Pre-AP Algebra 2
Unit 9 - Lesson 6 – Exponential Modeling

Objectives: Students will be able to model word problems with exponential functions and use logs to solve exponential models.

Materials: Hw #9-5 answers overhead; quiz #2; pair work and answer overhead; board collaborations; hw #9-6

Time	Activity
5 min	**Check Homework** Put the answers to hw #9-5 on the overhead.
20 min	**Quiz #2** Check homework while students are taking the quiz.
20 min	**Pair Work** Hand out the **Introduction to Modeling Exponential Functions** sheet for students to work on in pairs or groups. (15 minutes) Review answers on the overhead. (5 minutes)
35 min	**Pair Work** Board Collaborations! Each pair gets a mini-whiteboard, marker, and eraser. Pairs get 1 point for each question they get exactly right. The more points they earn the more bragging rights they acquire! Part 1: (10 minutes) On the overhead, display verbal situations with a given function, one at a time. For each problem, students must determine: a) the initial value (including units) b) if it is growth or decay c) the growth/decay percentage Part 2: (10 minutes) On the overhead, display verbal situations. Students must determine: a) the initial value (including units) b) if it is growth or decay c) the growth/decay factor d) the function that models the situation Part 3: (15 minutes) On the overhead, display a verbal situation with a given function. For each problem, there are two questions: one where the x is given and one where the y is given. Students must plug in to find the other, using logarithms to find the x when the y is given.

Homework #9-6: Exponential Modeling

Pre-AP Algebra 2 **Name: ______________________**
9-6 Pair Work

Introduction to Modeling Exponential Functions

1) In California, the sales tax is 8.25%. That means, when you buy something at the store, the price you pay will always be 8.25% more than the listed price.

   a. How much would the *just the tax* be if the listed price is $50?

   b. How much would you pay *in total* if the listed price is $50?

   c. What number can you multiply $50 by to get the same result directly? Show why it works (using the distributive property).

   d. If the listed price of an item is **x**, write a function **p(x)** for the price you actually pay.

2) To ***depreciate*** means to "go down in value over time". Certain items always depreciate. For example, when you buy a new car, it depreciates very quickly – that's why buying a car is not considered to be a good investment. A house will almost always ***appreciate***, so buying a house is usually a good investment.

   a. Suppose you have a car that you bought for $20,000. Over the first year, it depreciates by 15%. How much value has it *lost* after 1 year?

   b. How much is your car *worth* after 1 year?

   c. What number can you multiply 20,000 by to get the same result directly? Show why it works.

   d. The car continues to lose 15% of its value every year. How much is your car worth after 2 years? 3 years?

   e. Write a function **v(t)** for the value of the car over time, where **v** is measured in dollars and **t** is measured in years *(think about your work in part d)*.

   f. Use your function to determine the value of the car after 10 years.

Part 1:

Example Situation:
The value of a car over time is modeled by the function $\mathbf{v(t) = 35{,}000(0.76)^t}$, where v is in dollars and t is in years.

a) Initial value: \$35,000

b) Decay, because 0.76 is less than 1

c) 24% decrease (1 – 0.76 = 0.24 = 24%)

Situation 1:

The value of my sister's house in San Jose is modeled by $\mathbf{v(t) = 450{,}000(1.07)^t}$, where v is in dollars and t is in years.

a) Initial value: \$450,000

b) Growth, because 1.07 is greater than 1

c) 7% increase (1.07 – 1 = .07 = 7%)

Situation 2:
The number of websites on the Internet can be modeled by the equation $\mathbf{N(t) = 1.313(2)^t}$, where *N* is measured in millions of websites, and *t* is measured in years since January 1, 1993.

a) Initial value: 1.313 **million** = 1,313,000 websites

b) Growth, because 2 is greater than 1

c) 100% increase (2 – 1 = 1 = 100%)

Situation 3:
The following is a model for the amount of Ibuprofen (Advil®) in an adult's system over time, where *t* is measured in hours and *I* is measured in milligrams: $\mathbf{I(t) = 400(0.71)^t}$

a) Initial value: 400 milligrams or 400 mg

b) Decay, because 0.71 is less than 1

c) 29% decrease (1 – 0.71 = 0.29 = 29%)

Situation 4:
A waste product of producing nuclear energy is radioactive plutonium. A given quantity of plutonium is stored in a container. The amount *P* (in grams) of plutonium present in the container after *t* years can be modeled by $\mathbf{P = 100(0.99997)^t}$.

a) Initial value: 100 grams

b) Decay, because 0.99997 is less than 1

c) 0.003% decrease
(1 – 0.99997 = 0.00003 = 0.003%)

