Pre-AP Algebra 2
Unit 8 -Lesson 3 – End behavior of rational functions

Objectives:
Students will be able to:

- Determine end behavior by dividing and seeing what terms drop out as x → $\pm\infty$
- Know that there will be a horizontal asymptote at the ratio of the leading coefficients when the degrees of the numerator and denominator are equal
- Compare a "features analysis sketch" to a graph generated by graphically dividing lines

Materials: hw #8-2 answers handout; pair work handout; special note-taking template; hw #8-3;

Time	Activity
10 min	**Review Homework** Hand out solutions to the homework. Students check and grade, and ask questions if they need help. Problems to grade are circled.
20 min	**Pair Work** Preparation for the End Behavior lecture. Students practice dividing polynomials and substituting values into a term that drops out as x → ∞. Review answers on the overhead.
50 min	**Direct Instruction** Hand out special note-taking template **Background:** End behavior: what the function does as x gets really big or small. End behavior of a ***polynomial***: always goes to ±∞. **Examples:** 1) $f(x)=\frac{4x-6}{2x+6}$ Ask students to graph the function on their calculators. Do the same on the overhead calculator. Note the vertical asymptote and the intercepts, and how they relate to the function. Use polynomial division to write $f(x)=\frac{4x-6}{2x+6}=2-\frac{18}{2x+6}$. What happens as x → ±∞? The remainder term drops out and f(x) → 2. This is called a **horizontal asymptote**. On the calculator, add in the function g(x) = 2 and look at the graph. Zoom out and see what happens. On the sketch, draw in dotted lines for both asymptotes, plot the intercepts, and make the sketch. 2) $f(x)=\frac{x^2-5x+1}{2x-4}$ Sketch this on the calculator. What is the end behavior like? It goes to ±∞, but we can be more specific with a rational function. Use polynomial division to write $f(x)=\frac{x^2-5x+1}{2x-4}=\frac{1}{2}x-\frac{3}{2}-\frac{5}{2x-4}$. As x gets really large, the last terms drops off and f(x) → ½x – 3/2, which is a linear function. This is called a **slant asymptote**. On the sketch, draw in dotted lines for the slant and vertical asymptotes, plot the y-intercept, and make a sketch. 3) $f(x)=\frac{3x+6}{x^2-2x-15}$ Graph this on the calculator. What are the vertical asymptotes? Now, what happens as x gets really large or small? What would polynomial division give us? If we add in a $0x^2$ as a place holder, we

end up with $f(x) = 0 + \dfrac{3x+6}{x^2 - 2x - 15}$. This is a **horizontal asymptote**, at y = 0.

Concepts:

1) Given the rational function $f(x) = \dfrac{g(x)}{h(x)}$:

If the denominator has a larger degree:

- There is a horizontal asymptote at the line y = 0.

If the denominator and numerator have the same degree:

- There is a horizontal asymptote at the line y = k
- *k* is the ratio of the leading coefficients.

If the denominator has a smaller degree:

- There is no horizontal asymptote.
- Divide g(x) by h(x). The quotient (without the remainder) describes the end behavior function.
- If that quotient is a linear function, it is called a slant asymptote. *(If it is a quadratic function, it is called a parabolic asymptote, and so forth).*

2) Comparison of horizontal and vertical asymptotes:

- A vertical asymptote:
  - may never be crossed by the function
  - is created when x makes the denominator = 0 in a rational function
- A horizontal (or slant) asymptote:
  - may be crossed in the middle, but not at the ends
  - is created when x → ±∞ in a rational function

Homework #8-3: End behavior and quiz prep

Pre-AP Algebra 2 **Name: ___________________________**
8-3 Pair Work

Preparation for the End Behavior Lecture

1) Determine the end behavior of each **polynomial** function.

a. $f(x) = 3x^8 - 2x^2$

b. $f(x) = -2x^7 + 8x^2 - 5x + 1$

2) Do each division problem, either with regular or synthetic division.

