7.1 Apply the Pythagorean Theorem

Obj.: Find side lengths in right triangles.

Key Vocabulary
- Pythagorean triple - A Pythagorean triple is a set of three positive integers $a$, $b$, and $c$ that satisfy the equation $c^2 = a^2 + b^2$.
- Right triangle – A triangle with one right angle.
- Leg of a right triangle - In a right triangle, the sides adjacent to the right angle are called the legs.
- Hypotenuse - The side opposite the right angle is called the hypotenuse of the right triangle.

Pythagorean Theorem \[ \text{Pyth. Th.} \]
In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.

\[ c^2 = a^2 + b^2 \]

EXAMPLE 1 Find the length of a hypotenuse
Find the length of the hypotenuse of the right triangle.

Solution

\[
(x) = \sqrt{7^2 + 24^2} = \sqrt{49 + 576} = \sqrt{625} = 25
\]

EXAMPLE 2 Find the length of a leg
Door A 6 foot board rests under a doorknob and the base of the board is 5 feet away from the bottom of the door. Approximately how high above the ground is the doorknob?

Solution

\[
\begin{align*}
6^2 &= 5^2 + x^2 \\
36 &= 25 + x^2 \\
11 &= x^2 \\
\sqrt{11} &= x \\
\approx 3.3 &\text{ feet above the ground.}
\end{align*}
\]

In real-world applications, it is usually appropriate to use a calculator to approximate the square root of a number. Round your answer to the nearest tenth.
EXAMPLE 3 Find the area of an isosceles triangle
Find the area of the isosceles triangle with side lengths 16 meters, 17 meters, and 17 meters.

Solution

Step 1  Draw a sketch. By definition, the length of an altitude is the \text{height} of the triangle. In an isosceles triangle, the altitude to the base is also a perpendicular bisector. So, the altitude divides the triangle into two \text{right} triangles with the dimensions shown.

Step 2  Use the Pythagorean Theorem to find the height of the triangle.
\[c^2 = a^2 + b^2\]
\[17^2 = 8^2 + h^2\]
\[289 = 64 + h^2\]
\[225 = h^2\]
\[15 = h\]

Step 3  Find the area.
\[\text{Area} = \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}(16)(15) = 120\]
The area of the triangle is 120 square meters.

EXAMPLE 4 Find length of a hypotenuse using two methods
Find the length of the hypotenuse of the right triangle.

Solution

Method 1: Use a Pythagorean triple.

A common Pythagorean triple is 8, 15, 17. Notice that if you multiply the lengths of the legs of the Pythagorean triple by 3, you get the lengths of the legs of this triangle:
\[8 \cdot 3 = 24\] and \[15 \cdot 3 = 45\]. So, the length of the hypotenuse is \[17 \cdot 3 = 51\].

Method 2: Use the Pythagorean Theorem.
\[x^2 = 24^2 + 45^2\]
\[x^2 = 576 + 2025\]
\[x^2 = 2601\]
\[x = 51\]

The most common Pythagorean triples are in bold. The other triples are the result of multiplying each integer in a bold face triple by the same factor.
7.1 Cont.

**Checkpoint** Complete the following exercise.

1. Find the length of the hypotenuse of the right triangle.
   \[ x = 15 \]

2. A 5 foot board rests under a doorknob and the base of the board is 3.5 feet away from the bottom of the door. Approximately how high above the ground is the doorknob?
   about 3.6 feet

3. Find the area of the triangle.
   \[ 672 \text{ ft}^2 \]

4. Use a Pythagorean triple to find the unknown side length of the right triangle.
   50
7.2 Use the Converse of the Pythagorean Theorem

Obj.: **Use its converse to determine if a triangle is a right triangle.**

Key Vocabulary
- **Acute triangle** - A triangle with **three acute angle**
- **Obtuse triangle** - A triangle with **one obtuse angle**

**Converse of the Pythagorean Theorem**  
**Conv. Pyth. Th.**
If the **square** of the length of the **longest** side of a triangle is **equal** to the **sum** of the **squares** of the lengths of the other **two** sides, then the triangle **is** a **right** triangle.  
If \( c^2 = a^2 + b^2 \), then \( \triangle ABC \) is a **right** triangle.

