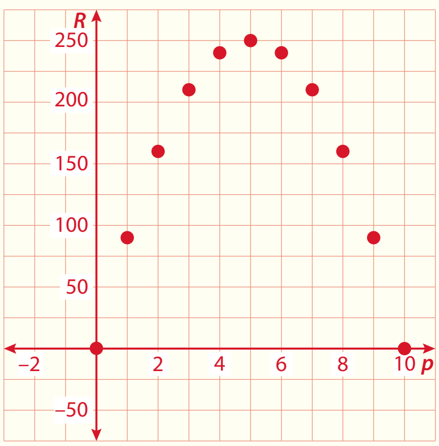
Have students work through the Investigation and the additional reading material after **Question 10.** Make sure that students pay particular attention to the new vocabulary that is introduced in this lesson.

Wrapping Up the Investigation:

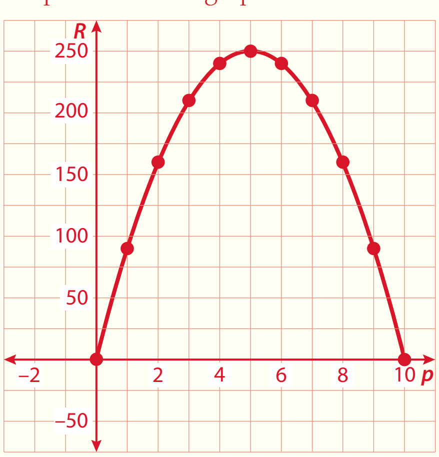
Once all groups have completed the Investigation, debrief their results by asking what different types of representations they used or observed in the lesson.

Make sure that all students understand the difference between the (mathematical) *domain* of a function and the *problem domain* that applies when a function is used to model a real-world situation.

4. Independent variable: selling price p; dependent variable: revenue R. The amount of revenue depends on the selling price of the shirt.
5. There is exactly one revenue amount for each price.
6.



7. Sample answer: Price values for each of the points on the curve result in revenues that fit the pattern of the graph.



8. No, the graph should not extend to the left because negative prices make no sense in this context. The graph could be extended to the right, but no T-shirts would be sold at prices greater than $10, so the R-values would all be 0.
9. There are two p-intercepts at the points (0, 0) and (10, 0). There is only one R-intercept. Its coordinates are (0, 0). These intercepts mean that either a price of $0 or $10 will produce $0 in revenue.
10. $5; $250

Selling Price ($)	Number of Sales	Total Revenue ($)
10	0	0
9	10	
8	20	
7		
6		
5		
4		
3		
2		
1		
0		

Note

Not all tables represent functions. The table below shows the relationship between time worked and wages earned at a local store. At this store, some employees are paid overtime after working 40 hours, and others are not. Notice that the input value of 50 hours has two output values, $280 and $385. Hence, this table does not represent a function.

Time Worked (hr)	Wages ($)
20	140
30	210
40	280
50	280
50	385

4. Consider the variables selling price p and revenue R from your table. Which variable is the independent variable? Which is the dependent variable? Explain.

5. Explain why your table suggests that revenue is a *function* of selling price.

6. Using your values from Question 3, construct a scatter plot of revenue R versus selling price p.

7. Connect the points in your graph with a smooth curve. Your curve now includes points for ordered pairs (p, R) that were not in your table. What could these points mean?

8. For this situation, would it make sense to extend the graph to the left and right of the plotted points? Explain.

9. Any point where a graph touches or crosses either of the coordinate axes is called an **intercept**. An intercept is usually given the name of the appropriate variable. In this case, it is possible to have a p-intercept, an R-intercept, or both. What are the coordinates of the intercepts on your graph? Interpret the meaning of these intercepts.

10. For what price should you sell the T-shirts in order to produce maximum revenue? What is the maximum revenue?

DOMAIN AND RANGE

The revenue function that you have been exploring can be written in symbolic form: $R = 100p - 10p^2$. This equation gives revenue R as a function of price p.

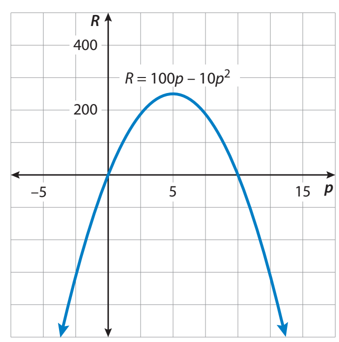
You can verify that this equation models the situation by inserting price values from your table into the equation. For example, when $p = \$10$, $R = 100(10) - 10(10^2) = 0$. And when $p = \$7$, $R = 100(7) - 10(7^2) = 210$.

First consider this function without any real-life restrictions. Any input value can be substituted for p in the function equation $R = 100p - 10p^2$ to produce a real output value. This includes all positive and negative numbers as well as 0. Hence, the domain of this function is unlimited. It includes all real numbers.

> The **domain** of a function is the set of all possible input values for which the function is defined.

As you saw from the graphs in Questions 6 and 7, there is a maximum value of 250 for R. So, the range of the function $R = 100p - 10p^2$ has an upper value of 250. But it has no lower limit. Negative values for R can result from certain input values such as $p = 12$, for which $R = 100(12) - 10(12^2) = -240$.

> The **range** of a function is the set of all possible output values that the function generates from its domain.



Examine the graph of the function $R = 100p - 10p^2$. Since the domain of the function includes all real numbers, the graph would extend indefinitely to the left and to the right.

If you reexamine this function as it applies to a real-world situation, you may find that input values that make sense for the independent variable may be only a portion of the domain of the function. This restricted domain is often called the **problem domain**. The problem domain of a function includes only numbers that

- are actually reasonable values for the independent variable and
- will produce reasonable values for the dependent variable.

TEACHING TIP

Throughout this text, the domain of a function will sometimes be referred to as the *mathematical domain* in order to distinguish it from the problem domain of the function.

LESSON 5.1

ADDITIONAL EXAMPLE

Consider the function $I = 0.04P$.

a. Determine the domain for the function.

b. Determine the problem domain for the function if P represents the principal invested at a local bank and I represents the total interest earned in one year.

a. The domain of the function is all real numbers.

b. The problem domain in this situation consists of all real numbers greater than or equal to 0. (There is no largest reasonable value for the independent variable.)

TEACHING TIP

Point out to students that not all functions have problem domains. It is only when a function represents a real-world situation that these restrictions on the domain of the function need to be considered.

COMMON ERROR

Exercise 2 If students confuse the independent and dependent variables, have them ask themselves which of the variables affects the outcome of the other. That variable is the independent variable. Also, remind them that the **in**put of the function is the **in**dependent variable.

Practice for Lesson 5.1 Answers

1. Sample answer: One advantage of graphs and tables over verbal descriptions, equations, and arrow diagrams is that both graphs and tables display many input-output pairs at the same time. This makes important properties of the function more visible.

2a. variables: time and grade; independent variable: time; dependent variable: grade

In the function $R = 100p - 10p^2$, it is mathematically possible to substitute any real number for p. But in the T-shirt revenue context, the selling price p only makes sense for 0, 10, and numbers between 0 and 10 that represent dollars and cents.

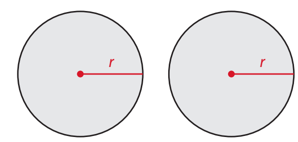
So the mathematical domain is all real numbers, but the problem domain is restricted to numbers between 0 and 10 inclusive.

EXAMPLE

Consider the function $A = 6.28r^2$.

a. Determine the domain for the function.

b. Determine the problem domain for the function if r represents the radius of a circular object and A represents the total area of these two circular objects.



Solution:

a. Any number can be substituted for r in the function, so the domain is all real numbers, including 0 and negative numbers.

b. The problem domain in this situation consists of all real numbers greater than 0. There is no largest reasonable value for the independent variable.

As you investigate different types of functions, you will be using several types of representations. For example, it is possible to represent a situation with a table and then represent the situation with a graph, as you did in the Investigation. Or, you can begin with an equation or graph and move to other representations, as you will see in later lessons in this chapter.

Practice for Lesson 5.1

1. Why might graphs and tables be preferred over other ways of representing a function?
2. For each situation, identify the two quantities that vary. Which is the independent variable? Which is the dependent variable?
   a. the amount of time spent studying and the grade earned on the test

b. the daily high temperatures in Texas for the month of August
c. the number of car accidents on a given interstate highway and the maximum speed limit

For Exercises 3–5, state whether the table represents a function. Then explain why or why not.

3.

Day	Time Spent Hiking (hours)
1	3
2	1
3	2
4	3
5	2

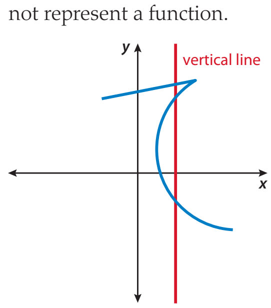
4.

x	y
1	7
−2	4
3	9
−4	−6
1	0

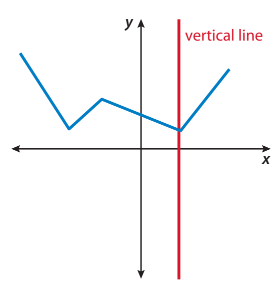
5.

x	y
2	3
4	4
6	4
8	5
10	6

6. The *vertical line test* can be used to determine whether a graph represents a function. This simple test states that if no vertical line can be drawn that intersects the graph more than once, the graph is a function. If such a vertical line can be drawn, the graph does not represent a function.



not a function



function

LESSON 5.1

2b. variables: date and temperature; independent variable: date; dependent variable: temperature

2c. variables: number of accidents and speed limit; independent variable: speed limit; dependent variable: number of accidents

3. Yes; for each day (input value), there is exactly one amount of time spent hiking (output value).

4. No; the x-value of 1 has two different y values, 7 and 0.

5. Yes; for each input value x, there is exactly one output value y.

TEACHING TIP

Exercise 6 Be sure to assign this problem as it explains how to use the vertical line test as a way to look at a graph and determine whether it is a graph of a function.

6a. function; No vertical line can be drawn that intersects the graph in 2 or more points.

6b. function; No vertical line can be drawn that intersects the graph in 2 or more points.

6c. not a function; Any vertical line that intersects the graph will intersect in two points (except for a vertical line drawn through the point at the left end of the graph).

6d. not a function; The graph is a vertical line, so any vertical line intersecting the graph intersects it in two or more points.

7. Yes, for each input value (time), there is exactly one output value (speed).

8. All real numbers from 0 seconds to 17 seconds inclusive.

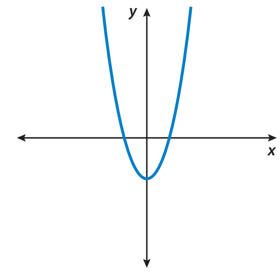
9. All real numbers from 0 meters per second to about 3.5 meters per second inclusive.

10. A, B, and D

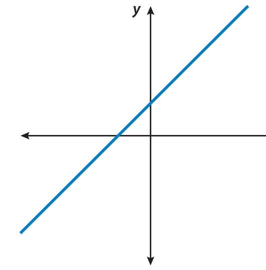
11. Yes; (0, 0), all points from about (8.7, 0) through about (9.9, 0), and (17, 0)

For Parts (a–d), determine whether the graph represents a function. Explain why or why not.

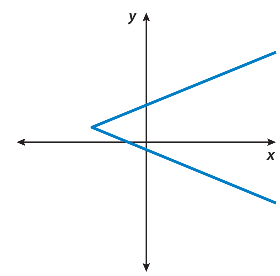
a.



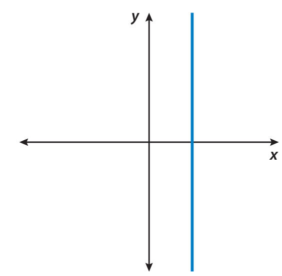
b.



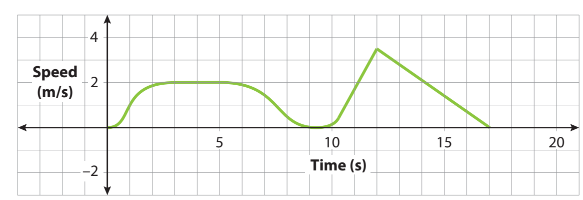
c.



d.



For Exercises 7–11, use the graph below that shows the speed of a person as she walks up a hill and then sleds down.



