Notes on Factors, Prime Numbers, and Prime Factorization

Factors are the numbers that multiply together to get another number.

A **Product** is the number produced by multiplying two factors.

All numbers have 1 and itself as factors.

A number whose only factors are 1 and itself is a **prime number.** Prime numbers have exactly two factors.

The smallest 168 prime numbers (all the prime numbers under 1000) are:

> 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107, 109, 113, 127, 131, 137, 139, 149, 151, 157, 163, 167, 173, 179, 181, 191, 193, 197, 199, 211, 223, 227, 229, 233, 239, 241, 251, 257, 263, 269, 271, 277, 281, 283, 293, 307, 311, 313, 317, 331, 337, 347, 349, 353, 359, 367, 373, 379, 383, 389, 397, 401, 409, 419, 421, 431, 433, 439, 443, 449, 457, 461, 463, 467, 479, 487, 491, 499, 503, 509, 521, 523, 541, 547, 557, 563, 569, 571, 577, 587, 593, 599, 601, 607, 613, 617, 619, 631, 641, 643, 647, 653, 659, 661, 673, 677, 683, 691, 701, 709, 719, 727, 733, 739, 743, 751, 757, 761, 769, 773, 787, 797, 809, 811, 821, 823, 827, 829, 839, 853, 857, 859, 863, 877, 881, 883, 887, 907, 911, 919, 929, 937, 941, 947, 953, 967, 971, 977, 983, 991, 997

There are infinitely many prime numbers. Another way of saying this is that the sequence of prime numbers never ends.

The number 1 is not considered a prime. The definition of a prime number is one that has exactly TWO factors: itself and 1. So the number 1, having only ONE factor, itself, does not meet the definition.

A number with three or more factors is a **composite number**.

Divisibility rules can be used to factor a number and to test the primality of a number. Some divisibility rules:

- All numbers are divisible by 1
- Any even number is divisible by 2. A number is divisible by 2 if it ends with a 0, 2, 4, 6, or 8.
- A number is divisible by 3 if the sum of its digits is divisible by 3.
- A number is divisible by 4 if the last two digits (tens and ones) are divisible by 4.
- A number is divisible by 5 if it ends in a 5 or 0.
- A number is divisible by 6 if it is also divisible by 2 and 3 (see these tests above).
- To test a number for divisibility by 7
    - Take the last digit in a number.
    - Double and subtract the last digit in your number from the rest of the digits.
    - Repeat the process for larger numbers.
- A number is divisible by 8 if the last 3 digits are divisible by 8.
- A number is divisible by 9 if the sum of its digits are divisible by 9
- A number is divisible by 10 if it ends in a zero.

A **prime factorization** is a factor string expressing a number as the product of only prime factors. Every number has exactly one prime factorization. This prime factorization can be written using exponents if any of its prime factors appear more than once in the string.

To factor a number

- Write 1 and the number itself separated by some space.
- Test the number for divisibility by 2. If it is, write 2 and the other number inside the first two.
- Continue testing and writing the factor pairs inside the previous pair.
- When you reach the middle, you are finished.

Example:

36 {1, 36}

36 {1, 2, 18, 36}

36 {1, 2, 3, 12, 18, 36}

36 {1, 2, 3, 4, 9, 12, 18, 36}

36 {1, 2, 3, 4, 6, 9, 12, 18, 36}

Another way to get the factors of a number is to list its factor pairs using a systematic approach:

1 x 36, 2 x 18, 3 x 12, 4 x 9, 6 x 6

So the factors of 36 are:

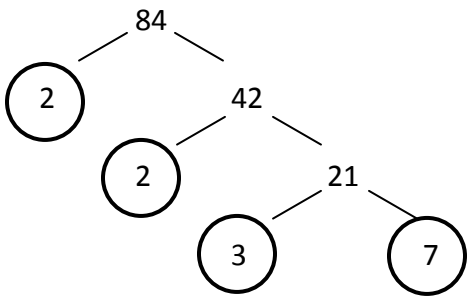
1, 2, 3, 4, 6, 9, 12, 18, 36

In the example above, 5 is not a factor of 36 so it is not in the factor list. 6 is in the middle so stop there, because 7 is not a factor of 36, nor is 8. The next number, 9, is already in the list, and every number greater than 9 has been included or eliminated when the lower factors were added.

To find the prime factorization of a number with a factor tree

1. Write the number to be prime factored at the top and draw two branches below it.
2. Write its smallest prime factor on the left and circle it. If the number is even, this will be 2. Write the companion factor on the right.
3. If the companion factor is composite, draw two branches below it and repeat step 2 for this factor.
4. Continue repeating steps 2 and 3 until the companion factor on the right is prime. Circle that prime also.
5. The prime factorization is the factors on the left and bottom of the tree that are circled.

Example:



84 is the number to be prime factored

2 is the smallest prime factor of 84 along with 42

2 is the smallest prime factor of 42 along with 21

3 is the smallest prime factor of 21 along with 7

7 is prime so stop. The prime factorization of 84 is:

2 x 2 x 3 x 7

To write the prime factorization using exponents, combine like prime factors: **2^2 x 3 x 7**