a. $\dfrac{4x^2 + 8x - 3}{2x + 1}$

b. $\dfrac{2x^3 - 4x^2 + 9}{x - 3}$

c. $\dfrac{6x - 2}{3x + 5}$

3) Let $f(x) = 7 + \dfrac{8}{x-4}$. **Enter the function into your calculator and use the Table screen to evaluate the x values.** *Don't forget that the answers in the table will be rounded, so you may need to highlight them to see the actual value.*

x	f(x)		x	f(x)		x	f(x)		x	f(x)
1			7			13			100	
2			8			14			500	
3			9			15			1,000	
4			10			16			9,000	
5			11			17			9,999	
6			12			50			99,999	

Based on your results, as x → +∞, f(x) → _______? **Why does this happen?**

As the denominator gets really ***large***, what happens to the quotient?

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Homework #8-3

Hw #8-3: End Behavior of Rational Functions

For problems 1 – 4, let f(x) = 2x + 3 and g(x) = 2x – 5. What kind of graph will each of the following functions produce?

1) s(x) = f(x) + g(x)

a. horizontal line b. parabola c. slanted line d. hyperbola

2) d(x) = f(x) – g(x)

a. horizontal line b. parabola c. slanted line d. hyperbola

3) p(x) = f(x)g(x)

a. horizontal line b. parabola c. slanted line d. hyperbola

4) q(x) = $\frac{f(x)}{g(x)}$

a. horizontal line b. parabola c. slanted line d. hyperbola

For problems 5 – 7, determine what happens on the graph of the function at the given value of x.

5) $f(x) = \frac{x^2 - 2x - 3}{x^2 - 3x + 2}$, when x = -1

a. x-intercept b. vertical asymptote c. neither

6) $f(x) = \frac{4x + 8}{3x + 5}$, when x = 1

a. x-intercept b. vertical asymptote c. neither

7) $f(x) = \frac{x^2 - 7x + 2}{x^2 - 4}$, when x = 2

a. x-intercept b. vertical asymptote c. neither

8) Determine the y-intercepts of the functions in questions #5 – 7.

5) 6) 7)

For each function:

a) Determine the x- and y-intercepts.
b) Determine the vertical and horizontal asymptotes.
c) Look at the graphs and match each one to the correct function.

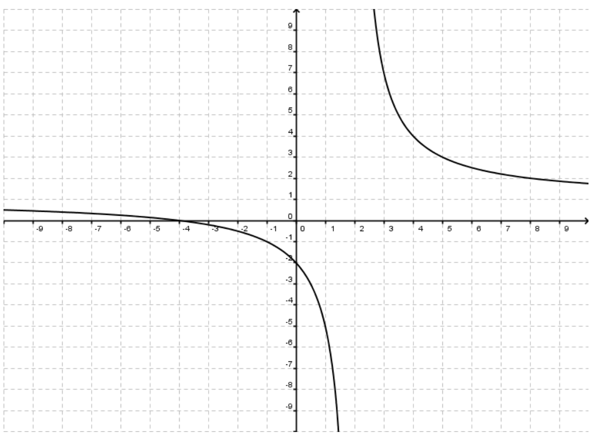
______9) $f(x)=\dfrac{x-4}{x-2}$

______10) $f(x)=\dfrac{4+x}{x-2}$

______11) $f(x)=\dfrac{2x-5}{x+2}$

______12) $f(x)=\dfrac{4x-1}{x+2}$

A



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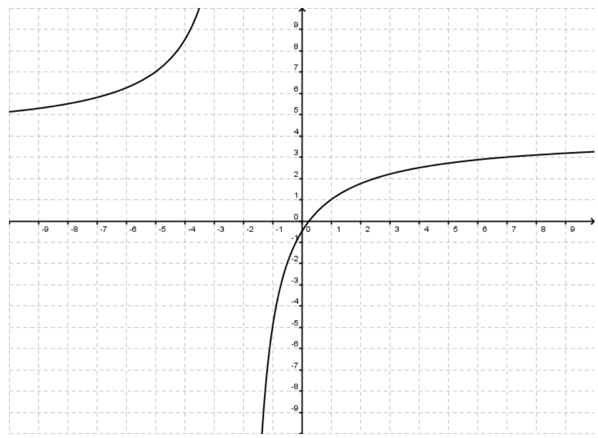