7. Does this graph represent a function? Explain.

8. What is the domain of the function?

9. What is the range of the function?

10. Which of these points are on the graph?
A. (1, 1) **B.** (3, 2) **C.** (2, 11) **D.** (5, 2)

11. Does the graph have any intercepts? If so, identify them and estimate their coordinates.

12. Examine the verbal description, the table, and the graph shown below.

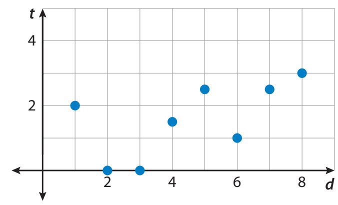
Verbal Description

On Thursday, a student studied 2 hours. On Friday and Saturday, he did not study. On Sunday through Thursday, he studied 1.5, 2.5, 1.0, 2.5, and 3.0 hours, respectively.

Table

Day	Time (hr)
1	2.0
2	0
3	0
4	1.5
5	2.5
6	1.0
7	2.5
8	3.0

Graph



a. Does the table represent a function? Explain why or why not.
b. State the domain and range of the function shown in the graph.
c. Do all three of these representations represent the same function? Explain.

13. Consider the function $y = \frac{1}{x}$.
a. Is the point (2, −2) on the graph of the function?
b. Determine the value of y when $x = 4$.
c. What is the domain of this function?

14. An electric company charges $15 per month plus 10 cents for each kilowatt-hour (kWh) of electricity used.
a. Use the verbal description of this function to complete the table below.

Electricity Used (kWh)	0	100	200	300	400	500	600	700	800	900	1,000
Total Cost (dollars)											

b. In this situation, identify the independent and the dependent variables.
c. Use your table to sketch a graph of this function.
d. Use the graph of the function to find the value of the independent variable when the dependent variable is $80.
e. Is the point (0, 0) on the graph of the function?
f. What is a reasonable problem domain for this function? Explain.
g. Is this a direct variation function? Why or why not?

TEACHING TIP

Exercise 13 Point out that the domain of $y = \frac{1}{x}$ does not include 0, since division by 0 is not defined. In this case, $x \neq 0$ is a restriction on the mathematical domain.

CONNECTION

Exercise 14 The kilowatt-hour is a unit of energy. To calculate the energy used by a device, look at the label for the wattage that is printed on it. To get kilowatt-hours, divide the number of watts by 1,000. Then multiply that result by the number of hours that the device is used.
For example, the energy used by a 100-watt light bulb in a month (about 750 hours) is

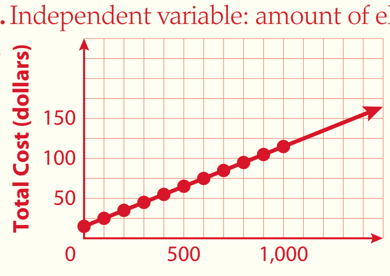
$$\frac{100 \text{ watts}}{1{,}000 \text{ watts/kilowatt}} \times 750 \text{ h} = 75 \text{ kWh}.$$

12a. Yes; for each day (input value), there is exactly one time (output value).
12b. domain: {1, 2, 3, 4, 5, 6, 7, 8}; range: {0, 1.0, 1.5, 2.0, 2.5, 3.0}
12c. Yes; all three represent the same relationship between the values in the domain and the range.
13a. No
13b. $y = \frac{1}{4}$
13c. All real numbers except 0

14a.

Electricity Used (kWh)	0	100	200	300	400	500	600	700	800	900	1,000
Total Cost (dollars)	15	25	35	45	55	65	75	85	95	105	115

14b. Independent variable: amount of electricity used; dependent variable: total cost
14c.



14d. 650 kilowatt-hours
14e. no
14f. Sample answer: All numbers greater than or equal to 0 and less than 5,000 kWh or any other upper limit to the domain that seems reasonable.
14g. Sample answer: No, the ratio of total cost to electricity used is not constant. Also, the graph of the function does not pass through the origin.

LESSON 5.2

5e Engage

Lesson Objective

- collect and analyze data that have a constant rate of change.

Vocabulary

- continuous
- discrete
- linear relationship

Materials List

Per group:

- a scale that measures in grams or tenths of an ounce
- a container such as a small glass bowl that weighs between 3 and 6 ounces and is large enough to hold 10 objects
- ten or more objects that are uniform in weight and each weigh between 0.2 and 0.4 ounces. For example, marbles, miniature candy bars, large metal washers, or 10-penny nails work well in this Activity.

Description

Preparation:

Have students work in groups of 2–4 students. Provide each group with a scale and at least 10 objects that are uniform in weight.

Teaching Option: The data collection portion of this lesson can be done as a demonstration where the teacher or a student collects the data in front of the class and records it in a table for the class to use.

During the Activity:

Before starting the data collection portion of this Activity, have students answer **Question 1**. You may want to discuss their answers before the weighing begins.

Have students carefully collect the desired data by first weighing the container without any objects.

Lesson 5.2 Activity Answers

1–7. See answers beginning on page 621.

Lesson 5.2 ACTIVITY: Linear Relationships

In Lesson 5.1, you learned that functions can be described in many different ways. In this lesson, you will collect data and use what you know about direct variation, slope, and rate of change to analyze the data.

In this Activity, you will be given a container and several objects. You will first weigh the container with no objects in it and record its weight in a table. Then you will place objects in the container, one at a time, and record the new weight.

1. What is the independent variable in this data collection activity? What is the dependent variable? Explain.

2. Weigh the container with no objects in it.

a. How much does it weigh? Record this weight in a table like the one below.

Number of Objects	0	1	2	3	4	5	6	7	8	9	10
Weight (oz)											

b. Place one object in the container. Check the weight and record it in your table.

c. Add a second object to the container and record the weight in your table. Repeat until the table is complete.

3. Use the data in your table to create a scatter plot of weight w versus the number of objects n in the container.

4. When the points in a scatter plot lie along a straight line, the relationship between the two variables is called a **linear relationship**. Is the relationship between the total weight of the container with objects and the number of objects linear? Why or why not?

5. What are the domain and range for this situation? Why?

6. Is the relationship between the total weight and the number of objects a proportional relationship? Explain.

7. Is this relationship a function? Explain.

Closing the Activity:

After students have completed the Activity, have them discuss what they know about the relationship between the number of objects and the total weight of the container and objects. Make note of their vocabulary by making a list of the words used. Terms used should include words such as independent variable, dependent variable, linear relationship, proportional relationship, range, domain, function, intercept, discrete variable, and continuous variable.

TEACHING TIP

Question 6 provides the opportunity to compare and contrast proportional relationships and linear relationships. Point out to students that all proportional relationships are linear, but not all linear relationships are proportional. Take time to look at a table and graph of this function as well as a table and graph of a direct proportion (**Exercise 12, page 147**). Examine the ratios of the dependent variable to the independent variable as well as the ratios of y-change to x-change between any two points, pointing out what each ratio tells you about the function.

8. Two quantities have a *positive relationship* when the dependent variable increases as the independent variable increases. If one quantity decreases when the other increases, the quantities have a *negative relationship*.

   Do you think this relationship is better described as a "positive relationship" or as a "negative relationship"? Explain.

Some variables in data sets are **discrete**, which means that only certain values (often integers) are possible for the data. For example, the independent variable in your table may be discrete because it can only take on whole number values because fractional parts of one of your objects may make no sense in this situation. However, if it makes sense for a variable to take on any real-number value, then the variable is **continuous**. This means that the actual values could include any real number up to the maximum number of objects in your data.

If your independent variable is discrete, then your scatter plot should show distinct points. If it is continuous, it makes sense to connect the points with a line.

9. Is your independent variable continuous or discrete? Explain.

There are times that it is helpful to connect the points in a scatter plot even when the variables are discrete, especially if you plan to use your data to make predictions. However, you should be aware that doing so changes the domain shown in the graph from *some* values to *all* values.

10. To make it easier to examine the relationship between the number of objects n and the weight w, draw a line through the points on your scatter plot. Describe your graph.

11. Find the slope of your graph in Question 10. Then write a sentence that interprets the slope as a rate of change.

12. What are the coordinates of the w-intercept of your graph? What is the meaning of the w-intercept in this situation?

13. Write an equation that models the weight w of the objects and container after n objects have been placed in the container.

14. Use your graph, table, or equation to predict the weight when there are twenty objects in the container.

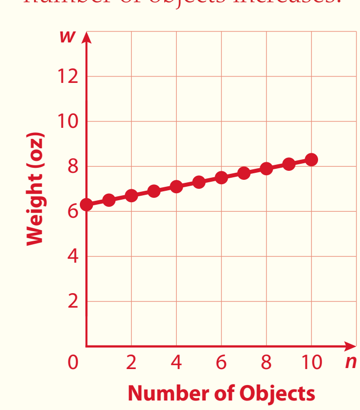
TEACHING TIP

The answer to **Question 9** depends on the objects used in the Activity. If the objects used are items such as mini-candy bars that can be cut into pieces, it may make sense to talk about a fractional part of an object. In other cases such as marbles, nails, and washers, it may not make sense.

TEACHING TIP

In **Question 13**, if students have difficulty writing an equation, remind them of the work they did in **Lesson 3.2**. Point out that it might be helpful for them to first write a sentence expressing the relationship between weight and the number of objects.

8. Positive relationship; As the independent variable increases in value, the dependent variable increases in value.
9. Answers will vary. If it makes sense to talk about fractional parts of an object (*e.g.*, small candy bars), the variable is continuous. If not (*e.g.*, marbles), the variable is discrete.
10. Sample answer: The graph appears to be linear with a w-intercept of 6.3 (the weight of the container). The graph increases from left to right as the number of objects increases.



11. Sample answer: For the pairs (2, 6.7) and (3, 6.9): $\frac{\Delta w}{\Delta n} = \frac{6.9 - 6.7}{3 - 2} = \frac{0.2}{1} = 0.2$ ounces per object; the weight on the scale increases at a rate of 0.2 ounces each time an object is added.
12. The coordinates of the w-intercept are (0, 6.3). This is the value of the dependent variable (weight) when there are 0 objects in the container.
13. Sample answer: $w = 0.2n + 6.3$
14. Sample answer: About 10.3 oz

TEACHING TIP

Exercises 1–11 Rather than using the data given in the exercises, provide groups of students with rope. Have them measure the original length of their rope with no knots and record the length in a table. Then have them tie knots, one at a time, and record the new length.

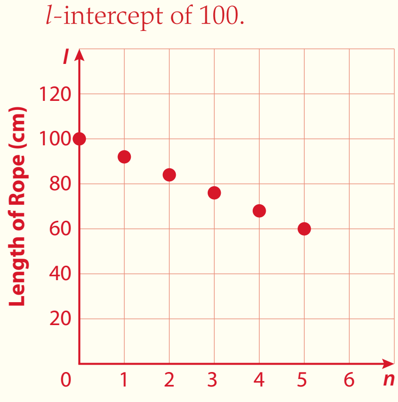
Once the data set is complete, have them answer **Exercises 1–11.**

TEACHING TIP

In **Exercises 1–11**, students apply and extend what they have learned about modeling data that have a linear pattern. Having worked with linear data whose model passes through the origin and with linear data whose model has a non-zero *y*-intercept, students now work with linear data whose model has a non-zero *y*-intercept and a decreasing rate of change, or negative slope.

Practice for Lesson 5.2 Answers

1. Independent variable: the number of knots; dependent variable: the length of the rope; Since the number of knots in the rope explains the length of the rope, it is the independent variable.
2. discrete; It makes no sense to talk about a fractional part of a knot.
3. whole numbers 0 to 5 (the number of knots tied)
4. Range: the lengths of the rope (60, 68, 76, 84, 92, and 100 cm)
5. Sample answer: The graph appears to be linear with an l-intercept of 100.



Practice for Lesson 5.2

For Exercises 1–11, use the following information.

The length of a rope was measured and recorded in the table below. Knots were tied in the rope, one at a time. After each knot was added, the length of the rope was measured and recorded. A total of five knots were tied.

Number of Knots	Length of Rope (cm)
0	100
1	92
2	84
3	76
4	68
5	60

1. What is the independent variable in this situation? What is the dependent variable? Explain.
2. Is the independent variable discrete or continuous? Explain.
3. What is the domain for this situation?
4. What is the range for this situation?
5. Create a scatter plot of the length l of the rope versus the number of knots n tied in the rope. Describe your graph.
6. Is this a proportional relationship? Explain.
7. Is this relationship a function? How do you know?
8. Do you think this relationship is better described as a "positive relationship" or as a "negative relationship"? Explain.
9. What are the coordinates of the l-intercept of your graph? What is the meaning of the l-intercept in this situation?
10. Find the rate of change for any two pairs of values in your table.
11. In symbolic form, this function can be written as $l = -8n + 100$.
    a. Use this equation to predict the length of the rope when 11 knots are tied in it.
    b. Use this equation to predict how many knots will have to be tied in the rope so that its length is 44 cm.