Part 2:

Example:
You drink a cup of coffee that has 120 milligrams of caffeine. Each hour, the amount of caffeine in your body decreases by about 12%.

a) Initial value: 120 mg
b) Decay
c) Factor: 0.88 (100% - 12% = 88% = 0.88)
d) $f(x) = 120(0.88)^x$

Situation 1:

The petting zoo director purchased 10 guinea pigs. Each month, the number of guinea pigs doubles.

a) Initial value: 10 guinea pigs

b) Growth!

c) Factor: 2 (100% + 100% = 200% = 2)

d) $f(x) = 10(2)^x$

Situation 2:

In 1971, computers were much less powerful, since the average number of transistors we were able to fit on a computer chip was only 2300. Since then, the average number of transistors that fit on a chip has grown by 59% each year.

a) Initial value: 2300 chips

b) Growth

c) Factor: 1.59 (100% + 59% = 159% = 1.59)

d) $f(x) = 2300(1.59)^x$

Situation 3:
You buy a new computer for \$2100. The computer depreciates at about 45% annually.

a) Initial value: \$2100

b) Decay

c) Factor: 0.55 (100% - 45% = 55% = 0.55)

d) $f(x) = 2100(0.55)^x$

Part 3:
Example:
When you drop a sugar cube into a glass of hot water, the amount of sugar remaining in the cube is given by the function $S(t) = 5(0.9)^t$, where *t* is measured in seconds and *S* is measured in grams.

a) About how many grams are left after 20 seconds?

0.61 grams

b) About how many seconds will it take for there to be only 2 grams of sugar left?

8.7 seconds

Situation 1:
In a Petri dish, there is a drop of water containing some bacteria. The number of bacteria in the dish over time can be modeled by the function $B(t) = 300(1.85)^t$, where *t* is measured in hours and *B* is measured in **millions** of bacteria.

a) About how many bacteria will there be after 1 **day**?

774,894,781 bacteria

b) About how many hours will it take for the number of bacteria to **triple**?

1.79 hours

Situation 2:
Your uncle bought a classic car for $22,000. Because it is a classic, and your uncle takes great care of it, its value appreciates at 7% a year. The function can be expressed as $V(t) = 22000(1.07)^t$, where *t* is measured in years and V in dollars.

a) How much will it be worth after 15 years?

$60699

b) How long will it take for the car to be worth $35000?

6.86 years

Pre-AP Algebra 2 **Name: ______________________**
Homework #9-6

HW #9-6: Exponential Models

Part 1: Percent Work

1) Find 28% of 13.

2) Find 127% of 204.

3) Find 200% of 200.

4) Find 7.2% of 25.

5) A $250 TV goes on sale for 35% off. What is the new price?

6) Your bill at a restaurant comes out to $55 but you want to leave the waiter an 18% tip. What is the total amount you pay?

7) At the start of the year, there are 140 freshmen. At the end of the year, there are 115. What percent did the class size decrease by?

8) At the start of the year, I had $1500 in my bank account. Now, I have $1625. By what percent did my total balance increase?

9) There are 120 bacteria in a Petri dish, and the number increases by 25% each hour. Fill in the table, rounding off to the nearest whole bacterium. (Think about how to use your calculator efficiently on this problem.)

