

Analyzing Numerical Data: Validating Identification Numbers

I.D Student Activity Sheet 12: Universal Product Codes

Identification numbers are present everywhere in society. Today's identification numbers are more sophisticated than those introduced years earlier (for example, Social Security numbers). Today's numbers have a check digit to partially ensure that they have been correctly scanned or entered into a computer.

Universal Product Codes (UPCs), typically in the form of barcodes, identify retail products.



The 12-digit UPC barcode consists of three parts:

- manufacturer number,
- product number, and
- check digit.

For example, the manufacturer number for the Dr. Pepper Company is 078000 and appears in the first six digits of all of the company's product UPC barcodes. GS1, formerly the Uniform Code Council, issues a company this six-digit number. Every item sold by a company requires a different five-digit product number. This includes specific products, their different sizes, their array of colors, their variety of flavors, and other distinguishing features. The last number is the **check digit**, which guards against entry errors and fraud.

The check digit in a UPC number (that is, the twelfth digit) is determined in the following manner:

- Multiply the first digit by 3.
- Add the second digit.
- Multiply the third digit by 3.
- Add the fourth digit.
- Continue this alternating process for the Digits 5 to 12.

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The check digit is chosen so that the calculation described previously totals a number whose final digit is 0. In the UPC number $a_1a_2a_3a_4a_5a_6a_7a_8a_9a_{10}a_{11}d$, the check digit is d , for which the sum

$$3a_1 + a_2 + 3a_3 + a_4 + 3a_5 + a_6 + 3a_7 + a_8 + 3a_9 + a_{10} + 3a_{11} + d$$

ends in 0. In this weighted sum, the weights are: {3, 1, 3, 1, 3, 1, 3, 1, 3, 1, 3, 1}.

When entering a code number, the *single-digit error* is most common (for example, keying in 8 instead of 3). Another common error is the transposition error, where the order of two adjacent digits is reversed (for example, writing 83 instead of 38). Systems have been established to detect and correct (when possible) these and other errors almost immediately.

1. Show that 0-58200-48826-5 is a valid UPC number.

2. Show that 0-52200-48826-5 is an invalid UPC number.

If someone made a single-digit error when entering this invalid number, can you tell which digit is incorrect? Why or why not?

Change one digit in this invalid number so the resulting number is valid.

3. Determine the check digit (d) for the UPC number 38137009213 d .

4. Suppose you entered 8 instead of 9 when recording the UPC number 1 55210 02149 6. Explain why the UPC method will detect this error.

Do you think the UPC method will detect all such single-digit errors? Either give several examples of numbers with an error that will not be detected or explain why you think all such single-digit errors will be detected.

5. Suppose you use the weights {7, 1, 7, 1, 7, 1, 7, 1, 7, 1, 7, 1} instead of {3, 1, 3, 1, 3, 1, 3, 1, 3, 1, 3, 1} in the UPC method. Do you think this altered method will detect single-digit errors? Either give several examples of numbers with an error that will not be detected or explain why you think all such single-digit errors will be detected.

6. Suppose you use the weights {2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1} instead of {3, 1, 3, 1, 3, 1, 3, 1, 3, 1, 3, 1} in the UPC method. Do you think this altered method will detect single-digit errors? Either give several examples of numbers with an error that will not be detected or explain why you think all such single-digit errors will be detected.

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7. When entering UPC numbers, will the error of transposing 2 and 7 be detected? That is, ... 72 ... is recorded as ... 27 How about 2 and 6? Make a conjecture about the values of a and b for which the error of transposing ab and ba is detected by the UPC method. Explain your reasoning in making your conjecture.
8. **REFLECTION:** In what ways is the UPC check digit calculation like the activities you explored in Section C, “Indices Using Weighted Sums and Averages”?
9. **EXTENSION**
- The European Article Numbering Code (EAN 13) is the worldwide standard. Conduct research on how the EAN 13 number is calculated and what types of errors this method detects. Prepare a report and presentation about your findings.
 - The International Standard Book Numbers (ISBNs) used to identify books have a check-digit scheme that detects all single-digit and transposition errors. Conduct research on how the ISBN number is calculated and why this method detects all single-digit and transposition errors. Prepare a report and presentation about your findings.
 - In a vehicle identification number (VIN), the check digit comes in the middle of the number instead of at the end. Conduct research on how the VIN is calculated and what types of errors this method detects. Prepare a report and presentation about your findings.
 - Conduct research on the meaning of various digits in credit card numbers and how check digits are determined. Prepare a report and presentation about your findings.
 - Imagine that you are employed by a company that does not have an identification code number for its employees and products. Prepare a report discussing various numbering alternatives and make a recommendation. Include a check digit in your code number. Indicate how successful your method of determining the check digit will be in detecting single-digit and transposition errors. The company can be your school.