6. Sample answer: This is not a proportional relationship even though the graph is linear because the ratio of the length of the rope to the number of knots is not constant, for example $\frac{92}{1} \neq \frac{84}{2} \neq \frac{76}{3}$. Also, the graph does not pass through the origin (0, 0).
7. Yes, for each input value, there is exactly one output value.
8. Negative relationship; As the independent variable increases in value, the dependent variable decreases in value.
9. The l-intercept has coordinates (0, 100). This is the value of the dependent variable (length of the rope) when there are 0 knots in the rope.
10. Sample answer: For the pairs (2, 84) and (3, 76): $\frac{\Delta l}{\Delta n} = \frac{84 - 76}{2 - 3} = \frac{8}{-1} = -8$ cm/knot

11a. 12 cm

11b. 7 knots

For Exercises 12–16, use the following information.

The table shows the relationship between the weight w of the tickets given out at a family arcade center and the number of tickets t. Tables similar to this allow workers to determine the number of tickets a customer has by weighing the tickets. For example, if the weight of your tickets is 40 grams, the arcade center knows that you have 184 tickets.

Weight (grams)	Number of Tickets
0	0
10	46
20	92
30	138
40	184
50	230

12. Is this a proportional relationship? Explain.

13. Is this relationship a function? How do you know?

14. What is the independent variable in this situation? Is it discrete or continuous in this context?

15. What is the dependent variable in this situation? Is it discrete or continuous in this context?

16. Find the rate of change for any two pairs of values in your table.

17. State whether each of the following statements is true or false. If the statement is false, give a counterexample.

a. All linear relationships are proportional relationships.

b. All proportional relationships are linear relationships.

TEACHING TIP

Exercises 12–16 provide a link to the context presented in the Chapter Opener. In these exercises, students will begin to explore a mathematical relationship that can be used to determine the number of tickets a person has from weighing the tickets. In **Lesson 5.3**, students will be asked to determine an equation to model this relationship.

12. Yes, the ratio of the number of tickets to the weight is constant, for example, $\frac{46}{10} = \frac{92}{20} = \frac{138}{30}$.

13. Yes, for each input value, there is exactly one output value.

14. weight; It is discrete.

15. number of tickets; It is discrete.

16. Sample answer: For (10, 46) and (20, 92), $\frac{\Delta t}{\Delta w} = \frac{92 - 46}{20 - 10} = \frac{46}{10} =$ 4.6 tickets per gram.

17a. False. Sample answer: Any equation of the form $y = mx + b$, where $b \neq 0$ provides a counterexample. Many students will likely use their equation from Question 13 of the Activity or the equation from Exercise 11.

17b. True

LESSON 5.3

5e Explore

Lesson Objective

- identify linear functions from tables, graphs, and equations.

Vocabulary

- linear function
- slope-intercept form
- x-intercept
- y-intercept

Materials List

none

Description

This lesson works best if students work in pairs. Before beginning the Investigation, ask students what they know about depreciation. Ask what kinds of items depreciate.

Have students work through the Investigation. Explain that at 0 years, the computer value is the purchase price. For **Question 8**, if they have forgotten how to determine the slope of a line when given two points on the line, have them return to **Lesson 2.5**.

Wrapping Up the Investigation:

Once all groups have completed the Investigation, use **Question 9** to debrief it. As students give you different pieces of information, write them down so that students can see how much information can be obtained from the different representations.

Make sure that students notice that from the table, they can tell that the rate of change in the function is constant, and from the graph, they can tell that the relationship is linear.

Point out the three different ways to identify a linear function.

Lesson 5.3 Investigation Answers

1. See table in Question #2.

9. Sample answers: The rate of change is equal to the coefficient of t in the equation. The rate of change that appears in the table is the slope of the line in the graph. The slope of the line is negative so the graph slopes downward. The relationship is a linear relationship, but it is not a proportional relationship because, for example, $\frac{2{,}880}{1} \neq \frac{2{,}160}{2} \neq \frac{1{,}440}{3}$. The constant term in the equation is the y-intercept of the graph.

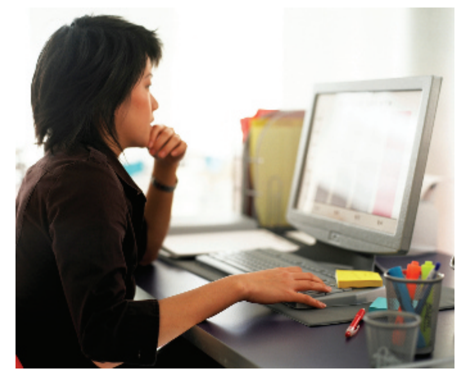
10. Yes, it is already written in the form $y = mx + b$, where the slope (m) is equal to –720 and the y-intercept (b) is 3,600.

Lesson 5.3 — INVESTIGATION: Identifying Linear Functions

In Chapter 2, you learned that a function is a relationship between input and output in which each input value has exactly one output value. In this lesson, you will focus on one particular type of function: the linear function. You will examine the characteristics of linear functions and learn to identify them by looking at equations, graphs, and tables.

Note
Depreciation is a decrease in the value of an asset over time.

According to the Internal Revenue Tax Code, a computer used for business purposes can be *depreciated* once a year over a period of 5 years. The function $v = -720t + 3{,}600$ can be used to model the value v in dollars of a computer that was purchased for $3,600, where t represents the age in years of the computer.



1. Use the given function to complete the second column of the table below.

Age (years)	Computer Value (dollars)	
0		
1		
2		
3		
4		
5		

2 **a.** Label the third column of your table "Depreciation per Year" as shown below. This column shows the average rate of change of the value of the computer with respect to age during each year.

Age (years)	Computer Value (dollars)	Depreciation per Year (dollars/year)
0	3,600	—
1	2,880	$\frac{2{,}880 - 3{,}600}{1 - 0} = -720$
2		
3		
4		
5		

b. Complete the table.

c. What do you notice about the rate of change of this function?

3. What is the domain of the function $v = -720t + 3{,}600$?

4. What is the problem domain of the function $v = -720t + 3{,}600$?

5. Construct a scatter plot of the data in your table.

6. Is this relationship a linear relationship? If so, draw a line through the points on your scatter plot.

7. Give the coordinates and interpret the meaning of any intercepts on your graph.

8. Determine the slope of your graph.

9. Compare and contrast all of your different representations of the function $v = -720t + 3{,}600$. What do you notice?

Recall

The ***y*-intercept** of a graph is the *y*-value of the point where the graph intersects the *y*-axis.

The ***x*-intercept** of a graph is the *x*-value of the point where the graph intersects the *x*-axis.

Any function whose equation can be written in the form $y = mx + b$, where m and b are real numbers, is a **linear function**. This form of the equation of a linear function is called **slope-intercept form**. It expresses the dependent variable y in terms of the independent variable x, the slope of the line m, and the y-intercept of the line whose coordinates are $(0, b)$.

10. Is the depreciation function $v = -720t + 3{,}600$ a linear function? Explain.

You now have three criteria that can be used to identify linear functions.

Linear functions are characterized by the following:
- equations of the form $y = mx + b$, where m and b are real numbers
- graphs that are non-vertical straight lines
- rates of change that are constant

TEACHING TIP

Question 2 The numerator in the third column of this table is found by subtracting the previous year's value from the current year's value. Since the value decreases, the depreciation is negative.

2a. Check students' tables.

2b.

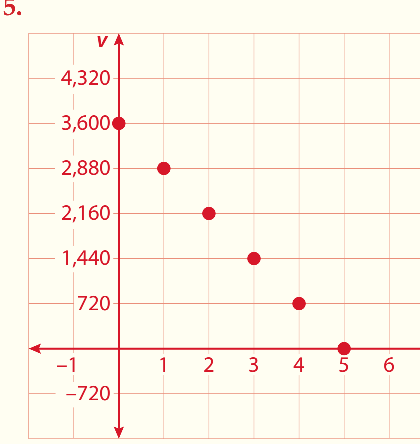
Computer Value (dollars)	Depreciation per Year (dollars/year)
3,600	–
2,880	$\frac{2{,}880 - 3{,}600}{1 - 0} = -720$
2,160	$\frac{2{,}160 - 2{,}880}{2 - 1} = -720$
1,440	$\frac{1{,}440 - 2{,}160}{3 - 2} = -720$
720	$\frac{720 - 1{,}440}{4 - 3} = -720$
0	$\frac{0 - 720}{5 - 4} = -720$

2c. The rate of change, the ratio of $\frac{\text{change in value}}{\text{change in age}}$ between any 2 consecutive years, is constant and equal to –720.

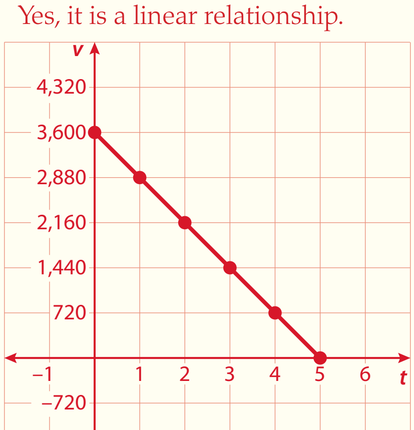
3. All real numbers, because any number can be used as an input for the function.

4. In the context of this problem, only the numbers 0, 1, 2, 3, 4, and 5 are meaningful.

5.



6. Yes, it is a linear relationship.



7. The v-intercept is the point (0, 3,600), and the t-intercept is the point (5, 0). The v-intercept indicates the value of the computer ($3,600) at time zero. The t-intercept indicates that at the end of 5 years, the value of the computer is $0.

8. The slope is –720 dollars per year.

9–10. See answers on page 148.

LESSON 5.3

COMMON ERROR

Exercise 4 If students say that this graph represents a linear function, ask them if the graph represents a function. If necessary, remind them that the graph is a vertical line where the input value has more than one output value.

Practice for Lesson 5.3 Answers

1. Yes, this represents a linear function because the rates of change are constant. $\frac{\Delta y}{\Delta x} = 2$
2. No, this does not represent a linear function because the rates of change are not constant.
3. Yes, this graph represents a linear function because the graph of this function is a non-vertical line.

TEACHING TIP

Exercises 9–12 There is no one right way for students to determine the correct equation. One way, however, is to find the slope of the line from the graph or the table and the *y*-intercept. Once these are calculated, the correct equation can be quickly found since each equation choice is written in slope-intercept form.

4. No, this graph does not represent a linear function because the graph of this relationship is a vertical line where the input value has more than one output.
5. Yes, this equation represents a linear function because the equation is written in the form $y = mx + b$, where the slope (m) is equal to 4 and the y-intercept (b) is equal to 7.
6. Yes, this equation represents a linear function because this equation can be written in the form $y = mx + b$ ($y = 2x – 4$), where the slope (m) is equal to 2 and the y-intercept (b) is equal to –4.
7. No, this equation does not represent a linear function because it cannot be written in the form $y = mx + b$.
8. No, this equation does not represent a linear function because it cannot be written in the form $y = mx + b$.
9. C
10. B

Practice for Lesson 5.3

For Exercises 1–8, state whether the table, graph, or equation represents a linear function. Explain why or why not.

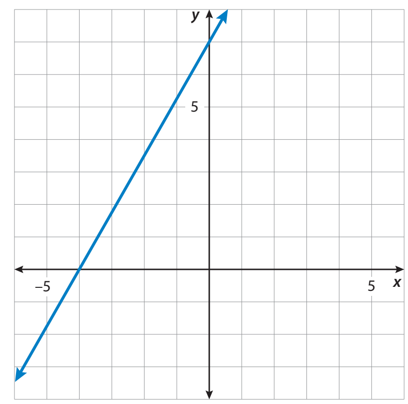
1.

x	−1	0	1	2	3
y	−3	−1	1	3	5

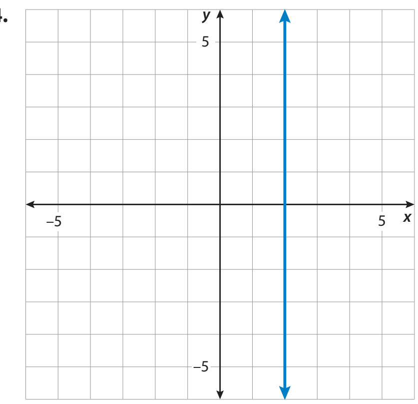
2.

x	−3	−1	1	3	5
y	9	1	1	3	25

3.



4.



5. $y = 4x + 7$

6. $6x - 3y = 12$

7. $x = 12$

8. $y = 2x^2 + 4$

For Exercises 9–12, choose the equation that best represents the linear function described in the given table or graph.