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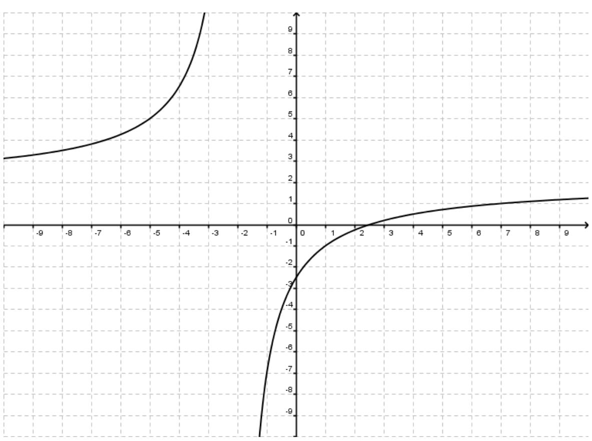

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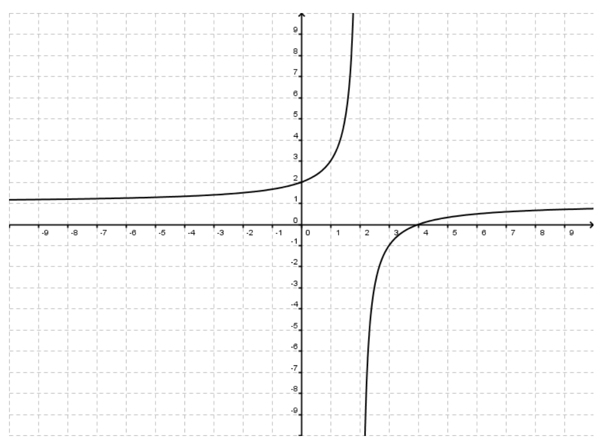


For each rational function:

- Use polynomial division to divide the numerator by the denominator (use scratch paper).
- Look at your result and write the **end behavior function** in the form “as x →±∞, f(x) → ________”
- Match the functions to the graphs based on their end behaviors.

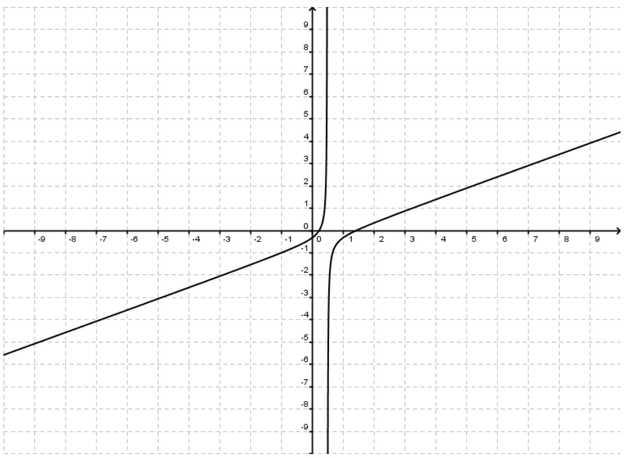
______13) $f(x)=\dfrac{x^3-2x^2+1}{2x-1}$