**Acute Triangle Theorem**  
**Acute ∆ Th.**
If the **square** of the length of the **longest** side of a triangle is **less** than the **sum** of the **squares** of the lengths of the other **two** sides, then the triangle \( ABC \) is an **acute** triangle.  
If \( c^2 < a^2 + b^2 \), then the triangle \( ABC \) is **acute**.

**Obtuse Triangle Theorem**  
**Obtuse ∆ Th.**
If the square of the **length** of the longest **side** of a triangle is **greater** than the **sum** of the **squares** of the **lengths** of the other **two** sides, then the **triangle** \( ABC \) is an **obtuse** triangle.  
If \( c^2 > a^2 + b^2 \), then triangle \( ABC \) is **obtuse**.

**EXAMPLE 1 Verify right triangles**
Tell whether the given triangle is a right triangle.

a. 

Solution
Let \( c \) represent the length of the longest side of the triangle. Check to see whether the side lengths satisfy the equation \( c^2 = a^2 + b^2 \).

\[
(3\sqrt{13})^2 \neq 6^2 + 9^2 \\
9 \cdot 13 \neq 36 + 81 \\
117 \neq 117
\]

The triangle **is not** a right triangle.

b. 

\[
29^2 \neq 24^2 + 16^2 \\
841 \neq 576 + 256 \\
841 \neq 832
\]

The triangle **is not** a right triangle.
EXAMPLE 2 Classify triangles
Can segments with lengths of 2.8 feet, 3.2 feet, and 4.2 feet form a triangle? If so, would the triangle be acute, right, or obtuse?

Solution
Step 1 Use the Triangle Inequality Theorem to check that the segments can make a triangle.
\[
\begin{align*}
2.8 + 3.2 &= 6 \\
2.8 + 4.2 &= 7 \\
3.2 + 4.2 &= 7.4 \\
6 &> 4.2 \\
7 &> 3.2 \\
7.4 &> 2.8
\end{align*}
\]

The side lengths 2.8 feet, 3.2 feet, and 4.2 feet form an acute triangle.

Step 2 Classify the triangle by comparing the square of the length of the longest side with the sum of squares of the lengths of the shorter sides.
\[
\begin{align*}
c^2 &> a^2 + b^2 \\
4.2^2 &> 2.8^2 + 3.2^2 \\
17.64 &> 7.84 + 10.24 \\
17.64 &< 18.08
\end{align*}
\]

EXAMPLE 3 Use the Converse of the Pythagorean Theorem
Lights You are helping install a light pole in a parking lot. When the pole is positioned properly, it is perpendicular to the pavement. How can you check that the pole is perpendicular using a tape measure?

Solution
To show a line is perpendicular to a plane you must show that the line is perpendicular to two lines in the plane.
Think of the pole as a line and the pavement as a plane. Use a 3-4-5 right triangle and the Converse of the Pythagorean Theorem to show that the pole is perpendicular to different lines on the pavement.
First mark 3 feet up the pole and mark on the pavement 4 feet from the pole.

CONCEPT SUMMARY
For Your Notebook
Methods for Classifying a Triangle by Angles Using its Side Lengths

**Conv. Pyth. Th**
If \( c^2 = a^2 + b^2 \), then \( \angle C = 90^\circ \) and \( \triangle ABC \) is a right triangle.

**Acute ΔTh.**
If \( c^2 < a^2 + b^2 \), then \( \angle C < 90^\circ \) and \( \triangle ABC \) is an acute triangle.

**Obtuse ΔTh.**
If \( c^2 > a^2 + b^2 \), then \( \angle C > 90^\circ \) and \( \triangle ABC \) is an obtuse triangle.
Checkpoint In Exercises 1 and 2, tell whether the triangle is a right triangle.

1. not a right triangle

2. right triangle

3. Can segments with lengths of 6.1 inches, 9.4 inches, and 11.3 inches form a triangle? If so, would the triangle be acute, right, or obtuse?

Yes; obtuse

4. In Example 3, could you use triangles with side lengths 50 inches, 120 inches, and 130 inches to verify that you have perpendicular lines? Explain.

Yes; A triangle with side lengths 50 inches, 120 inches, and 130 inches is a right triangle. The right triangle shows that you have perpendicular lines.
7.3 Use Similar Right Triangles

Obj.: Use properties of the altitude of a right triangle.