9.

x	*y*
−4	0
0	8
4	16

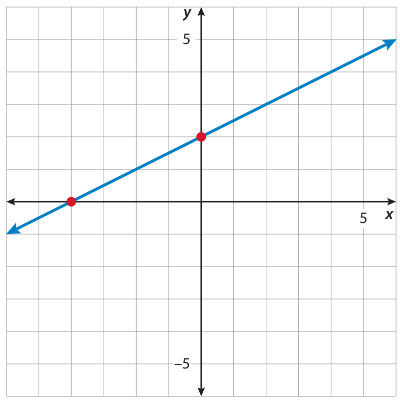
A. $y = -2x + 8$
B. $y = 2x + 4$
C. $y = 2x + 8$
D. $y = -2x + 4$

10.

x	*y*
0	6
2	4
4	2

A. $y = 2x + 6$
B. $y = -x + 6$
C. $y = x + 6$
D. $y = -2x + 6$

11.



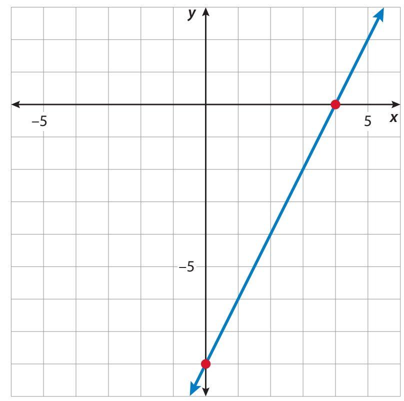
A. $y = \frac{1}{2}x + 2$

B. $y = 2x + 2$

C. $y = \frac{1}{2}x - 4$

D. $y = 2x - 4$

12.



A. $y = 2x + 8$

B. $y = -2x - 8$

C. $y = 2x + 4$

D. $y = 2x - 8$

13. In Lesson 5.2, you answered questions about the table on the right, which shows the relationship between the weight w of the tickets given out at a family arcade center and the number of tickets t.

a. Is the number of tickets a linear function of the weight of the tickets? Explain why or why not.

b. What is the slope of the line that could be used to describe this function?

c. Give the coordinates and interpret the meaning of any intercepts of the line that could be used to describe this function.

d. Write an equation that represents the linear function described in the table.

e. If a person turns in a pile of tickets that weighs 135 grams, about how many tickets are there?

Weight (grams)	Number of Tickets
0	0
10	46
20	92
30	138
40	184
50	230

11. A

12. D

13a. Yes, this represents a linear function as the rates of change are constant. $\frac{\Delta t}{\Delta w} = 4.6$ tickets per gram

13b. 4.6 tickets per gram

13c. Sample answer: The intercept on the vertical axis is the origin (0, 0). This intercept indicates that when there is no weight, there are no tickets.

13d. $t = 4.6w$

13e. about 621 tickets

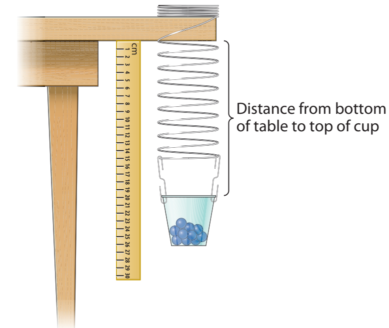
TEACHING TIP

Exercise 14 If time permits and materials are available, you may want to conduct this experiment as a whole-class demonstration and have students use the data collected to answer the questions.

To do so, you will need a Slinky®, a yardstick or meter stick, and a container such as a small paper cup or film canister. Paper clips can be used to attach the container to the Slinky. Marbles or other objects of uniform weight are needed to add to the container.

14a. The number of marbles placed in the cup is the independent variable and the distance from the bottom of the table to the top of the cup is the dependent variable.
14b. Yes, the rates of change are approximately constant.
14c. 2 cm per marble
14d. The intercept on the vertical axis is the point (0, 17.2). The intercept on the horizontal axis is not in the problem domain of the function. The intercept on the vertical axis indicates the distance from the bottom of the table to the top of the cup when there are no marbles in the cup.
14e. Sample answer: $y = 2x + 17.2$

14. A small Slinky with a cup attached at the bottom is suspended from the edge of a table. Marbles are added to the cup, one at a time, and the distance from the bottom of the table to the top of the cup is measured.



The data collected are shown in the table below.

Number of Marbles	Distance from Bottom of Table to Top of Cup (cm)
0	17.2
1	19.2
2	21.1
3	23.3
4	25.2
5	27.1

a. What is the independent variable in this relationship? What is the dependent variable?
b. Is this relationship a linear function? Explain.
c. What is the slope of the line that could be used to describe this function?
d. Give the coordinates and interpret the meaning of any intercepts of the line that could be used to describe this function.
e. Write an equation that represents the linear function described in the table.

Lesson 5.4

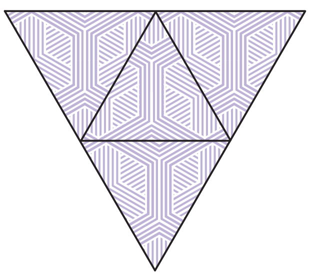
R.A.P.

Fill in the blank.

1. The set of all possible input values for the independent variable of a function is called the __________ of the function.
2. Any function that can be written in the form $y = mx + b$ is called a(n) __________ function.
3. The __________ surface area of a three-dimensional figure is the surface area of the figure, excluding the area of the bases.

Choose the correct answer.

4. If the ratio of the corresponding sides of a rectangular solid is 3 : 4, what is the ratio of the corresponding surface areas?
   A. $\frac{3}{4}$ **B.** $\frac{4}{3}$ **C.** $\frac{9}{16}$ **D.** $\frac{27}{64}$



5. Which solid can be folded from the net shown on the left?
   A. triangle
   B. triangular prism
   C. triangular pyramid
   D. quadrilateral
6. What is the sum of the measures of the angles of any pentagon?
   A. 5° **B.** 108° **C.** 540° **D.** 900°
7. In which quadrant of the coordinate plane does the point (−2, 1) lie?
   A. I **B.** II **C.** III **D.** IV

Write the decimal as a percent.

8. 0.015 **9.** 0.6 **10.** 1.8 **11.** 5.0

Write the percent as a decimal.

12. 6% **13.** 65% **14.** 128% **15.** 0.7%

LESSON 5.4

5e Evaluate

Lesson Objective

- solve problems that require previously learned concepts and skills.

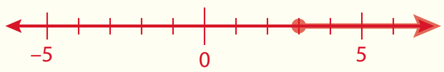
Exercise Reference

Exercise 1: Lesson 5.1
Exercise 2: Lesson 5.3
Exercises 3–4: Lesson 4.7
Exercise 5: Lesson 4.6
Exercise 6: Lesson 4.1
Exercise 7: Appendix O
Exercises 8–15: Appendix E
Exercises 16–18: Appendix J
Exercises 19–20: Appendix F
Exercises 21–22: Lesson 3.6
Exercise 23: Lesson 5.1
Exercise 24: Lesson 4.2
Exercises 25–26: Lesson 2.5
Exercise 27: Lesson 2.3
Exercises 28–29: Lesson 4.4
Exercise 30: Lesson 4.6
Exercise 31: Appendix N

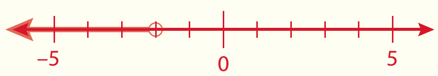
Lesson 5.4 R.A.P. Answers

1. domain
2. linear
3. lateral
4. C
5. C
6. C
7. B
8. 1.5%
9. 60%
10. 180%
11. 500%
12. 0.06
13. 0.65
14. 1.28
15. 0.007

16. Distributive Property
17. Commutative Property of Multiplication
18. Zero Property of Multiplication
19. 20%
20. 600%
21. $t \geq 3$



22. $x < -2$



23. Yes, for each value in the domain, there is exactly one value in the range.
24. $n = \frac{4 - m}{2}$ or $n = 2 - \frac{m}{2}$
25. 2
26. undefined
27. $n = \frac{2}{3}m$
28. a cylinder
29. $V = Bh = \pi r^2 h = \pi(16)(8) \approx 402.1 \text{ cm}^3$
30. $SA \approx 301.6 \text{ cm}^2$
31. mean: 9.2 s; median: 7.5 s; range: 15 s

Identify the property illustrated by the equation.

16. $8(4 + 2) = 8(4) + 8(2)$

17. $5 \times 6 = 6 \times 5$

18. $0 \cdot a = 0$

Solve.

19. What percent of 45 is 9?

20. What percent of 12 is 72?

Solve for the variable. Check your solution and graph it on a number line.

21. $t + 1 \geq 4$

22. $8 - 3x > 14$

23. Is the relationship shown in the table a function? Explain.

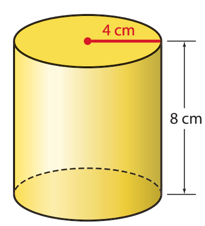
x	−2	−1	0	1	2
y	4	1	0	1	4

24. Solve the equation $16 = 4(2n + m)$ for n.

25. Find the slope of the line that passes through the points (3, 8) and (−1, 0).

26. Find the slope of the line that passes through the points (−4, 5) and (−4, −2).

27. The variable n is directly proportional to m. If $n = 10$ when $m = 15$, then write an equation that gives n as a function of m.



For Exercises 28–30, use the solid figure shown on the left.

28. Identify the figure.

29. Find the volume of the figure.

30. Find the surface area of the figure.

31. Find the mean, median, and range of the times shown in the table.

Trial Number	1	2	3	4	5	6	7	8	9	10
Time (s)	8	14	5	3	18	6	7	15	10	6

Lesson 5.5 Graphing Linear Equations

A graph of a function can provide a quick visual summary of the relationship that exists between two variables. In this lesson, you will use your knowledge of plotting points and slope to graph linear equations.

A **linear equation** is an equation of a line. When looking at a graph of a linear equation, it is important to remember this very important fact.

Every point on the graph represents a solution to the linear equation, and every solution to the equation can be represented by a point on the graph.

GRAPH AN EQUATION USING A TABLE

The linear equation $C = 20 + 55t$ can be used to represent the relationship between the cost C in dollars of renting a jet ski and the rental time t in hours.

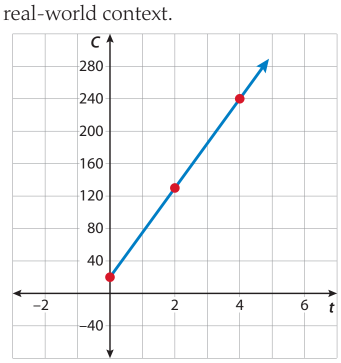
One way to sketch the graph of this linear equation is to create a table of values in the domain of the relationship. Then plot the ordered pairs and connect the points.

Note

Since you know that this relationship is linear, you need only two points to determine the line. However, it is a good idea to have a third point to help you avoid mistakes.

t	$20 + 55t$	C	Ordered Pair (t, C)
0	$20 + 55(0)$	20	(0, 20)
2	$20 + 55(2)$	130	(2, 130)
4	$20 + 55(4)$	240	(4, 240)

This graph shows the three points from the table and a line drawn through them. Notice that the line has not been extended to the left of the vertical axis since it is meaningless to talk about time being less than zero in this real-world context.



LESSON 5.5

5e Explain

Lesson Objectives

- graph a linear equation using a table.
- graph a linear equation in slope-intercept form.
- graph a linear equation using the intercepts.

Vocabulary

- linear equation

Description

In this lesson students use what they know about the slope of a line and plotting points to graph an equation. Three techniques are introduced in this lesson:

- creating a table of ordered pairs and plotting the points
- using slope-intercept form of a linear equation
- using the x- and y-intercepts

TEACHING TIP

Make sure students fully understand the blue-highlighted statement on page 155 of this lesson. The importance of being able to connect the points on a graph with the solutions to an equation should be stressed.

TEACHING TIP

Explain that the graph of $C = 20 + 55t$ does not extend to the left of the y-axis because negative values for t are not in the problem domain; that is, they make no sense in this problem. In other words, you cannot rent a jet ski for a negative amount of time.

LESSON 5.5

TEACHING TIP

Point out that the slope-intercept method only works for linear equations that are in the form $y = mx + b$.

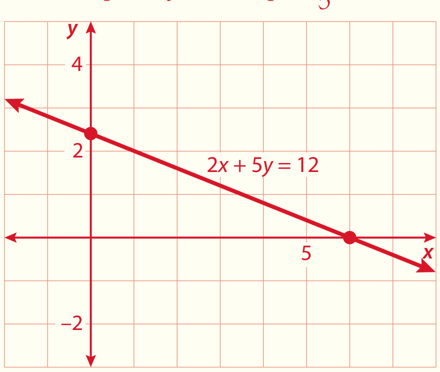
TEACHING TIP

Ask students to explain why the equation of a horizontal line *can* be written in slope-intercept form and the equation for a vertical line *cannot*.

