Review: Expected Value, Variance, Transformations, Combinations, Binomial & Geometric Probability

1. Which of the following could be quantified as a binomial random variable?

A) The number of persons in an emergency room.
B) The weights of truck arriving at a weight station.
C) Whether or not a student wears glasses.
D) The square foot areas of houses being built in Preston Hollow.
E) The first time you get a speeding ticket.

2. Suppose the probability that a softball player gets a hit in any single at-bat is .300. Assuming that her chance of getting a hit on a particular time at bat is independent of her other at bats, what is the probability that she will not get a hit until her fourth time at bat in a game?

(A) ${}_4C_3(.3)^1(.7)^3$
(B) ${}_4C_3(.3)^3(.7)^1$
(C) ${}_4C_1(.3)^3(.7)^1$
(D) $(.3)^3(.7)^1$
(E) $(.3)^1(.7)^3$

$P(H^c \cap H^c \cap H^c \cap H)$

3. A 12-sided die has faces, which are numbered from 1 to 12. Assuming that the die is fair (that is, each face is equally likely to appear each time), which if the following would give the exact probability of getting 10 3s out of 50 rolls

$P(X=3) = 1/12 = .083$

A) ${}_{50}C_{10}(.083)^{40}(.917)^{10}$
B) ${}_{50}C_{10}(.083)^{10}(.917)^{40}$
C) ${}_{12}C_{10}(.083)^{10}(.917)^{2}$
D) $10\ (.083)3(.917)^7$
E) $(.083)^{10}(.917)^{40}$

4. Which of the following is NOT a condition for a geometric setting?

(A) There are only two possible outcomes for each trial. ✓
(B) The probability of success is the same for each trial. ✓
(C) The trials are independent. ✓
(D) There are a fixed number of observations. X
(E) The variable of interest is the number of trials requires to reach the first success. ✓

5. Forty percent of patrons at a particular coffee bar order a pastry along with their venti nonfat extra-hot no foam with whip three-pump vanilla latte. What is the expected number and standard deviation of the next 60 customers who will order a pastry to go with their latte?

$p=.4$

$E(X) = 60(.4)$

24

$SD(X) = \sqrt{60(.4)(.6)}$

A) 24, 3.79
B) 24, 14.4
C) 4.90, 3.79
D) 4.90, 14.4
E) 2.4, 3.79

6. If X is a binomial random variable with n = 10 and p = 0.25, then

a) $\sigma_x = 1.875$
b) $\sigma_x = \sqrt{2.5}$
c) $\sigma_x = \sqrt{1.875}$
d) $\sigma_x = 2.5$
e) 4

$\sqrt{10(.25)(.75)}$

$\sqrt{1.875}$

7. Suppose X and Y are random variables with E(X) = 500, Var (X) = 50, E(Y) = 400 and Var (Y) = 30. Given that X and Y are independent, what is the expected value and variance of the random variable X – Y?

(A) $E(X-Y) = 900, Var\ (X-Y) = 20$
(B) $E(X-Y) = 900, Var\ (X-Y) = 80$
(C) $E(X-Y) = 100, Var\ (X-Y) = 80$
(D) $E(X-Y) = 100, Var\ (X-Y) = 20$
(E) There is insufficient information to answer this question.

8. Ten percent of the population is left-handed. In a class of 100 students, write binomial probability for the statement "there are at most 12 left-handed students in the class."

a. P(x < 12)
b. P(x ≤ 12)
c. P(x = 12)
d. P(x > 12)

$p = .1$ $P(x \leq 12) =$

9. Find the probability that in 200 tosses of a fair six-sided die, a five will be obtained at most 40 times.

a. 0.1223
b. 0.0894
c. 0.9106
d. 0.8777

$p = \frac{1}{6}$ $P(x \leq 40)$

10. The probability that the Red River will flood in any given year has been estimated from 200 years of historical data to be one in four. This means:

a) The Red River will flood every four years.
b) In the next 100 years, the Red River will flood exactly 25 times.
c) In the last 100 years, the Red River flooded exactly 25 times.
d) In the next 100 years, the Red River will flood about 25 times.
e) In the next 100 years, it is very likely that the Red River will flood exactly 25 times.

$\frac{1}{4}$

11. In a large population of college students, 20% of the students have experienced feelings of math anxiety. If you take a random sample of 10 students from this population, the probability that exactly 2 students have experienced math anxiety is

(a) 0.3020 (b) 0.2634 (c) 0.2013 (d) 0.5 (e) 1

p n $P(x=2) =$

12. ~~In a certain large population, 40% of households have a total annual income of over $70,000.~~ In a certain large population, 40% of households have a total annual income of over $70,000. A simple random sample of 4 of these households is selected. What is the probability that 2 or more of the households in the survey have an annual income of over $70,000?

(a) 0.3456 (b) 0.4000 (c) 0.5000 (d) 0.5248 (e) answer cannot be computed

p $n=4$ $P(x \geq 2)$ $1-P(x \leq 1) =$

13. Twenty percent of all trucks undergoing a certain inspection will fail the inspection. Assume that trucks are independently undergoing this inspection, one at a time. The expected number of trucks inspected before a truck fails inspection is

(a) 2
(b) 4
(c) 5
(d) 20
(e) The answer cannot be computed from the information given.

