Pre-AP Algebra 2
Lesson 2-3 – Using Systems to Solve Problems

Objectives: The students will be able to:
- solve word problems that are modeled by 3x3 systems.

Materials: Hw #2-3 answers overhead; Do Now worksheet and answers overhead; “article” overhead; word problem relay packets (7); Matrices investigation

<table>
<thead>
<tr>
<th>Time</th>
<th>Activity</th>
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<tbody>
<tr>
<td>5 min</td>
<td>Review Homework</td>
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<td>Show answers to homework #2-3 on overhead.</td>
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<tr>
<td>5 min</td>
<td>Homework Presentations</td>
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<td>The homework is relatively short; show answers on the overhead and take any questions as we go.</td>
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<tr>
<td>20 min</td>
<td>Do Now</td>
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<td>Pass out the 2x2 word problems sheet. Students work in pairs to solve the problems. Students should only work on the challenge if they finish the other two.</td>
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<td>Show answers on the overhead.</td>
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<tr>
<td>20 min</td>
<td>Direct Instruction</td>
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<td>Put the swim team “article” on the overhead.</td>
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<td>Read it together and ask students to recall the process for solving a 2x2 system of equation word problem. As a class, generate the variables and the equations.</td>
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<td>Students will find the last equation most difficult: “Having as many third-place finishers as first- and second-place finishers combined”. Write the equal sign first; we are balancing “third place finishers” and “first and second place finishers combined”, so ( z = x + y ).</td>
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<td>Point out that you can rewrite the last equation to be ( x + y - z = 0 ) to use elimination; but it might be easier to plug in ( x + y ) for ( z ) into the first two equations to reduce it to a 2x2 system. Ask students to choose one of the methods and to finish solving the problem. Solution: 4 first-place, 8 second-place, and 12 third-place winners.</td>
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<tr>
<td>30 min</td>
<td>Word Problem Relay</td>
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<td>Students form groups of 3 or 4. Each group picks a runner to come and get the first problem from me. They work on it in their groups, and when they are done, bring the answer to me to check. If they are wrong, they get a 20 second penalty (runner must stay with me for 20 seconds), and then go back to keep working. I may give a hint if I feel like it! If they are right, they get the next problem to work on. Whichever team gets the most problems correct will get a 3-point bonus to their homework score.</td>
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Homework #2-3:
HW #2-2 Answers

Part 1
1) Infinitely Many Solutions
2) (-1, 1, 0)

Part 2
3) No Solution
4) (2, 0, -10)
5) (5, -2, 3)

Part 3
4)
6)

Bonus:
(w, x, y, z) = (1, 2, -1, 3)
2x2 Systems of Equations Word Problems

1) I am thinking of two numbers. If you take half of the first number and add it to one-third of the second number, the sum is two. Also, the second number is 3 more than 6 times the first number. What is the product of my two numbers?

2) For a wedding, Celia bought several dozen roses and several dozen carnations. The roses cost $15 per dozen, and the carnations cost $8 per dozen. Celia bought a total of 17 dozen flowers and paid a total of $192. How many roses did she buy?

Challenge!

3) Make a set of axes and sketch a quadrilateral (a 4-sided polygon) whose corners are at the following points: (-2, 0); (1, 5); (4, 3); and (3, -1). Now, draw in the diagonals (the lines that connect the opposite corners). At what point do the diagonals intersect? Show how to solve this problem algebraically.
Here is a news clipping from a recent issue of the Denton News. Let’s read it and determine how many DHS swimmers finished in each place.

**Denton, TX** – In yesterday’s swim meet, **Denton High School** dominated in the individual events, with 24 individual-event placers scoring a total of 56 points. A first-place finish scores 5 points, second-place scores 3 points, and a third-place finish earns 1 point. Having as many third-place finishers as first- and second-place finishers combined really shows the team’s depth (no pun intended)!
Word Problem Relay #1

A news clipping from a recent issue of the Denton Record Chronicle:

Denton, TX — Denton High School prevailed in Saturday’s track meet with the help of 20 individual-event placers earning a combined total of 68 points. A first-place finish earns 5 points, a second-place finish earns 3 points, and a third-place finish earns 1 point. DHS had a strong second-place showing, with as many second-place finishers as first- and third-place finishers combined.