ADDITIONAL EXAMPLE

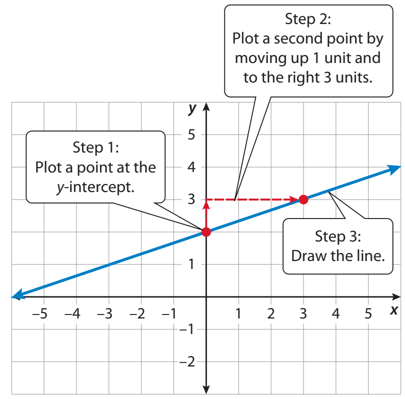
Determine the intercepts of $2x + 5y = 12$. Then use them to graph the equation.

x-intercept: 6; y-intercept: $2\frac{2}{5}$



GRAPH AN EQUATION IN SLOPE-INTERCEPT FORM

When a linear equation is in slope-intercept form, a graph can be quickly made. For example, to sketch the graph of $y = \frac{1}{3}x + 2$, identify the slope and the y-intercept. The slope is equal to $\frac{1}{3}$. The y-intercept is 2, so the point (0, 2) is on the graph.



To graph the equation:

- Plot the point (0, 2).
- From the point at the y-intercept, use the slope of $\frac{1}{3}$ to identify another point. Move up 1 unit and to the right 3 units. Then plot a point.
- Draw a line through the two points.

GRAPH AN EQUATION USING INTERCEPTS

Another quick way to graph a linear equation is to use the x- and y-intercepts.

- First find the two intercepts. (To find the x-intercept, let $y = 0$ in the equation. To find the y-intercept, let $x = 0$.)
- Plot a point at each intercept.
- Draw a line through the two points.

EXAMPLE

Determine the intercepts of $2x - 3y = 9$. Then use them to graph the equation.

Solution:

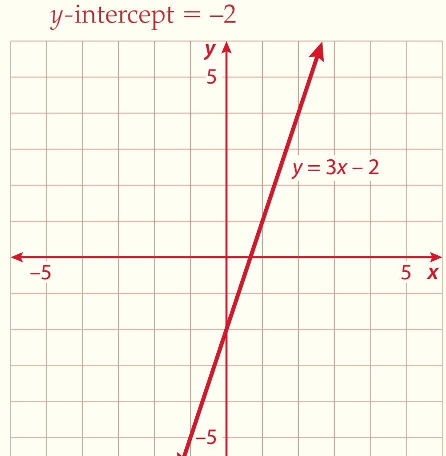
To find the x-intercept, let $y = 0$.

$$2x - 3y = 9$$
$$2x - 3(0) = 9$$
$$2x = 9$$
$$x = 4.5$$

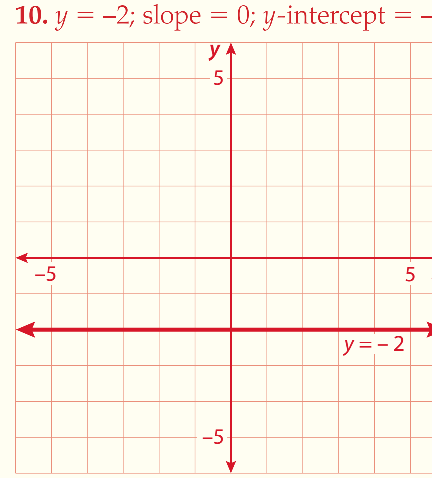
To find the y-intercept, let $x = 0$.

$$2x - 3y = 9$$
$$2(0) - 3y = 9$$
$$-3y = 9$$
$$y = -3$$

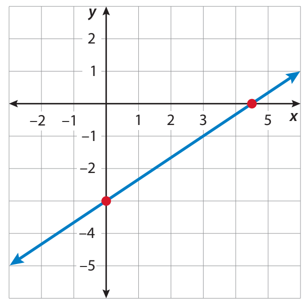
9. $y = 3x - 2$; slope $= 3$; y-intercept $= -2$



10. $y = -2$; slope $= 0$; y-intercept $= -2$



Now plot points at the intercepts and draw a line through the two points.



Practice for Lesson 5.5

1. Use a graphing calculator. Enter the equations $y = x + 2$, $y = -2x + 2$, and $y = \frac{1}{2}x + 2$ in the [Y=] list on the function screen. Then graph them in an appropriate viewing window that shows the important features of each graph. Write a description of what you see. How are these graphs the same? How are they different?
2. Use a graphing calculator. Enter the equations $y = 2x$, $y = 2x + 1$, and $y = 2x - 3$ in the [Y=] list on the function screen. Then graph them in an appropriate viewing window. Write a description of what you see. How are these graphs the same? How are they different?

For Exercises 3–6, write the equation in slope-intercept form. Then identify the slope and *y*-intercept.

3. $2x - 3y = 12$
4. $2x + 4y = 4$
5. $6 - 2y = 0$
6. $2x - y = 8$

For Exercises 7–8, make a table of values and graph the equation.

7. $y - 3x = -4$
8. $x + 2y = 1$

For Exercises 9–10, write the equation in slope-intercept form, identify the slope and *y*-intercept, and graph the equation.

9. $3x - y = 2$
10. $2y + 4 = 0$

TEACHING TIP

Exercises 1 and 2 require the use of a graphing calculator. In **Exercise 1,** students explore how changing the value of *m* in the equation $y = mx + b$ affects the graphs of the lines. In **Exercise 2**, students explore how changing the value of *b* affects the graphs.

TEACHING TIP

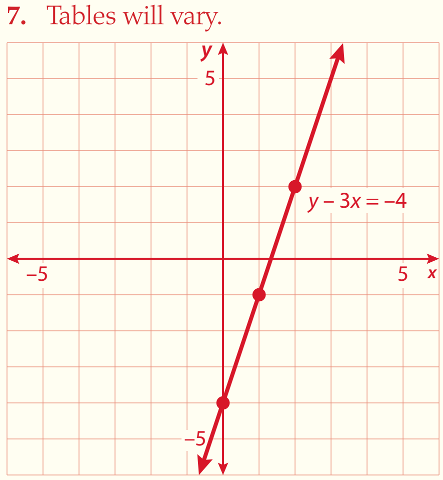
Exercises 1 and 2 refer to an "appropriate" viewing window. Encourage students to consider the functions and select a window that shows the important features of the graphs, such as intercepts and possible intersections of the functions. In these two exercises, a $[-10, 10] \times [-10, 10]$ screen with an Xscl and a Yscl of 1 is appropriate.

COMMON ERROR

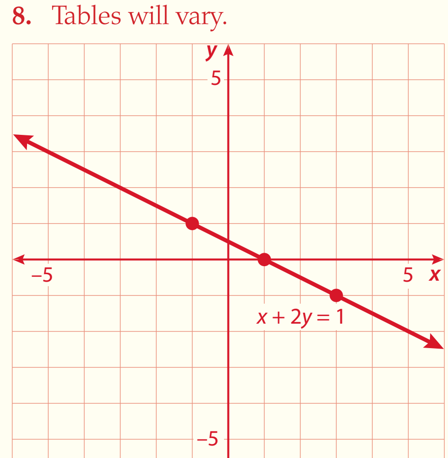
Exercises 3–6, 9–10 Some students may forget to write the equation in slope-intercept form before trying to identify the slope and *y*-intercept.

Practice for Lesson 5.5 Answers

1. Sample answer: All three graphs have the same *y*-intercept. The slopes are different. The graph of the line with the negative slope slants down from left to right. The two lines with positive slopes slant up from left to right. The greater the absolute value of the slope is, the steeper the line is.
2. Sample answer: All three graphs have the same slope. The *y*-intercepts are different. The graph of $y = 2x + 1$ is the same as the graph of $y = 2x$, but shifted up 1 unit. The graph of $y = 2x - 3$ is the same as the graph of $y = 2x$, but shifted down 3 units. All three lines are parallel.
3. $y = \frac{2}{3}x - 4$; slope $= \frac{2}{3}$; *y*-intercept $= -4$
4. $y = -\frac{1}{2}x + 1$; slope $= -\frac{1}{2}$; *y*-intercept $= 1$
5. $y = 3$; slope $= 0$; *y*-intercept $= 3$
6. $y = 2x - 8$; slope $= 2$; *y*-intercept $= -8$
7. Tables will vary.



8. Tables will vary.



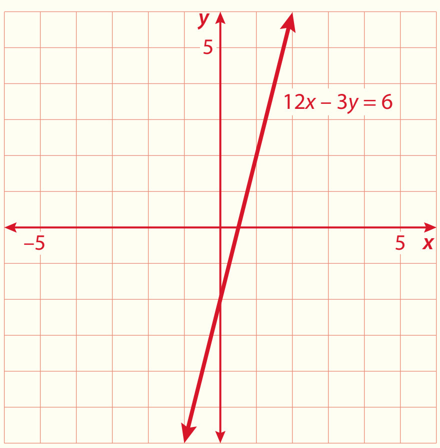
9–10. See answers can on page 156.

LESSON 5.5

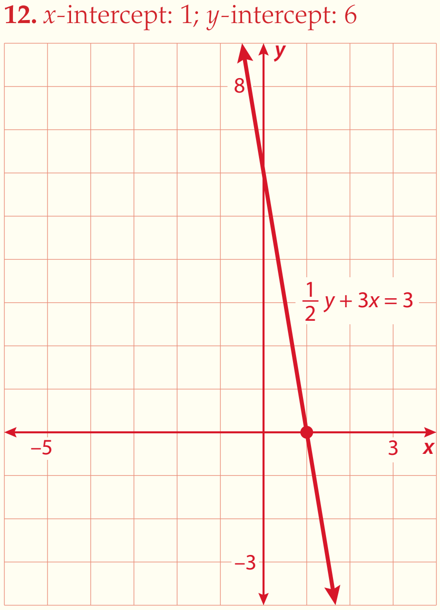
TEACHING TIP

Exercise 13 Take this opportunity to review the terms *continuous* and *discrete* variables. In this situation, *n* is discrete, so the graph is actually a set of points. Remind students that the points can be connected with a line if they feel that it helps to show the general trend of the context.

11. x-intercept: $\frac{1}{2}$; y-intercept: –2



12. x-intercept: 1; y-intercept: 6



For Exercises 11–12, identify the intercepts and graph the equation.

11. $12x - 3y = 6$

12. $\frac{1}{2}y + 3x = 3$

13. The equation $C = 40 + 5n$ can be used to represent the relationship between the yearly cost C in dollars for a student who has a membership to the Museum of Nature and Science in Dallas, Texas and the number of times n the student attends the IMAX® Theater at the museum.

a. Graph the equation.

b. Use your graph to determine the cost of a student attending the theater four times.

14. A cable television company charges a monthly fee of \$10, plus \$4 for each movie rented for their movie plan.

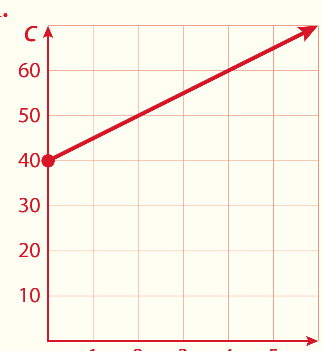
a. Write an equation that gives the total monthly cost C for n movie rentals.

b. Graph the equation.

c. Use your graph to determine the cost for a month if 8 movies are rented.

15. Explain the statement "Not all linear equations represent linear functions."

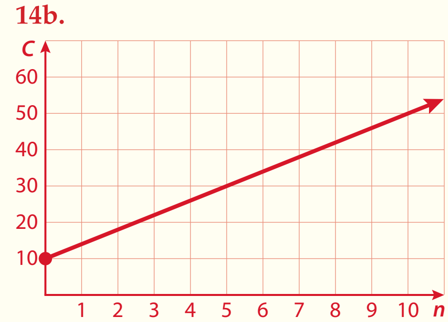
13a.



13b. \$60

14a. $C = 10 + 4n$

14b.



14c. \$42

15. Sample answer: A vertical line can be represented with an equation of the form $x = a$, where a is a real number. This line is not a function because there is more than one output for each input value.

Lesson 5.6

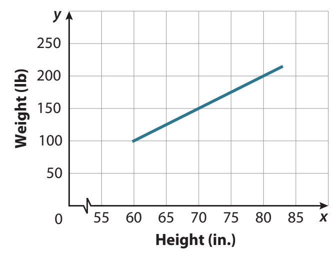
INVESTIGATION: Writing Equations of Lines

In Lesson 5.5, you learned to graph linear equations. In this lesson, you will learn to find an equation of a line given information such as two points on the line or a point on the line and the slope.

Note

A *rule of thumb* can be thought of as a useful principle or method that is not intended to be strictly accurate or reliable in every situation.