$p = .2$ $E(x) = \mu_x = \frac{1}{p} = \frac{1}{.2} =$

Geometric

$-n$

14. A survey asks a random sample of 1500 adults in Ohio if they support an increase in the state sales tax from 5% to 6%, with the additional revenue going to education. Let X denote the number in the sample that say they support the increase. Suppose that 40% of *all* adults in Ohio support the increase. The probability that X is more than 650 is

$p = .4$ $\quad P(X > 650) = 1 - P(X \le 650) = .004$

(a) less than 0.0001.
(b) less than 0.001.
(c) less than 0.01.
(d) 0.9960.
(e) none of these.

15. A pretzel company calculated that there is a mean of 72.5 broken pretzels in each production run with a standard deviation of 7.1. If the distribution is approximately normal, find the probability that there will be fewer than 60 broken pretzels in a run.

a) 0.00 b) 0.04 c) 0.06 d) 0.10 e) None of these

$Z = \frac{60 - 72.5}{7.1}$ $\quad -1.761$

$P(X < 60) = .039$

16. A random sample of students at UTD is taken to determine the number of courses (x) a student is registered to take.

x	1	2	3	4	5	6	7
p(x)	.03	.05	.1	.21	.43	.12	.06

a) What is P(x = 3)? $.1$

b) What is $P(x \le 6)$? $.94$

c) What is the probability that a selected student is taking at least 2 courses? $P(X \ge 2) =$ add up 2 – 7 OR $1 - P(x=1)$ $.97$

d) What is the probability that a selected student is taking at least 5 courses? $P(X \ge 5) = .43 + .12 + .06 = .61$

e) What is P(3 ≤ x ≤ 5) ? $= .1 + .21 + .43 = .74$

17. A probability distribution is described below:

x	2	3	4
p(x)	.6	.1	.3

must add to 1

a) What is the missing probability?

b) Find the mean(expected value) of the distribution. $E(X) = 2(.6) + 3(.1) + 4(.3) = 2.7$

c) Find the variance. $Var(X) = (2-2.7)^2(.6) + \cdots (3-2.7)^2(.3) = .81$

d) Find the standard deviation. $SD(X) = \sqrt{.81} = .9$

18. Suppose that the distribution in #17 represents X = the number of people at a table in a restaurant who have ordered an all-you-can-eat-buffet for which there is a table charge of \$5 plus \$6 per person. Define a new random variable Y = the total cost of the meal. Write the equation for Y. What is the expected value of Y? What are the variance and standard deviation of Y?

$Y = 5 + 6X$

$E(Y) = 5 + 6E(X) = 5 + 6(2.7) = \$21.20$

$Var(Y) = 6^2 \, var(X) = 36(.81) = 29.16$

$SD(Y) = \$5.40$

$-p$

19. According to government data, 20% of employed women have never been married.

a) If 10 employed women are selected at random, what is the probability that exactly 2 have never been married?

$P(X=2) = .302$

b) That at most 2 have never been married?

$P(X \le 2) = .678$

c) That at least 8 never been married?

$P(X \ge 8) = 1 - P(X \le 7) = .00008$

20. Camp Wee-O-Wee has found that 8% of young campers get poison ivy each season. If 273 children are registered for the summer season, about how many can be expected to get poison ivy.

n p

$E(X) = 273(.08) = 21.84$ campers

21. An accountant believes that 90% of the company's invoices are free of errors. If the claim is correct, what is the probability that of 25 invoices, seven will contain errors?

$p = .1$

n x

$P(X=7) = .007$

22. Sara Bellum is taking Western Civilization this semester on a pass/fail basis. The department teaching the course has a history of passing 82% of the students who take the course each term.

p

a) What is the probability that Sara first passes on the second try at taking the course?

$P(X=2) = .1476$

geo

b) What is the probability that Sara needs three or more tries to pass Western Civilization?

$P(X \ge 3) = 1 - P(X \le 2) = .0324$

23. Assume the heights of high school basketball players are normally distributed. For boys the mean is 74 inches with a standard deviation of 4.5 inches, while girl players have a mean height of 70 inches with a standard deviation of 3 inches. At a mixed 2-on-2 tournament teams are randomly pairing boys with girls as teammates.

$B \sim N(74, 4.5)$

$G \sim N(70, 3)$

a.) On average, how much taller do you expect the boy to be?

$E(B-G) = 74 - 70 = 4$ inches

b.) What will be the standard deviation of the difference in teammates' heights?

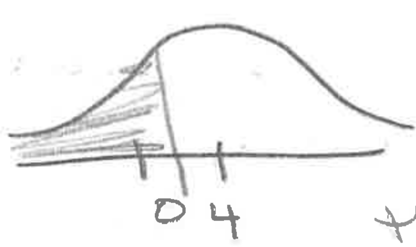
$SD(B-G) = \sqrt{4.5^2 + 3^2} = \sqrt{29.25} = 5.408$ inches

c.) On what percent of the teams would you expect the girl to be taller than the boy?