How many DHS athletes finished in each place?
Word Problem Relay #2

An advertisement from the back page of the Denton Chronicle:

$1300
Sofa and love seat

$1400
Sofa and two chairs

$1600
Sofa, love seat, one chair

Dan’s Furniture Store

How much does each piece of furniture cost individually?
Word Problem Relay #3

The table below shows the percent of comedies, drama, and action videos available at a video store. Assume that the store has a collection of 3,405 general videos to be rented, 1,070 children’s videos to be rented, and 1,225 videos for sale. Write and solve a system of equations to find out how many comedies, dramas, and action movies are at the store.

<table>
<thead>
<tr>
<th>Store Section</th>
<th>Comedy</th>
<th>Drama</th>
<th>Action</th>
</tr>
</thead>
<tbody>
<tr>
<td>General rental</td>
<td>55%</td>
<td>65%</td>
<td>60%</td>
</tr>
<tr>
<td>Children’s rental</td>
<td>25%</td>
<td>10%</td>
<td>20%</td>
</tr>
<tr>
<td>Videos for sale</td>
<td>20%</td>
<td>25%</td>
<td>20%</td>
</tr>
</tbody>
</table>
Word Problem Relay #4

You have $25 to spend on picking 21 pounds of three different types of apples in an orchard. The Empire apples cost $1.40 a pound, the Red Delicious apples cost $1.10 per pound, and the Golden Delicious apples cost $1.30 per pound. You want to buy twice as many Red Delicious apples as the other two types combined. How many pounds of each apple should you buy?
Word Problem Relay #5

You have $25 to spend on picking 21 pounds of three different types of apples in an orchard. The Empire apples cost $1.40 a pound, the Red Delicious apples cost $1.10
How do you beat the system?

Part 1: What’s a matrix? Not the movie with Keanu Reeves

A matrix is a rectangular arrangement of numbers into rows and columns. It’s dimension is notated by (#rows x #columns)

Example Dimensions = (2, 3)

The numbers inside the matrix are its elements. To add or subtract matrices, you just add or subtract their corresponding elements. So, they must be the same dimensions.

Examples:

NOTE: Your calculator can do this too.

Opens the Matrix Menu

The first screen lists the NAMES you can use for your matrices (you could fit 10)

The MATH menu lists a lot of stuff you can do with matrices that you probably won’t have to do for a very long time.

The EDIT menu allows you to enter the dimensions and the elements of each matrix.

Let’s try it.

Enter each of these into your calculator. To stop editing a matrix and to begin performing operations with matrices, enter [2nd] [MODE] to quit the previous task.

Enter [2nd] [x⁻¹] to recall the name of the matrix you are going to use.

Find the following.

1.

2.

3. Is matrix addition commutative?
4. 

5. 

6. What happens when a matrix is being multiplied by a constant?

7. 

8. 

9. Why did you get the answer you did?

10. 

11. 

12. Is matrix multiplication commutative?

13. 

14. 

15. Are \[ A \] and \[ B \] the same? What do you notice?

16. 

17. 

18. Are \[ A \] and \[ B \] the same? What do you notice?

19. 

Part 2: Now let’s see how matrices can be used to “beat the system”
Before we do, let’s remember some background knowledge. To solve the equation \( x \) for \( y \), you had to multiply both sides of the equation by \(-1\), the **multiplicative inverse** of \( y \).

1. First solve the following system using any of the methods we have done so far.

   Solution: (___,___)

   Now put the following matrices in your calculator.

   and

2. Find \( \) . Write what appears in your screen.

3. Compare your answer from the calculator to the solution you got above. What do you notice?

4. How did \( x \) compare to the original equations?

5. What about \( y \)?

6. What would you put for \( x \) and \( y \) on the following system?

   and

7. Find

8. Solution: (_____, _____)
Part 3: Setting up other equations to use matrices.

1. Notice that both of these equations are not in standard form. What should the second equation look like?

2. Now you can set up your matrices.

   and

3. Find

4. Solution: (______, ______)

5. Notice that both of these equations are not in standard form. What should the second equation look like?

6. Now you can set up your matrices.

   and

7. Find

8. Solution: (______, ______)

9. What? You got an error? What do you think that means?

10. Actually it means one of two things. Either the lines are parallel and there is no solution, or the two equations represent the same line so they have infinite points in common. In this case which one is it?
Part 3: Can we use this for systems with more than 2 variables?
You betcha. Try this one.

1. What would the dimensions have to be for ?___________ ?___________

2. Now you can set up your matrices.
   and

3. Find

4. Solution:

5. Now for your final trick try this nasty system of equations.

   Solution:

   For 5 bonus quiz points, solve it by hand.