People are concerned about good nutrition and exercise as they strive to lead a healthier life. In their attempts to change their lifestyles, they often encounter graphs that relate weights and heights. The graph below reflects an old and outdated *rule of thumb* that was used in the past for finding an ideal weight from a given height.



1. Use your prior knowledge to describe the graph and the relationships represented.
2. According to the graph, what was considered an ideal weight for a person who was 6′ 8″ tall? Explain how you found your answer.
3. Find what was considered to be an ideal weight for a person who was 5′ 8″, for a person who was 5′ 3.5″, and for a person who was 7′ 4″. Explain any problems you encountered in finding these weights.

The problems that you encountered in Question 3 can be resolved if you find an equation for the line. You can do this by finding the slope of the line and the y-intercept.

4 **a.** Find the slope of the line.
b. What does the slope of the line tell you about the relationship between a person's height and his or her ideal weight?

5. The y-intercept of this graph is −200. For this context, what does the y-intercept tell you?
6. Now that you have found the slope of the line and the y-intercept, find an equation of the line.

7 Use your equation from Question 6 to predict the ideal weight for each of the given heights.
a. 5′ 6.5″ **b.** 4′ 10″ **c.** 7′ 7″

Lesson 5.6 Investigation Answers

1. Sample answers: This graph represents a function that is linear. It shows a positive relationship. The slope of the line is positive. The graph is continuous. The problem domain is from 60 inches to about 83 inches. It is not a direct variation function. No intercepts are shown on the graph.
2. First convert the height into inches: 80″; Find 80″ along the horizontal axis, and then move up to find the point on the graph. The ideal weight is the y-coordinate of that point, 200 lb.
3. The approximate ideal weights were: 5′8″ (140 lb); 5′3.5″ (118 lb); 7′ 4″ (240 lb). The problems encountered involve accuracy and the limitations of the graph.

4a. $m = 5$

4b. A slope of 5 means that for every 1-inch change in height, there is a corresponding 5-pound change in weight.

5. A person with a height of 0 inches should have a weight of –200 pounds. Of course, this makes no sense in this context.
6. $y = 5x – 200$

7a. 132.5 lb

7b. 90 lb

7c. 255 lb

LESSON 5.6

5e Explore

Lesson Objectives

- write an equation of a line given its slope and y-intercept.
- use the definition of *slope* to write an equation of a line.

Vocabulary

- point-slope form

Materials List

none

Description

Before beginning this Investigation, have students read the first paragraph. Ask them if anyone knows of any "rules of thumb." As an example, one rule says that when an adult's arms are stretched out straight to the side, the distance from his or her nose to the tips of his or her fingers is about one yard.

Have students look at the graph. Make sure that students understand that this graph represents an old and outdated rule of thumb that is no longer used and should not be used to predict their ideal weights. Doctors today take into account many variables and use several different measures to decide on a range of weights for individual people.

Question 1 can be used as a class discussion before groups of students begin the Investigation. This question provides a way to do an extensive review of what students should know about linear functions. Creating a list of all of these concepts will help students realize how much they actually know. As you discuss this question, you may need to ask students leading questions such as "Which variable is the independent variable? How do you know?" or "Is this a positive or negative relationship?"

TEACHING TIP

If students struggle finding an equation for **Question 6**, it might be helpful to remind them about the slope-intercept form of an equation.

Wrapping Up the Investigation:

Once all groups have completed the Investigation, use **Question 8** to debrief them. As you talk about whether the graph is a good model, point out that the rule of thumb that was used to create the graph did not take into account things such as a person's sex, age, or frame size. This model is particularly poor for very tall or very short people.

8. Answers will vary. Sample answer: This is not a good model because it does not take into account the sex, age, bone structure, or frame size of a person. For tall people, the ideal weight seems to be too low.

TEACHING TIP

The point-slope form of the equation of a line is written as $y - k = m(x - h)$ for two reasons. One is to avoid the use of subscripts that are confusing to some students. The other reason is in the hope that once students begin to think in terms of translations (Chapter 9), they will see that this is simply a translation of the graph $y = mx$, a line with a slope of m that passes through the origin.

If you prefer to use the more traditional equation, $y - y_1 = m(x - x_1)$, feel free to introduce it here.

ADDITIONAL EXAMPLE 1

Find an equation for a line that passes through the point (–1, 4) and has a slope of 2. $y = 2x + 6$

8. Do you think that the line in the graph provides a good model for predicting ideal weight from any given height? Explain.

WRITING AN EQUATION GIVEN THE SLOPE AND A POINT

One of the challenges of the Investigation was to accurately find the y-intercept of the graph so that you could find the equation for the line. There are times that you will need to find the equation of a line and do not have a graph, the slope of the line, or the y-intercept. But you may have other information, such as points on the line.

A linear equation in the form $y - k = m(x - h)$, where m is the slope of the line and (h, k) is a point on the line, is written in **point-slope form**.

This equation can be derived from the definition for slope, $m = \frac{y_2 - y_1}{x_2 - x_1}$. If you replace the point (x_1, y_1) with the coordinates of the point on the line (h, k) and (x_2, y_2) with any point on the line (x, y), you get $m = \frac{y - k}{x - h}$ or $y - k = m(x - h)$.

EXAMPLE 1

Find an equation for a line that passes through the point (1, 5) and has a slope of −2.

Solution:

Use the point-slope form.

Point-slope form	$y - k = m(x - h)$
Substitute (1, 5) for (h, k) and −2 for m.	$y - 5 = -2(x - 1)$
Simplify.	$y = -2x + 7$

WRITING AN EQUATION GIVEN TWO POINTS

To find the equation of a line when given two points on the line

- find the slope of the line.
- Then use either one of the two points and the slope in the point-slope form.

EXAMPLE 2

Find an equation for a line that passes through the points (2, 1) and (3, −4).

Solution:

Find the slope of the line.

Definition of slope $m = \frac{y_2 - y_1}{x_2 - x_1}$

Substitute. $= \frac{1 - (-4)}{2 - 3}$

Simplify. $= \frac{5}{-1}$

Simplify. $= -5$

Use the point-slope form

Point-slope form $y - k = m(x - h)$

Substitute (2, 1) for (h, k) and −5 for m. $y - 1 = -5(x - 2)$

Simplify. $y = -5x + 11$

Practice for Lesson 5.6

Tibia Length (cm)	Height (cm)
38	174
44	189

1. According to scientists, the relationship between a person's height and the length of his or her tibia, the inner and larger of the two bones of the lower leg, is approximately linear.
   a. Use the data in the table to create a linear model that relates a person's height h to the length of his or her tibia t. Express your model as an equation in slope-intercept form.
   b. What does the slope of the line mean in this context?
   c. Does the y-intercept have any meaning in this context? Explain.
   d. What is a reasonable problem domain for this function?
   e. Use your model to predict the height of a person whose tibia is 40 cm long.
   f. Use your model to predict the length of your tibia. (Hint: You will need to convert your height to centimeters before using your model.)
2. Water boils at 100° Celsius or 212° Fahrenheit. It freezes at 0° Celsius, or 32° Fahrenheit. The relationship between degrees Celsius and degrees Fahrenheit is linear. Use this information to write an equation that can be used to calculate the temperature in degrees Celsius C for any temperature given in degrees Fahrenheit F.

ADDITIONAL EXAMPLE 2

Find an equation for a line that passes through the points (1, 5) and (–2, –1).
$y = 2x + 3$

COMMON ERROR

Word problems such as those in **Exercises 1, 2, 11, and 12** may be difficult for some students because they cannot picture what the problems say. In these cases, suggest to the students that they graph or draw a picture of the information before trying to solve the problem.

Practice for Lesson 5.6

1a. $h = 2.5t + 79$

1b. The slope 2.5 indicates that a 1-centimeter increase in the length of the tibia would correspond to a 2.5-centimeter increase in height.

1c. The y-intercept of 79 indicates that a person with a tibia length of 0 would be 79 cm tall. This makes no sense in this context.

1d. Answers will vary. Sample answer: A tibia of a small person might be 20 cm, while the tibia of a very tall person might be as long as 60 cm.

1e. 179 cm

1f. Answers will vary. To convert a person's height from inches to centimeters, multiply the height by 2.54 cm/in.

2. $C = \frac{5}{9}(F - 32)$

LESSON 5.6

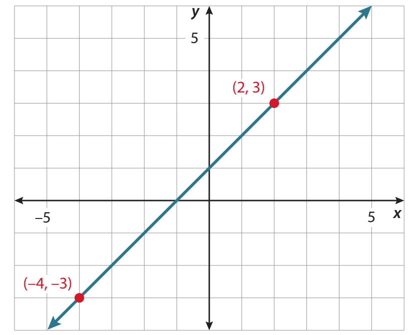
3. $y = x + 1$
4. $y = -2x + 3$
5. $y = \frac{1}{4}x + 3$
6. $y = 5$
7. $y = -3x + 13$
8. $y = 3x - 6$
9. $y = 2x + 1$
10. $x = 3$
11a. Temperature; the number of chirps per minute depends on the temperature.
11b. $c = 5t - 30$
11c. 95 chirps
11d. 21°C
12. $C = 1.5m + 1.35$

TEACHING TIP

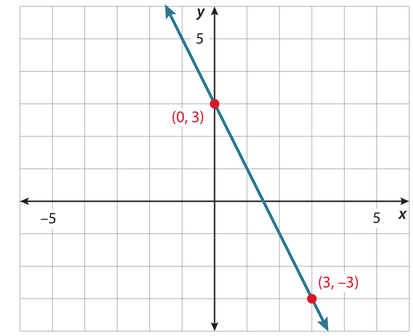
Exercise 12 This problem may be confusing for students due to the mix of fractional and decimal values. There are many ways to approach this problem. One way to solve the problem is to convert the values to decimals, use the points (0.1, 1.50) and (0.2, 1.65), and then find an equation of the line that passes through them. Accept all reasonable approaches.

For Exercises 3–4, find an equation for the line shown in the graph. Express your answer in slope-intercept form.

3.



4.



For Exercises 5–6, find an equation for the line that passes through the two given points.

5. $(-4, 2)$ and $(4, 4)$ **6.** $(-1, 5)$ and $(3, 5)$

For Exercises 7–10, find an equation for the line with the given properties.

7. The slope is -3 and it passes through the point (4, 1).
8. The x-intercept is 2 and the y-intercept is -6.
9. The line passes through the points (2, 5) and $(-3, -5)$.
10. The slope is undefined and it passes through the point (3, 5).
11. The air temperature t affects the rate at which a cricket chirps c.

Temperature (°C)	10	14	18
Chirps per Minute	20	40	60

a. Which variable is the independent variable? Explain.
b. Use the information in the table to write an equation to predict the number of chirps c for any temperature t.
c. How many chirps per minute would be expected at a temperature of 25°C?
d. At what temperature would the crickets chirp 75 times per minute?

12. A taxi ride costs \$1.50 for the first $\frac{1}{10}$-mile and \$0.15 for each additional $\frac{1}{10}$-mile. Write an equation that models the cost of the fare C in terms of the number of miles traveled m.

Lesson 5.7

Slopes of Parallel and Perpendicular Lines

If two lines are parallel, what do you know about their slopes? If two lines are perpendicular, what do you know about their slopes? In this lesson, you will investigate the slopes of parallel and perpendicular lines and use that information to find the equations of lines parallel (or perpendicular) to a given line.

PARALLEL LINES

Recall that **parallel lines** are lines in a plane that do not intersect.

Two distinct, non-vertical lines are parallel if and only if they have the same slope. All vertical lines are parallel.

EXAMPLE 1

Find an equation for a line that is parallel to the line $y = -3x + 5$ and passes through the point $(2, -4)$.

Solution:

The slope of the given line is -3, so any line parallel to it will have a slope of -3.

Use the point-slope form and the point $(2, -4)$ to find an equation.

Point-slope form	$y - k = m(x - h)$
Substitute $(2, -4)$ for (h, k) and -3 for m.	$y - (-4) = -3(x - 2)$
Simplify.	$y = -3x + 2$

LESSON 5.7

5e Explain

Lesson Objectives

- write an equation of a line that passes through a given point and is parallel to a given line.
- write an equation of a line that passes through a given point and is perpendicular to a given line.

Vocabulary

- parallel lines
- perpendicular lines

Description

This lesson provides students an opportunity to use what they learned in Chapter 4 and to work with geometry and algebraic concepts in the same lesson.

TEACHING TIP

Remind students that the phrase "if and only if" provides a way to write two converse statements as a single statement. So, the "if and only if" statement about parallel lines means:

- If two distinct, non-vertical lines are parallel, then they have the same slope.
- If two distinct, non-vertical lines have the same slope, then they are parallel.