Key Vocabulary
- Altitude of a triangle - An altitude of a triangle is the perpendicular segment from a vertex to the opposite side or to the line that contains the opposite side.
- Geometric mean - The geometric mean of two positive numbers \( a \) and \( b \) is the positive number \( x \) that satisfies \( \frac{a}{x} = \frac{x}{b} \). So, \( x^2 = ab \) and \( x = \sqrt{ab} \).
- Similar polygons - Two polygons are similar polygons if corresponding angles are congruent and corresponding side lengths are proportional.

Alt. of rt. \( \triangle \rightarrow 3\sim\triangle \)
If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle and to each other. \( \triangle CBD \sim \triangle ABC \), \( \triangle ACD \sim \triangle ABC \), and \( \triangle CBD \sim \triangle ACD \).

Geometric Mean (Altitude) Theorem
In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments.
The length of the altitude is the geometric mean of the lengths of the two segments.

Geometric Mean (Leg) Theorem
In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments.
The length of each leg of the right triangle is the geometric mean of the lengths of the hypotenuse and the segment of the hypotenuse that is adjacent to the leg.

EXAMPLE 1 Identify similar triangles
Identify the similar triangles in the diagram.
Solution
Sketch the three similar right triangles so that the corresponding angles and sides have the same orientation.

\( \triangle FJG \sim \triangle GJH \sim \triangle FGH \)
EXAMPLE 2 Find the length of the altitude to the hypotenuse

Stadium A cross section of a group of seats at a stadium shows a drainage pipe \( BD \) that leads from the seats to the inside of the stadium. What is the length of the pips?

Solution

Step 1: Identify the similar triangles and sketch them.

\[
\Delta ADB \sim \Delta BDC \sim \Delta ABC
\]

Step 2: Find the value of \( x \). Use the fact that \( \triangle BDC \sim \triangle ABC \) to write a proportion.

\[
\frac{BD}{AB} = \frac{BC}{AC}
\]

Corresponding side lengths of similar triangles are in proportion.

Substitute.

\[
\frac{20}{x} = \frac{30}{10\sqrt{13}}
\]

Cross Products Property

Approximate.

\[
(10\sqrt{13})x = 20(30)
\]

\[
x = \frac{20(30)}{10\sqrt{13}} = 16.6
\]

The length of the pipe is about 16.6 feet.

EXAMPLE 3 Use a geometric mean

Find the value of \( y \). Write your answer in simplest radical form.

Solution

Write a proportion.

\[
\frac{15}{y} = \frac{4}{x}
\]

Substitute.

\[
y = \frac{60}{y^2}
\]

Cross Products Property

Take positive square roots.

Simplify.

Notice that \( \triangle FEG \) and \( \triangle FDE \) both contain the side with length \( y \), so these are the similar pair of triangles to use to solve for \( y \).

EXAMPLE 4 Find a height using indirect measurement

Overpass To find the clearance under an overpass, you need to find the height of a concrete support beam. You use a cardboard square to line up the top and bottom of the beam. Your friend measures the vertical distance from the ground to your eye and the distance from you to the beam. Approximate the height of the beam.

Solution

By Theorem 7.6, you know that 6.9 is the geometric mean of \( x \) and 5.

\[
\frac{x}{6.9} = \frac{5}{5}
\]

Write a proportion.

\[
x \approx 9.5
\]

Solve for \( x \).

So, the clearance under the overpass is

\[
5 + x \approx 5 + 9.5 = 14.5 \text{ feet.}
\]
7.3 Cont.

**Checkpoint** Complete the following exercise.

1. Identify the similar triangles in the diagram.

\[ \triangle NPL \sim \triangle LPM \sim \triangle NLM \]

2. Identify the similar triangles. Then find the value of \( x \).

\[ \triangle GFD \sim \triangle DFE \sim \triangle GDE; \quad x = 4.8 \]

3. Find the value of \( y \). Write your answer in simplest radical form.

\[ y = 6\sqrt{6} \]

4. The distance from the ground to Larry's eyes is 4.5 feet. How far from the beam in Example 4 would he have to stand in order to measure its height?

about 6.7 feet
7.4 Special Right Triangles

Obj.: Use the relationships among the sides in special right triangles.

Key Vocabulary
• Isosceles triangle - A triangle with at least two congruent sides.