______14) $f(x)=\dfrac{3x^2-5x+1}{6x-3}$

______15) $f(x)=\dfrac{x+2}{4x^2-4x+1}$

______16) $f(x)=\dfrac{4x^3-5x^2}{2x^3+x-1}$

A



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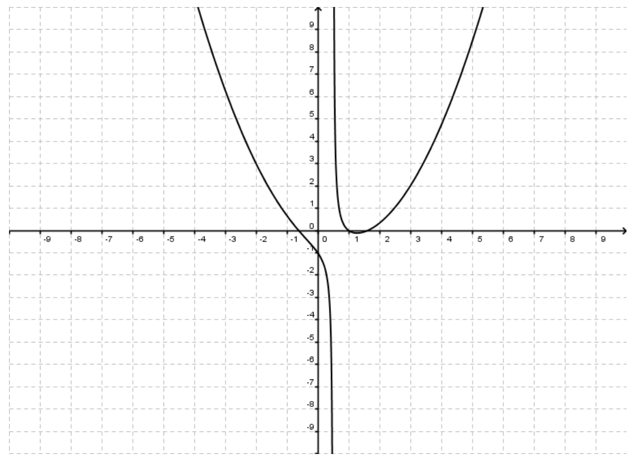

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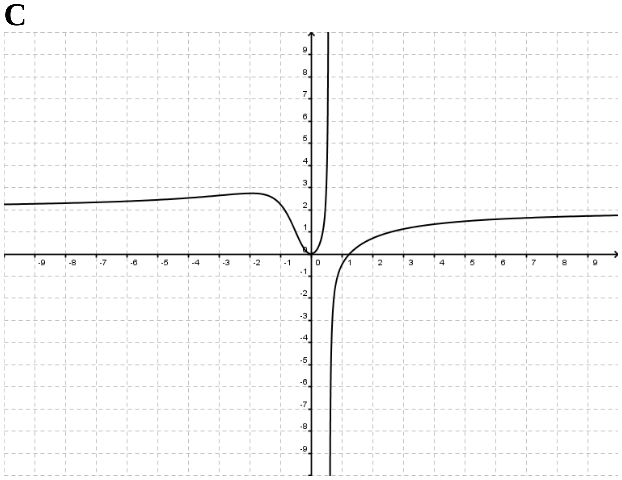

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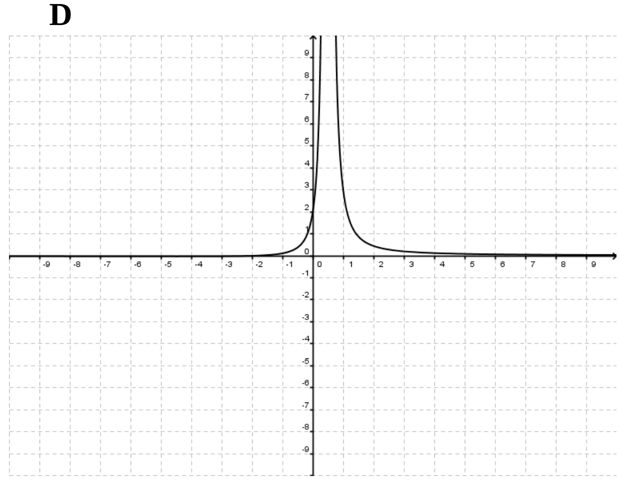


17) **<u>Follow all steps carefully!</u>**

a. Determine the vertical and horizontal asymptotes and draw them in as dotted lines.
b. Determine the x- and y-intercepts and plot those points.
c. Select two more x-values **on each side of the vertical asymptote** and plug them into the function to determine their matching y-values. Plot those 4 points.
d. Sketch the graph of the function.
e. Write down the domain and range, using interval notation.
f. Graph the function on the calculator to check your work.

$$f(x) = \frac{6x-9}{2x+2}$$

vertical asymptote: horizontal asymptote:

x-intercept: y-intercept:

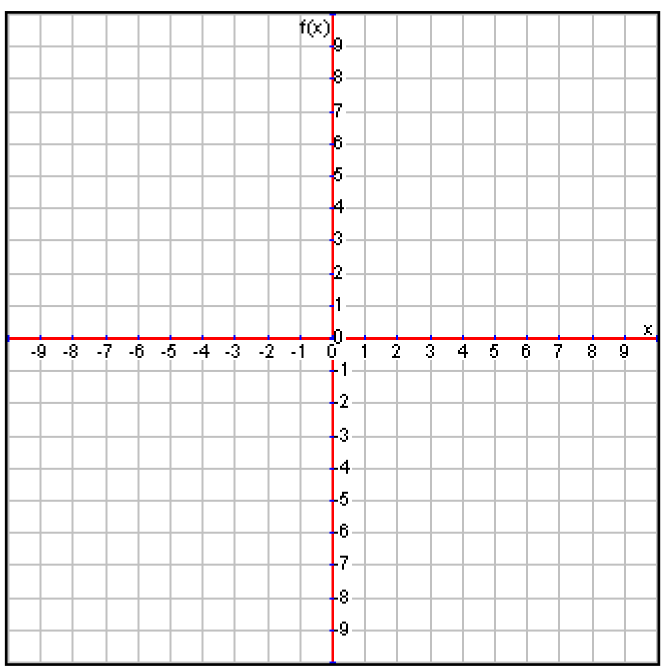
points:

x	f(x)

domain:

range:

graph:

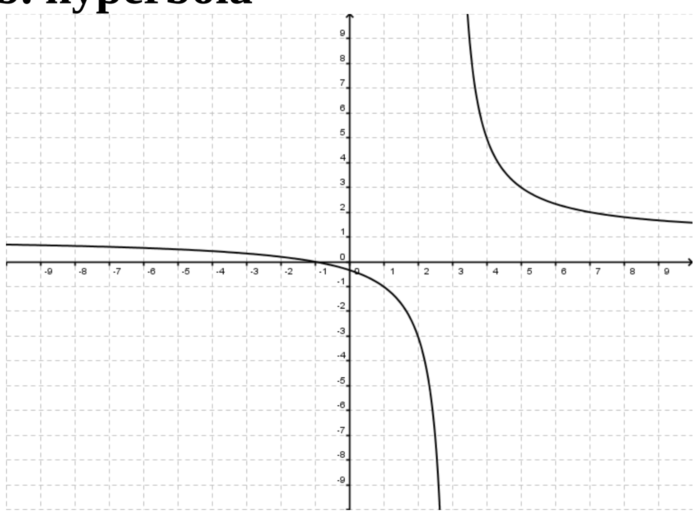


HW 8-2 Answer Sheet

1) a. $x = 4$ b. (3.5, 0) c. (0, 1.75) d. $x \neq 4$
2) Domain: $x \neq -2, x \neq 3$

3) Line A is the denominator because it's x-intercept is at the vertical asymptote. Line B is the numerator, because it's x-intercept is the same as the hyperbola.

4) a. x-int: (-1, 0) y-int: (0, -1/3)
   V.A.: $x = 3$ Domain: $x \neq 3$
   b. hyperbola



5) a. x-int: none y-int: (0, 2)
   V.A.: $x = -1$ Domain: $x \neq -1$

   b. x-int: (2, 0) y-int: (0, 2/25)
   V.A.: $x = 5, x = -5$ Domain: $x \neq 5, x \neq -5$

   c. x-int: (5, 0), (-4, 0) y-int: (0, 20/27)
   V.A.: $x = -3/2, x = -3, x = 3$
   Domain: $x \neq -3/2, x \neq -3, x \neq 3$

   d. x-int: (2, 0) y-int: (0, -1/2)
   V.A.: none Domain: all real #s

HW 8-2 Answer Sheet

1) a. b. c. d.

2)

3)

4) a. b. c.

5) a.

   b.

   c.

   d.

Lesson Name: *End Behavior of Rational Functions* **Date:** ____________________ **Student:** ____________________________

Concepts	Examples	Background Information
Given the rational function $f(x)=\frac{g(x)}{h(x)}$: **If the denominator has a higher degree:** **If the denominator and numerator have the same degree:** **If the denominator has a smaller degree:** Comparison of types of asymptotes: • A vertical asymptote: • A horizontal (or slant) asymptote:	1) $f(x)=\frac{4x-6}{2x+6}$ 2) $f(x)=\frac{x^2-5x+1}{2x-4}$	What does **end behavior** mean?

Concepts	Examples	Background Information
	3) $f(x)=\dfrac{3x+6}{x^2-2x-15}$	