TEACHING TIP

There are other ways to find an equation for **Example 1**. For instance, an equation can be found by using the slope-intercept form of a linear equation, $y = mx + b$, substituting 2 for x and -4 for y, then solving for b. Feel free to share alternative methods with students.

ADDITIONAL EXAMPLE 1

Find an equation for a line that is parallel to the line $y = 5x - 2$ and passes through the point $(-1, 4)$.
$y = 5x + 9$

LESSON 5.7

ADDITIONAL EXAMPLE 2

Find an equation for a line that is perpendicular to the line $y = \frac{2}{3}x - 2$ and passes through the point (2, –4).

$y = -\frac{3}{2}x - 1$

ADDITIONAL EXAMPLE 3

Consider the points $D(1, 4)$, $E(2, 2)$, and $F(6, 0)$. Are lines DE and EF parallel, perpendicular, or neither? Explain.

Neither; these two lines share the same point $E(2, 2)$, so they cannot be parallel. Since the two lines both have negative slopes, they cannot be perpendicular.

Note

In order for two numbers to be *negative reciprocals* of each other, they must be opposite in sign *and* reciprocals of each other. The product of two numbers that are negative reciprocals is −1. For example, $\frac{2}{3}$ and $-\frac{3}{2}$ are negative reciprocals, so $\frac{2}{3}\left(-\frac{3}{2}\right) = -1$.

PERPENDICULAR LINES

Recall that two lines are **perpendicular** if they meet to form right angles (90°).

> Two distinct, non-vertical lines are perpendicular if and only if their slopes are negative reciprocals of each other. Vertical and horizontal lines are also perpendicular.

For example, a line with a slope of 2 and a line with a slope of $-\frac{1}{2}$ are perpendicular because 2 and $-\frac{1}{2}$ are negative reciprocals of each other.

EXAMPLE 2

Find an equation for a line that is perpendicular to the line $y = -3x + 5$ and passes through the point $(2, -1)$.

Solution:

The slope of the given line is −3, so any line perpendicular to it will have a slope of $\frac{1}{3}$.

Use the point-slope form.

Point-slope form	$y - k = m(x - h)$
Substitute $(2, -1)$ for (h, k) and $\frac{1}{3}$ for m.	$y - (-1) = \frac{1}{3}(x - 2)$
Simplify.	$y = \frac{1}{3}x - \frac{5}{3}$

EXAMPLE 3

Consider the points $A(2, 3)$, $B(4, 5)$, and $C(6, 3)$. Are lines AB and BC parallel, perpendicular, or neither?

Solution:

The slope of line $AB = \frac{3 - 5}{2 - 4} = \frac{-2}{-2}$ or 1.

The slope of line $BC = \frac{5 - 3}{4 - 6} = \frac{2}{-2}$ or −1.

The slopes are negative reciprocals of each other, so $\overleftrightarrow{AB} \perp \overleftrightarrow{BC}$.

Practice for Lesson 5.7

For Exercises 1–2, find an equation for the line that passes through the given point and is parallel to the graph of the given line.

1. $(3, -1)$; $y = 2x - 4$

2. $(4, -2)$; $x - y = 3$

For Exercises 3–4, find an equation for the line that passes through the given point and is perpendicular to the given line.

3. $(-4, 2)$; $y = \frac{2}{3}x + 1$

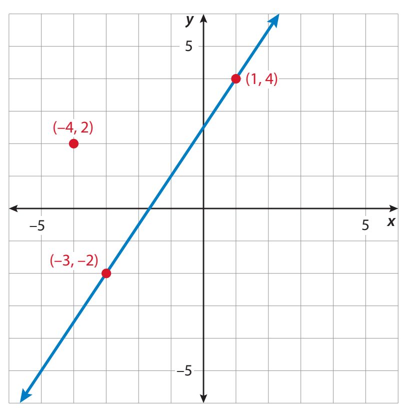
4. $(2, -5)$; $x + 5y = 1$

For Exercises 5–6, determine whether the two lines are parallel, perpendicular, or neither.

5. Line 1: $2x - 5y = -8$
Line 2: $4y + 10x = 7$

6. Line 1: contains the points $(1, 1)$ and $(-1, -5)$
Line 2: has a slope of 3 and a y-intercept of -6

7. Write an equation for the line that passes through the point $(-4, 2)$ and is parallel to the line shown in the graph.



8. Write an equation of the line that is perpendicular to the graph of $y = 5x + 15$ and passes through the origin.

COMMON ERROR

Exercises 3–4 Students may think that the negative reciprocal of a number must be negative. Remind them that it means "opposite in sign."

Practice for Lesson 5.7 Answers

1. $y = 2x - 7$

2. $y = x - 6$

3. $y = -\frac{3}{2}x - 4$

4. $y = 5x - 15$

5. perpendicular

6. parallel

7. $y = \frac{3}{2}x + 8$

8. $y = -\frac{1}{5}x$

LESSON 5.7

TEACHING TIP

Exercise 10 Students should recall that a parallelogram is a quadrilateral with opposite sides parallel.

TEACHING TIP

Exercise 11 Suggest to students that they use the information given to graph square *ABCD* and then draw in the diagonals.

9. $y = -3x + 12$

10. Yes, $\overline{AB}$ and $\overline{CD}$ have the same slope $\left(-\frac{5}{3}\right)$, so they are parallel. $\overline{BC}$ and $\overline{AD}$ have the same slope $\left(\frac{2}{7}\right)$, so they are parallel. Since both pairs of opposite sides of quadrilateral *ABCD* are parallel, it is a parallelogram.

11a. slope of $\overline{AC} = 2$; slope of $\overline{BD} = -\frac{1}{2}$

11b. They are perpendicular.

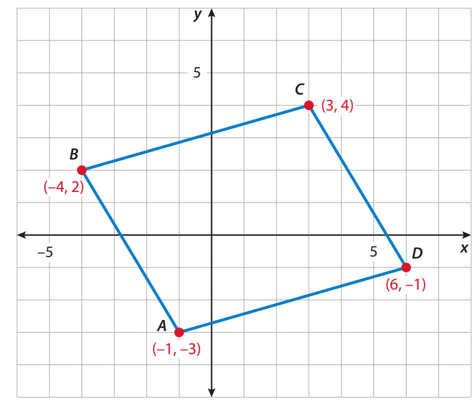
12. Sample answer: The slope of a horizontal line is 0 and the slope of a vertical line is undefined. So, there is no product of the two slopes.

13a. $x = 5$

13b. No, one of the lines is horizontal with a slope of 0 and the other is a vertical line with an undefined slope. Vertical and horizontal lines are perpendicular.

9. Write an equation of the line that has an x-intercept of 4 and is parallel to the line that passes through (5, 0) and (4, 3).

10. Determine whether *ABCD* is a parallelogram. Explain your reasoning.



11. Square *ABCD* has vertices at $A(2, 4)$, $B(4, -2)$, $C(-2, -4)$, and $D(-4, 2)$.

a. Find the slopes of the diagonals of the square.

b. What is the relationship between the two diagonals?

12. You know that vertical and horizontal lines are perpendicular. Explain why the product of their slopes is not equal to −1.

13 **a.** Find an equation of the line that is perpendicular to the line $y = 4$ and passes through the point $(5, -1)$.

b. Are the slopes of these lines negative reciprocals of each other?

CHAPTER 5

Modeling Project

It's in the News

Look on the Internet, in newspapers, and in magazines for at least two graphs of lines. Based on what you know, write a report on what you found. Be sure to include copies of the graphs you found, the equations of the lines, and interpretations of the slopes of the lines as well as the y-intercepts. Be sure to show your work.

Graphs can be drawn in such a way that they are misleading or deceptive. Look for possible deceptive graphs. If you find one, indicate what makes the graph deceptive. Discuss whether you think that the deception is intentional or not.

As you write your report, do not forget to identify where you found your graphs. If any of them come from a website, include the URL of the site.

MODELING PROJECT

5e Elaborate

Materials List

- newspapers
- magazines
- Internet

Description

Students are asked to search magazines, newspapers, and the Internet for graphs of lines. Once they have found a graph, they find the equation of the line and interpret the slope and y-intercept. Each report should include at least two lines. This project works best when students work individually.

Sample Answers

Answers will vary depending on the lines students find. Student answers should show the calculations used to determine the equations of the lines. Answers should also reflect the contexts shown in the graphs.

CHAPTER
REVIEW

5e Evaluate

Chapter 5 Review

You Should Be Able to:

Lesson 5.1

- represent functions in multiple ways.
- identify the domain and range of a function.
- identify the problem domain of a situation.

Lesson 5.2

- collect and analyze data that have a constant rate of change.

Lesson 5.3

- identify linear functions from tables, graphs, and equations.

Lesson 5.4

- solve problems that require previously learned concepts and skills.

Lesson 5.5

- graph a linear equation using a table.
- graph a linear equation in slope-intercept form.
- graph a linear equation using the intercepts.

Lesson 5.6

- write an equation of a line given its slope and y-intercept.
- use the definition of slope to write an equation of a line.

Lesson 5.7

- write an equation of a line that passes through a given point and is parallel to a given line.
- write an equation of a line that passes through a given point and is perpendicular to a given line.

Key Vocabulary

intercept (p. 138)
domain (p. 139)
range (p. 139)
problem domain (p. 139)
linear relationship (p. 144)
discrete variables (p. 145)
continuous variables (p. 145)
linear function (p. 149)
slope-intercept form (p. 149)
y-intercept (p. 149)
x-intercept (p. 149)
linear equation (p. 155)
point-slope form (p. 160)
parallel lines (p. 163)
perpendicular lines (p. 164)

Chapter 5 Test Review

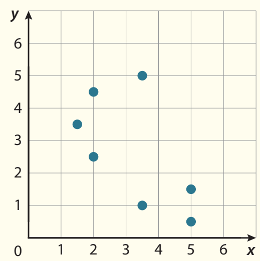
For Exercises 1–5, determine whether the relationship is a function. Explain why or why not.

1.

x	−2	4	7	11	15
y	1	−2	11	5	0

2. {(1, 6), (2, 6), (3, 6), (4, 6)}

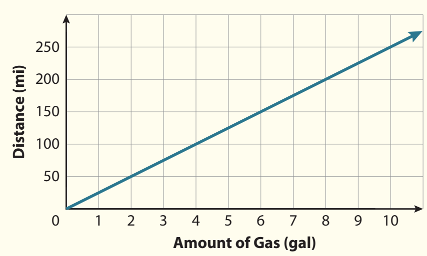
3.



4. $y = -3$

5. $x = -3$

6. The graph below represents the relationship between the distance d a car travels and the amount of gasoline g it uses.



a. Use the graph to complete the table.

Amount of Gas (gal)	0	2	4	6	8
Distance Traveled (mi)					

b. Is the relationship between the amount of gas used and the distance traveled a function? Explain.

c. What is the independent variable in this situation?

d. Write an equation to find the distance a car travels d for any amount of gas used g.

Chapter 5 Test Review Answers

1. Yes; each x-value has only one y-value.

2. Yes; each x-value has only one y-value.

3. No; the x-value of 2 has two different y-values (as do two other x-values).

4. Yes; the equation represents a horizontal line, and each x-value has exactly one y-value.

5. No; the equation represents a vertical line, and the x-value of –3 has an infinite number of different y-values.

6a.

Amount of Gas (gal)	0	2	4	6	8
Distance Traveled (mi)	0	50	100	150	200

6b. Yes; For each amount of gas g, the distance traveled d is different.

6c. In this case, either variable could be considered to be the independent variable. The graph shows the amount of gas as the independent variable.

6d. $d = 25g$

CHAPTER REVIEW

7a. (2, –2), (1, –1), (–2, 0)

7b. $y = -\frac{1}{5}$

7c. All real numbers except 4.

8. Yes, the rates of change are constant. $\frac{\Delta y}{\Delta x} = 2$

9. No; this graph is a vertical line where the input value has more than one output value.

10. Yes; this equation, written in the form $y = mx + b$, is $y = 0x + 6$, where the slope m is equal to 0 and the y-intercept b is equal to 6.

11. Graphs 1 and 2 have the greatest y-intercepts, and both have negative slopes. Graph 1 is steeper, so it has the equation $y = -1.4x + 10$. Graph 2 has the equation $y = -0.5x + 10$. Graphs 3 and 4 have positive y-intercepts that are less than 10. The slope of graph 3 is less than the slope of graph 4, so graph 3 has the equation $y = 0.8x + 5$. Graph 4 has the equation $y = 1.2x + 5$. Graph 5 is the only one with a negative y-intercept, so its equation is $y = 2.3x - 5$.

7. Consider the function $y = \frac{x+2}{x-4}$.

a. Which of the following points are on the graph of the function?