45°-45°-90° Triangle Theorem
In a 45°-45°-90° triangle, the hypotenuse is $\sqrt{2}$ times as long as each leg.

\[ \text{hypotenuse} = \text{leg} \cdot \sqrt{2} \]

30°-60°-90° Triangle Theorem
In a 30°-60°-90° triangle, the hypotenuse is twice as long as the shorter leg, and the longer leg is $\sqrt{3}$ times as long as the shorter leg.

\[ \text{hypotenuse} = 2 \cdot \text{shorter leg}, \quad \text{longer leg} = \text{shorter leg} \cdot \sqrt{3} \]

EXAMPLE 1 Find hypotenuse length in a 45°-45°-90° triangle
Find the length of the hypotenuse.

Solution

\[ a. \] By the Triangle Sum Theorem, the measure of the third angle must be 45°. Then the triangle is a 45°-45°-90° triangle, so by Theorem 7.8, the hypotenuse is $\sqrt{2}$ times as long as each leg.

\[ \text{hypotenuse} = \text{leg} \cdot \sqrt{2} \]

\[ = 6\sqrt{2} \]

\[ \text{Substitute.} \]

EXAMPLE 2 Find leg lengths in a 45°-45°-90° triangle
Find the lengths of the legs in the triangle.

Solution

\[ \text{By the Base Angles Theorem and the Corollary to the Triangle Sum Theorem, the triangle is a 45°-45°-90° triangle.} \]

\[ \text{hypotenuse} = \text{leg} \cdot \sqrt{2} \]

\[ \frac{9\sqrt{2}}{\sqrt{2}} = x \cdot \sqrt{2} \]

\[ \frac{9\sqrt{2}}{\sqrt{2}} = x \cdot \frac{\sqrt{2}}{\sqrt{2}} \]

\[ 9 = x \]

\[ \text{Simplify.} \]
EXAMPLE 3 Find the height of an equilateral triangle
Music You make a guitar pick that resembles an equilateral triangle with side lengths of 32 millimeters. What is the approximate height of the pick?
Solution

\[
\text{Draw the equilateral triangle described. Its altitude forms the longer leg of two } \frac{30^\circ}{60^\circ} - \frac{60^\circ}{90^\circ} \text{ triangles. The length } h \text{ of the altitude is approximately the height of the pick.}
\]

\[
\text{longer leg} = \text{shorter leg} \cdot \sqrt{3} = 16 \cdot \sqrt{3} \approx 27.7 \text{ mm}
\]

EXAMPLE 4 Find lengths in a 30°-60°-90° triangle
Find the values of \(x\) and \(y\). Write your answer in simplest radical form.
Solution

Step 1 Find the value of \(x\).

\[
\text{longer leg} = \text{shorter leg} \cdot \sqrt{3}
\]

\[
\frac{8}{\sqrt{3}} = x \sqrt{3} \quad \text{Substitute.}
\]

\[
\frac{8}{\sqrt{3}} \div \frac{\sqrt{3}}{\sqrt{3}} = x \quad \text{Divide each side by } \sqrt{3}.
\]

\[
\frac{8\sqrt{3}}{3} = x \quad \text{Multiply numerator and denominator by } \sqrt{3}.
\]

Step 2 Find the value of \(y\).

\[
\text{hypotenuse} = \frac{2}{2} \cdot \text{shorter leg}
\]

\[
y = \frac{2}{2} \cdot \frac{8\sqrt{3}}{3} = \frac{16\sqrt{3}}{3}
\]

EXAMPLE 5 Find a height
Windshield wipers A car is turned off while the windshield wipers are moving. The 24 inch wipers stop, making a 60° angle with the bottom of the windshield. How far from the bottom of the windshield are the ends of the wipers?
Solution

The distance \(d\) is the length of the longer leg of a 30°-60°-90° triangle.

\[
\text{The length of the hypotenuse is } 24 \text{ inches.}
\]

\[
\text{hypotenuse} = \frac{2}{2} \cdot \text{shorter leg} \quad \text{30°-60°-90° Triangle Theorem}
\]

\[
\frac{24}{2} = s
\]

\[
12 = s
\]

\[
d = \frac{12\sqrt{3}}{2} \quad \text{Substitute.}
\]

\[
d = 20.8 \quad \text{Approximate.}
\]

The ends of the wipers are about 20.8 inches from the bottom of the windshield.
7.4 Cont.