$P(B-G < 0) = P(Z < -.74) = .2296$

$.23$

$$Z = \frac{0-4}{5.408} = -.74$$



on 23% of the teams the girl is taller than the boy

Linda is a sales associate at a large auto dealership. At her commission rate of 25% of gross profit on each vehicle she sells, Linda expects to earn $350 for each car sold and $400 for each truck or SUV sold. She estimates her car sales on a sunny Saturday as follows:

X=Cars sold	0	1	2	3
Probability	.3	.4	.2	.1

She estimates her truck or SUV sales on a sunny Saturday as follows:

Y=Trucks/SUVs sold	0	1	2
Probability	.4	.5	.1

Find the following:

1. The mean and standard deviation of the number of cars sold.

$E(X) = 1.1$ cars

$SD(X) = \sqrt{.89} = .943$ cars

2. The mean and standard deviation of the number of trucks/SUVs sold.

$E(Y) = .7$ trucks

$SD(Y) = \sqrt{.41} = .640$ trucks

3. The expected number of total vehicles sold.

$E(X+Y) = 1.1 + .7 = 1.8$ vehicles

4. The standard deviation of the total vehicles sold. Are these two random variables independent?

$SD(X+Y) = \sqrt{.89 + .41} = \sqrt{1.3} = 1.14$ vehicles

5. How many cars would you expect Linda to sell on two sunny Saturdays? * It Didn't Ask

$E(X_1 + X_2) = 1.1 + 1.1 = 2.2$ cars but $SD(X_1 + X_2) = \sqrt{1.78} = 1.334$

6. How many cars would you expect Linda to sell on a super busy sunny Saturday when they have twice as much business? E(2x)? What is the standard deviation?

$E(2X) = 2(1.1) = 2.2$ cars $SD(2X) = 1.886$

7. Find the mean and standard deviation for the number of trucks/SUVs on this super busy Saturday?

$E(2Y) = 2(.7) = 1.4$ trucks

$SD(2Y) = 1.28$ trucks

1. A computer repair shop has two work centers. The first center examines the computer to see what is wrong, and the second center repairs the computer. Let x_1 and x_2 be random variables representing the lengths of time in minutes to examine a computer (x_1) and to repair a computer (x_2). Assume x_1 and x_2 are independent random variables. Long-terms history has shown the following times:

Examine computer x_1: $\mu_1 = 28.1\ minutes,\quad \sigma_1 = 8.2\ minutes$

Repair computer x_2: $\mu_2 = 90.5\ minutes,\quad \sigma_2 = 15.2\ minutes$

a.) Let W = $x_1 + x_2$ be a random variable representing the total time to examine and repair the computer. Compute the mean, variance and standard deviation of W.

$E(W) = 28.1 + 90.5 = 118.6$ minutes

$Var(W) = 8.2^2 + 15.2^2 = 298.28$

$SD(W) = \sqrt{298.28} = 17.271$ minutes

b.) Suppose it costs \$1.50 per minute to examine the computer and \$2.75 per minute to repair the computer. Then $Z = 1.50x_1 + 2.75x_2$ is a random variable representing the service charges. Compute the mean, variance and standard deviation of Z.

$E(Z) = 1.5(28.1) + 2.75(90.5) = \$291.03$

$Var(Z) = 1.5^2(8.2^2) + 2.75^2(15.2^2) = 1898.53$

$SD(Z) = \sqrt{1898.53} = \$43.57$

2. The weight of medium-sized tomatoes selected at random from a bin at Kroger is a random variable with a mean $\mu = 10\ ounces$ and a standard deviation $\sigma = .5\ ounce$.

$Y = X_1 + X_2 + X_3 + X_4$

a. Suppose we pick four tomatoes from the bin at random and put them in a bag. Let Y = the weight of the 4 tomatoes. Find the expected value (mean) and standard deviation of the random variable Y.

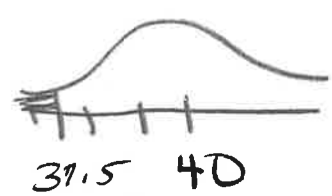
$E(Y) = 10 + 10 + 10 + 10 = 40$ ounces

$SD(Y) = \sqrt{.5^2 + .5^2 + .5^2 + .5^2} = \sqrt{1} = 1$ ounce

b. What is the probability that the 4 randomly chosen tomatoes weigh less than 37.5 ounces?

$P(Y < 37.5) = P(Z < -2.5) = .006$

$Z = \frac{37.5 - 40}{1} = -2.5$



c. Suppose we pick two tomatoes at random from the bin. The difference in the weights is the random variable $D = x_1 - x_2$. Find the mean and standard deviation (in ounces) of D.

$E(D) = 10 - 10 = 0$ ounces

$SD(D) = \sqrt{.5^2 + .5^2} = \sqrt{.5} = .707$ ounces