(2, −2) (1, −1) (−4, 0) (−2, 0)

b. Determine the value of y when $x = -1$.

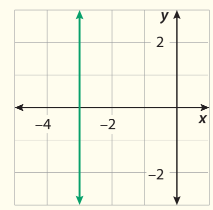
c. What is the domain of this function?

For Exercises 8–10, state whether the table, graph, or equation represents y as a linear function of x. Explain why or why not.

8.

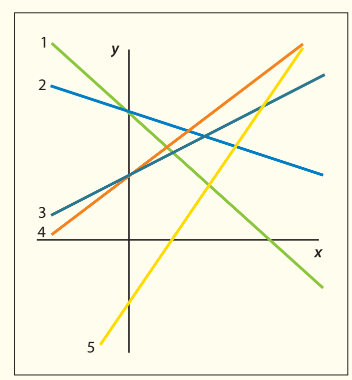
x	−4	−2	0	2	3
y	6	10	14	18	20

9.



10. $y = 6$

11. The following functions were graphed with a computer drawing program: $y = 1.2x + 5$, $y = 0.8x + 5$, $y = 2.3x - 5$, $y = -0.5x + 10$, and $y = -1.4x + 10$.



The order in which the lines were drawn is unknown, and the axes do not show a scale. Explain how you can identify the graph of each equation.

For Exercises 12–14, graph the equation.

12. $y + 4x = 8$

13. $5y - 2x = 10$

14. $x + 2 = 0$

For Exercises 15–19, write an equation for the line with the given properties.

15. Slope of $-\frac{5}{2}$ and passes through the point (8, 4)

16. Passes through the point (5, 1) and is parallel to the graph of $y = -x + 7$

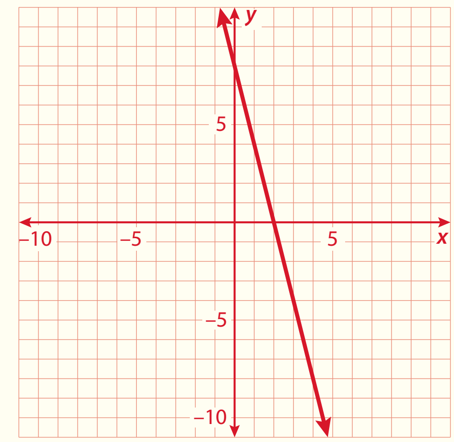
17. Passes through the points (−4, 12) and (2, 3)

18. Perpendicular to the graph of $y = \frac{1}{4}x - 3$ and has a y-intercept of 1

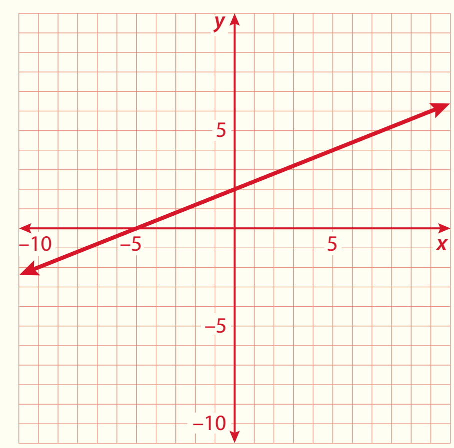
19. x-intercept of −5 and a y-intercept of 10

20. Suppose that a popular country-western singer can sell out the Cotton Bowl, which has 25,704 seats, when the ticket price is set at \$50. If the ticket price is raised to \$80, only 16,000 are estimated to attend. Assume that the relationship between ticket sales and price is linear. Write an equation to model the relationship.

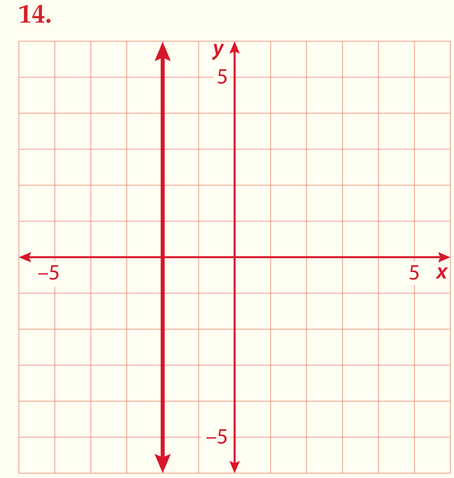
12.



13.



14.



15. $y = -\frac{5}{2}x + 24$

16. $y = -x + 6$

17. $y = -\frac{3}{2}x + 6$

18. $y = -4x + 1$

19. $y = 2x + 10$

20. Sample answer: $y = -323.5x + 41{,}879$

CHAPTER 5 EXTENSION

5e Elaborate

Lesson Objectives

- graph absolute value functions.
- graph piecewise-defined functions.
- graph step functions.

Vocabulary

- absolute value function
- greatest integer function
- piecewise-defined function
- step function

Description

In this Extension, piecewise-defined functions are defined. Then absolute value functions are defined as a subset of the piecewise-defined functions. Students are shown how to graph $y = |x|$ and then are asked questions relating to the function.

TEACHING TIP

If necessary, remind students that the quantity $|x|$ equals either x or $-x$, depending on whether x is positive or negative. Show examples of x values that are negative and $-x$ values that are positive. Students often think $-x$ always represents a negative number.

CHAPTER 5

Chapter Extension

Special Functions

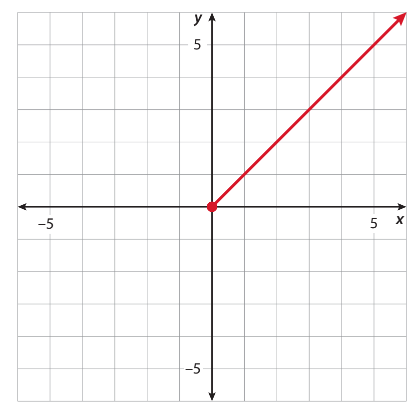
The **absolute value function**, $y = |x|$, is actually a **piecewise-defined function**. That is, it is made up of pieces of different functions. Its domain is split into smaller intervals. Over each interval, the function has the characteristics of the piece defined for that interval.

The function $y = |x|$ can be defined piecewise by the following equation:

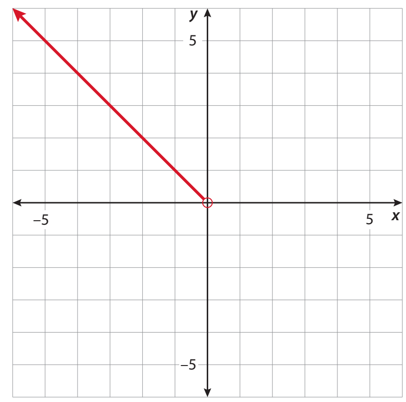
$$y = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

Since the absolute value function is a combination of two functions, it takes on the characteristics of each of them for the two different intervals.

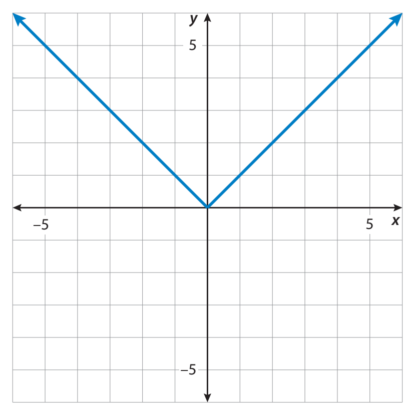
- When $x \geq 0$, the function is defined by the direct variation function $y = x$. Recall that the graph of this function is a straight line with a slope of 1.



- When $x < 0$, the function is defined by the linear function $y = -x$. This function is a decreasing function whose graph is a straight line with a slope of -1.



The graph of $y = |x|$ includes all the points on these two graphs.



1. Look at the graph of $y = |x|$. Explain how you know that it is the graph of a function.

2 **a.** When $x > 0$, is the graph increasing or decreasing?
b. When $x < 0$, is the graph increasing or decreasing?

3 **a.** What happens to the graph as x increases without bound? (Hint: to answer this question, look at the graph to see what happens to it as you move farther and farther to the right along the horizontal axis.)

CHAPTER 5 EXTENSION

Chapter 5 Extension Answers

1. Sample answer: For each input value x, there is exactly one output value y. It passes the vertical line test.
2a. increasing
2b. decreasing
3a. The graph increases.

TEACHING TIP

Questions 2–3 are included here because they foreshadow what you want students to observe as they begin to explore the end behaviors of functions that are not linear. This may be confusing to students at first, but it is worth the time to have students look at these characteristics and behaviors of different functions.

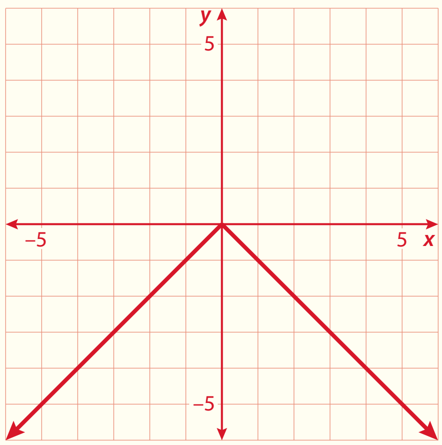
TEACHING TIP

In **Question 2b,** some students may not understand how the graph can be *decreasing* when $x < 0$ and yet in **Question 3b,** be *increasing* as x decreases without bound. To explain this, have students look at the graph of $y = |x|$ when $x < 0$. In **Question 2b**, students should be looking at the *slope* of the line in the piece of the graph that is to the left of the origin. In **Question 3b**, they should be looking at the *end behavior* of the function as x decreases. It may help to ask students what would happen to the graph when x goes from –100 to –200. They should see that the graph increases.

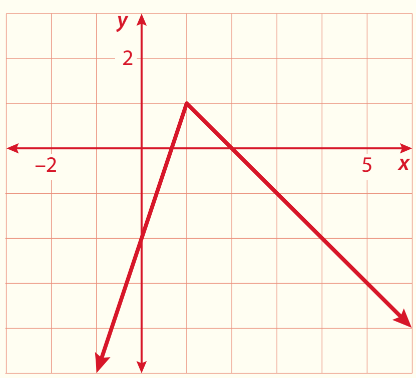
CHAPTER 5 EXTENSION

3b. The graph increases.

4.



5.



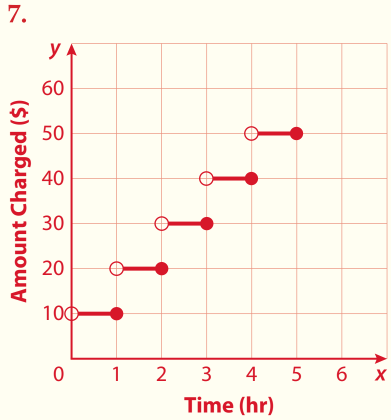
6a. the set of integers

6b. 3

6c. –4

6d. 3

7.



b. What happens to the graph as x decreases without bound? (Hint: to answer this question, look at the graph to see what happens to it as you move farther and farther to the left along the horizontal axis.)

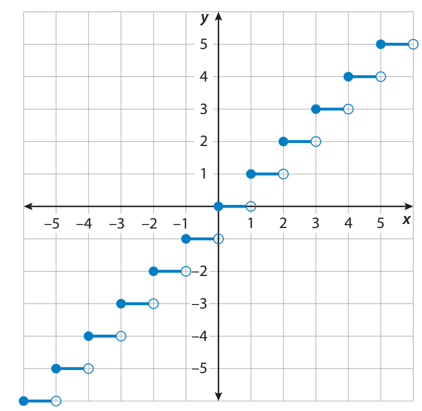
4. Graph the function $y = -|x|$.

5. Graph the piecewise-defined function:

$$y = \begin{cases} 2 - x & \text{if } x \geq 1 \\ 3x - 2 & \text{if } x < 1 \end{cases}$$

6. A **step function** is another special type of function that is made up of line segments or rays. The **greatest integer function**, written as $y = [\![x]\!]$, is one example of a step function. For all real numbers x, this function returns the greatest integer that is less than or equal to x. Examine the following table and the graph of $y = [\![x]\!]$.

$y = [\![x]\!]$	
x	y
−1	−1
−0.5	−1
0	0
0.25	0
0.75	0
1	1
1.3	1
1.8	1
2	2
2.1	2



a. The domain of the greatest integer function is the set of all real numbers. What is the range?

b. Evaluate $y = [\![x]\!]$ for $x = 3.5$.

c. Evaluate $y = [\![x]\!]$ for $x = -3.5$.

d. Evaluate $y = [\![2x]\!]$ for $x = 1.6$.

7. A babysitter charges $10 per hour or any fraction of an hour. Draw a graph of this situation.