**Checkpoint** Find the value of the variable.

1. \[ x = 4 \]

2. \[ x = 12 \]

**Checkpoint** In Exercises 3 and 4, find the value of the variable.

3. \[ x = 6 \]

4. \[ h = 6\sqrt{3} \]

5. In Example 5, how far from the bottom of the windshield are the ends of the wipers if they make a 30° angle with the bottom of the windshield?

12 inches
7.5 Apply the Tangent Ratio

Obj.: Use the tangent ratio for indirect measurement.

Key Vocabulary

- **Trigonometric ratio** - A trigonometric ratio is a ratio of the lengths of two sides in a right triangle. You will use trigonometric ratios to find the measure of a side or an acute angle in a right triangle.

- **Tangent** - The ratio of the lengths of the legs in a right triangle is constant for a given angle measure. This ratio is called the tangent of the angle.

**Tangent Ratio** \[ \tan \]

Let \( \triangle ABC \) be a right triangle with acute \( \angle A \). The tangent of \( \angle A \) (written as \( \tan A \)) is defined as follows:

\[
\tan A = \frac{\text{length of leg opposite } \angle A}{\text{length of leg adjacent to } \angle A} = \frac{BC}{AC}
\]

**EXAMPLE 1 Find tangent ratios**

Find \( \tan S \) and \( \tan R \). Write each answer as a fraction and as a decimal rounded to four places.

**Solution**

\[
\tan S = \frac{\text{opp. } \angle S}{\text{adj. to } \angle S} = \frac{ST}{RT} = \frac{60}{25} = \frac{12}{5} = 2.4
\]

\[
\tan R = \frac{\text{opp. } \angle R}{\text{adj. to } \angle R} = \frac{ST}{RT} = \frac{25}{60} = \frac{5}{12} \approx 0.4167
\]

**EXAMPLE 2 Find a leg length**

Find the value of \( x \).

**Solution**

Use the tangent of an acute angle to find a leg length.

\[
\tan 31^\circ = \frac{\text{opp. } \angle}{\text{adj.}} \quad \text{Write ratio for tangent of } 31^\circ.
\]

\[
\tan 31^\circ = \frac{17}{x} \quad \text{Substitute.}
\]

Multiply each side by \( x \).

\[
x \cdot \tan 31^\circ = 17
\]

Divide each side by \( \tan 31^\circ \).

\[
x = \frac{17}{\tan 31^\circ}
\]

Use a calculator to find \( \tan 31^\circ \).

\[
x \approx 0.6009
\]

Simplify.

\[
x \approx 28.3
\]
EXAMPLE 3 Estimate height using tangent

Lighthouse  Find the height $h$ of the lighthouse to the nearest foot.

Solution

\[
\tan 62^\circ = \frac{\text{opp.}}{\text{adj.}} \\
\tan 62^\circ = \frac{h}{100}
\]

Write ratio for $\tan 62^\circ$.

Substitute. \[100 \cdot \tan 62^\circ = h\]

Multiply each side by 100.

Use a calculator and simplify. \[188 \approx h\]

EXAMPLE 4 Use a special right triangle to find a tangent

Use a special right triangle to find the tangent of a $30^\circ$ angle.

Solution

Step 1  Choose $\sqrt{3}$ as the length of the shorter leg to simplify calculations. Use the $30^\circ$-$60^\circ$-$90^\circ$ Triangle Theorem to find the length of the longer leg.

\[\text{longer leg} = \text{shorter leg} \cdot \sqrt{3}\]

\[x = \sqrt{3} \cdot \sqrt{3} = 3\]

Step 2  Find $\tan 30^\circ$.

\[\tan 30^\circ = \frac{\text{opp.}}{\text{adj.}}\]

Write ratio for tangent of $30^\circ$.

\[\tan 30^\circ = \frac{\sqrt{3}}{3}\]

Substitute. \[\frac{\sqrt{3}}{3} \approx 0.5774\]

ACTIVITY  Right Triangle Ratio

**Materials:** metric ruler, protractor, calculator

**STEP 1** Draw a $30^\circ$ angle and mark a point every 5 centimeters on a side as shown. Draw perpendicular segments through the 3 points.

**STEP 2** Measure the legs of each right triangle. Copy and complete the table.

<table>
<thead>
<tr>
<th>Triangle</th>
<th>Adjacent leg</th>
<th>Opposite leg</th>
<th>Opposite leg/Adjacent leg</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\triangle ABC$</td>
<td>5 cm</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>$\triangle ADE$</td>
<td>10 cm</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>$\triangle AFG$</td>
<td>15 cm</td>
<td>?</td>
<td>?</td>
</tr>
</tbody>
</table>

**STEP 3** Explain why the proportions $\frac{BC}{DE} = \frac{AC}{AE}$ and $\frac{BC}{AC} = \frac{DE}{AE}$ are true.

**STEP 4** Make a conjecture about the ratio of the lengths of the legs in a right triangle. Test your conjecture by using different acute angle measures.
7.5 Cont.

Checkpoint Find \( \tan B \) and \( \tan C \). Write each answer as a fraction and as a decimal rounded to four places.

1. \[
\begin{align*}
\tan B &= \frac{24}{7} \approx 3.4286, \\
\tan C &= \frac{7}{24} \approx 0.2917
\end{align*}
\]

Checkpoint In Exercises 2 and 3, find the value of \( x \). Round to the nearest tenth.

2. \[
\begin{align*}
x &= 63^\circ \\
x &= \approx 6.6
\end{align*}
\]

3. \[
\begin{align*}
x &= 21 \\
x &= \approx 34.9
\end{align*}
\]

4. In Example 4, suppose the length of the shorter leg is 1 instead of \( \sqrt{3} \). Show that the tangent of \( 30^\circ \) is still equal to \( \frac{\sqrt{3}}{3} \).

\[
\text{longer leg} = \text{shorter leg} \cdot \sqrt{3}
\]

\[
\begin{align*}
x &= 1 \cdot \sqrt{3} = \sqrt{3} \\
\tan 30^\circ &= \frac{\text{opp.}}{\text{adj.}} = \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}
\end{align*}
\]
7.6 Apply the Sine and Cosine Ratios

Obj.: **Use the sine and cosine ratios.**

Key Vocabulary
- **Sine, cosine** - The sine and cosine ratios are trigonometric ratios for acute angles that involve the lengths of a leg and the hypotenuse of a right triangle.
- **Angle of elevation** - If you look up at an object, the angle your line of sight makes with a horizontal line is called the angle of elevation.
- **Angle of depression** - If you look down at an object, the angle your line of sight makes with a horizontal line is called the angle of depression.

Sine and Cosine Ratios → sin & cos

Let \( \triangle ABC \) be a right triangle with acute \( \angle A \). The sine of \( \angle A \) and cosine of \( \angle A \) (written \( \sin A \) and \( \cos A \)) are defined as follows:

\[
\sin A = \frac{\text{length of leg opposite } \angle A}{\text{length of hypotenuse}} = \frac{BC}{AB}
\]

\[
\cos A = \frac{\text{length of leg adjacent } \angle A}{\text{length of hypotenuse}} = \frac{AC}{AB}
\]

**EXAMPLE 1 Find sine ratios**

Find \( \sin U \) and \( \sin W \). Write each answer as a fraction and as a decimal rounded to four places.

**Solution**

\[
\sin U = \frac{\text{opp. } \angle U}{\text{hyp.}} = \frac{16}{34} = \frac{8}{17} \approx 0.4706
\]

\[
\sin W = \frac{\text{opp. } \angle W}{\text{hyp.}} = \frac{30}{34} = \frac{15}{17} \approx 0.8824
\]

**EXAMPLE 2 Find cosine ratios**

Find \( \cos S \) and \( \cos R \). Write each answer as a fraction and as a decimal rounded to four places.

**Solution**

\[
\cos S = \frac{\text{adj. to } \angle S}{\text{hyp.}} = \frac{45}{53} \approx 0.8491
\]

\[
\cos R = \frac{\text{adj. to } \angle R}{\text{hyp.}} = \frac{28}{53} \approx 0.5283
\]
EXAMPLE 3 Use a trigonometric ratio to find a hypotenuse
Basketball You walk from one corner of a basketball court to the opposite corner. Write and solve a proportion using a trigonometric ratio to approximate the distance of the walk.
Solution
\[
\sin 62^\circ = \frac{\text{opp.}}{\text{hyp.}} \quad \text{Write ratio for sine of } 62^\circ.
\]
\[
\sin 62^\circ = \frac{94}{x} \quad \text{Substitute.}
\]
\[
x \cdot \sin 62^\circ = 94 \quad \text{Multiply each side by } x.
\]
\[
x = \frac{94}{\sin 62^\circ} \quad \text{Divide each side by } \sin 62^\circ.
\]
\[
x \approx \frac{94}{0.8829} \quad \text{Use a calculator to find } \sin 62^\circ.
\]
\[
x \approx 106.5 \quad \text{Simplify.}
\]

The distance of the walk is about 106.5 feet.

EXAMPLE 4 Find a hypotenuse using an angle of depression
Roller Coaster You are at the top of a roller coaster 100 feet above the ground. The angle of depression is 44°. About how far do you ride down the hill?
Solution
\[
\sin 44^\circ = \frac{\text{opp.}}{\text{hyp.}} \quad \text{Write ratio for sine of } 44^\circ.
\]
\[
\sin 44^\circ = \frac{100}{x} \quad \text{Substitute.}
\]
\[
x \cdot \sin 44^\circ = 100 \quad \text{Multiply each side by } x.
\]
\[
x = \frac{100}{\sin 44^\circ} \quad \text{Divide each side by } \sin 44^\circ.
\]
\[
x \approx \frac{100}{0.6947} \quad \text{Use a calculator to find } \sin 44^\circ.
\]
\[
x \approx 143.9 \quad \text{Simplify.}
\]

You ride about 144 feet down the hill.

EXAMPLE 5 Find leg lengths using an angle of elevation
Railroad A railroad crossing arm that is 20 feet long is stuck with an angle of elevation of 35°. Find the lengths x and y.
Solution
Step 1 Find x.
\[
\sin 35^\circ = \frac{\text{opp.}}{\text{hyp.}} \quad \text{Write ratio for sine of } 35^\circ.
\]
\[
\sin 35^\circ = \frac{x}{20} \quad \text{Substitute.}
\]
\[
20 \cdot \sin 35^\circ = x \quad \text{Multiply each side by } 20.
\]
\[
x \approx 11.5 \quad \text{Use a calculator to simplify.}
\]
Step 2 Find y.
\[
\cos 35^\circ = \frac{\text{adj.}}{\text{hyp.}} \quad \text{Write ratio for cosine of } 35^\circ.
\]
\[
\cos 35^\circ = \frac{y}{20} \quad \text{Substitute.}
\]
\[
20 \cdot \cos 35^\circ = y \quad \text{Multiply each side by } 20.
\]
\[
y \approx 16.4 \quad \text{Use a calculator to simplify.}
\]

EXAMPLE 6 Use a special right triangle to find a sine and cosine
Use a special right triangle to find the sine and cosine of a 30° angle.
Solution
Use the 30°-60°-90° Triangle Theorem to draw a right triangle with side lengths of 1, \(\sqrt{3}\), and 2. Then set up sine and cosine ratios for the 30° angle.
\[
\sin 30^\circ = \frac{\text{opp.}}{\text{hyp.}} = \frac{1}{2} = 0.5000
\]
\[
\cos 30^\circ = \frac{\text{adj.}}{\text{hyp.}} = \frac{\sqrt{3}}{2} \approx 0.8660
\]
Checkpoint: Find \( \sin B \), \( \sin C \), \( \cos B \), and \( \cos C \). Write each answer as a fraction and as a decimal rounded to four places.

1. \[
\begin{align*}
\sin B &= \frac{21}{29} \approx 0.7241, \\
\sin C &= \frac{20}{29} \approx 0.6897, \\
\cos B &= \frac{20}{29} \approx 0.6897, \\
\cos C &= \frac{21}{29} \approx 0.7241
\end{align*}
\]

2. In Example 3, use the cosine ratio to approximate the width of the basketball court.
   about 50 feet

3. Suppose the angle of depression in Example 4 is 72°. About how far would you ride down the hill?
   about 105 feet

4. In Example 5, suppose the angle of elevation is 40°. What are the new lengths \( x \) and \( y \)?
   \( x \approx 12.9 \), \( y \approx 15.3 \)

5. Use a special right triangle to find the sine and cosine of a 60° angle.
   \( \sin 60° \approx 0.8660 \)
   \( \cos 60° = 0.5000 \)
7.7 Solve Right Triangles

Obj.: **Use inverse tangent, sine, and cosine ratios.**

Key Vocabulary
- **Solve a right triangle** - To solve a right triangle means to find the measures of all of its **sides** and **angles**.
- **Inverse tangent** - An inverse **trigonometric** ratio, abbreviated as \( \tan^{-1} \).
- **Inverse sine** - An inverse **trigonometric ratio**, abbreviated as \( \sin^{-1} \).
- **Inverse cosine** - An inverse trigonometric ratio, **abbreviated** as \( \cos^{-1} \).

**Inverse Trigonometric Ratios**

Let \( \angle A \) be an **acute** angle.

- **Inverse Tangent**
  
  If \( \tan A = x \), then \( \tan^{-1} x = m \angle A \).

- **Inverse Sine**
  
  If \( \sin A = y \), then \( \sin^{-1} y = m \angle A \).

- **Inverse Cosine**
  
  If \( \cos A = z \), then \( \cos^{-1} z = m \angle A \).

**EXAMPLE 1** Use an inverse tangent to find an angle measure

Use a calculator to approximate the measure of \( \angle A \) to the nearest tenth of a degree.

**Solution**

Because \( \tan A = \frac{16}{20} = \frac{4}{5} = 0.8 \),

\( \tan^{-1} 0.8 \approx 38.65980825 \ldots \).

So, the measure of \( \angle A \) is approximately **38.7°**.

**EXAMPLE 2** Use an inverse sine and an inverse cosine

Let \( \angle A \) and \( \angle B \) be acute angles in a right triangle. Use a calculator to approximate the measures of \( \angle A \) and \( \angle B \) to the nearest tenth of a degree.

**Solution**

- **a.** \( \sin A = 0.76 \)
- **b.** \( \cos B = 0.17 \)

  \( m \angle A = \sin^{-1} 0.76 \approx 49.5° \)

  \( m \angle B = \cos^{-1} 0.17 \approx 80.2° \)

**EXAMPLE 3** Solve a right triangle

Solve the right triangle. Round decimal answers to the nearest tenth.

**Solution**

**Step 1** Find \( m \angle B \) by using the **Triangle Sum Theorem**.

\[ 180° = 90° + 23° + m \angle B \]

\[ 67° = m \angle B \]

**Step 2** Approximate \( BC \) using a **sine** ratio.

\[ \sin 23° = \frac{BC}{40} \]

\[ 40 \cdot \sin 23° = BC \approx 15.6 \approx BC \]

**Step 3** Approximate \( AC \) using a **cosine** ratio.

\[ \cos 23° = \frac{AC}{40} \]

\[ 40 \cdot \cos 23° = AC \approx 36.8 \approx AC \]

The angle measures are \( 23°, 67°, \) and \( 90° \). The side lengths are \( 40 \) feet, about \( 15.6 \) feet, and about \( 36.8 \) feet.
EXAMPLE 4 Solve a real-world problem
Model Train You are building a track for a model train. You want the track to incline from the first level to the second level, 4 inches higher, in 96 inches. Is the angle of elevation less than $3^\circ$?

Solution

Use the tangent and inverse tangent ratios to find the degree measure $x$ of the incline.

$$\tan x^\circ = \frac{\text{opp.}}{\text{adj.}} = \frac{4}{96} \approx 0.0417$$

$$x \approx \tan^{-1} 0.0417 \approx 2.4^\circ$$

The incline is about $2.4^\circ$, so it is less than $3^\circ$.

Checkpoint Complete the following exercise.

1. In Example 1, use a calculator and an inverse tangent to approximate $m\angle C$ to the nearest tenth of a degree.

   $$m\angle C \approx 51.3$$

2. Find $m\angle D$ to the nearest tenth of a degree if $\sin D = 0.48$.

   $$m\angle D \approx 28.7^\circ$$

3. Solve a right triangle that has a $50^\circ$ angle and a 15 inch hypotenuse.

   Angles: $90^\circ$, $50^\circ$, and $40^\circ$; Side lengths: 15 in., about 9.6 in., and about 11.5 in.

4. In Example 4, suppose another incline rises 8 inches in 120 inches. Is the incline less than $3^\circ$?

   No, the incline is about $3.8^\circ$. 