Hour	**0**	**1**	**2**	**3**	**4**	**5**	**6**
# Bacteria	120						

Part 2: Exponential Models

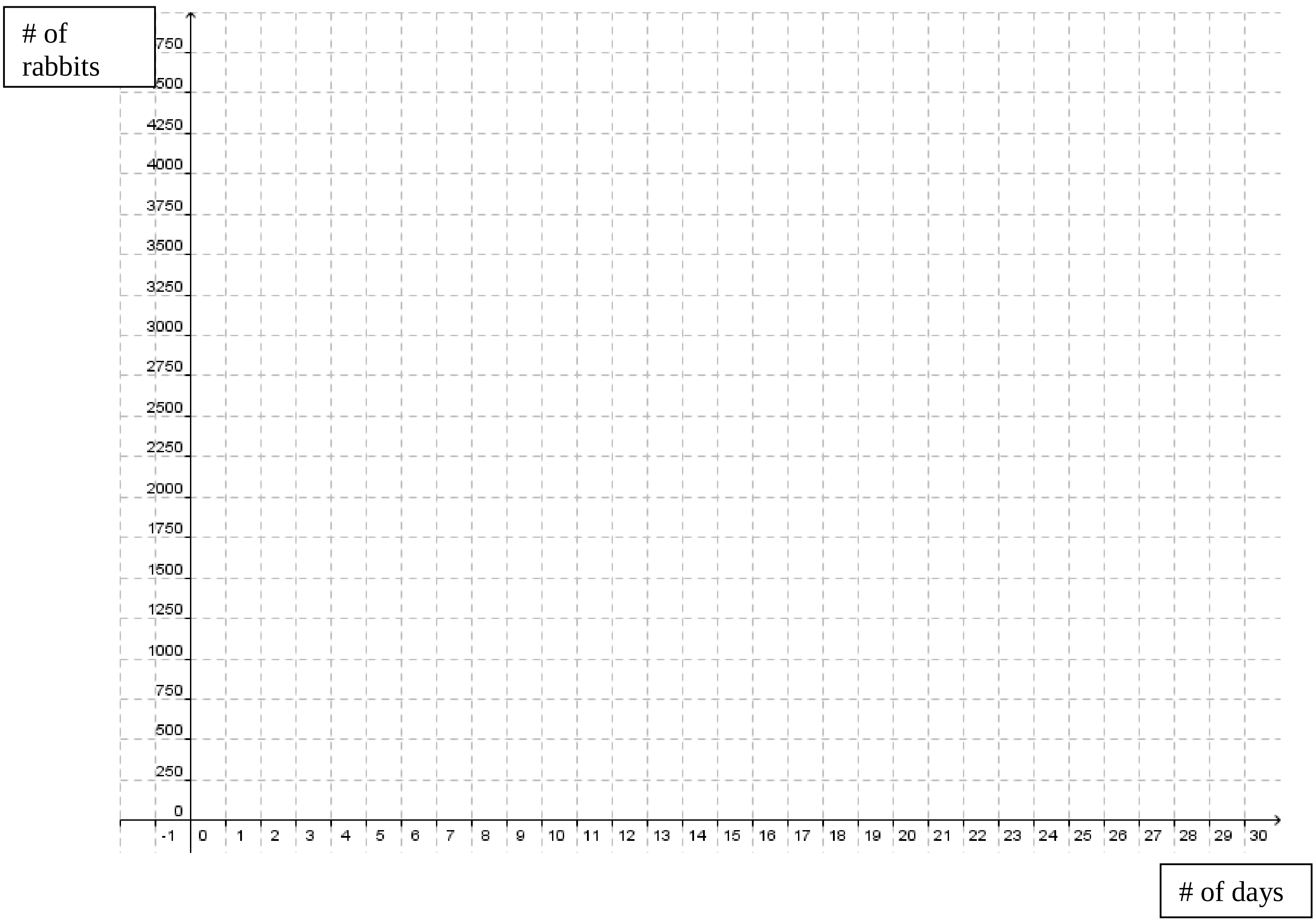
1)

I just bought a rabbit farm with 6 rabbits (Mopsy, Flopsy, Cottontail, Twitchy, Pinky, and George). But I am told that the population will grow very quickly. The guy I bought the farm from said that the growth function was exponential and could be modeled by $P = 6(1.248)^d$ where **P** is the number of rabbits and **d** is the number of days.

a. How many rabbits (round to the nearest rabbit!) will I have after:

5 days	10 days	15 days
20 days	25 days	30 days

b. Plot these points (as well as the initial value) on the axes below, and make a smooth curve.



c. After how many days will there be 1000 rabbits?

i) Estimate this by looking at your graph.

ii) Calculate this by using the given function and logarithms. Compare your results.

2)
For each situation, determine the initial value (including units), if it is decay or growth, and what the decay/growth percent is.

a) The amount of money in my account can be modeled by $A(t) = 500(1.035)^t$, where *t* is the time in years and *A* is the amount in dollars.

b) The amount of Aspirin in an adult's bloodstream (assuming they swallowed a single pill) can be modeled by $A(t) = 200(0.65)^t$, where *t* is measured in hours and *A* is measured in milligrams (mg).

c) Due to global warming, the size of the remaining glaciers is quickly decreasing. Assume that the volume of the Great Northern Glacier can be modeled by $V(t) = 25000(0.76)^t$, where *t* is measured in years and *V* is measured in cubic kilometers (km^3).

3)
Write an exponential function to model each situation. Then, use the model to answer the questions.

a) Mr. Greene likes to drink a cup of green tea every morning. One cup has about 35 mg of caffeine in it, and the amount of caffeine left in the bloodstream decreases by 23% every hour.

   a. Function:

   b. How much caffeine will be in his system after 4 hours?

   c. How long will it take for there to be only 5 mg of caffeine left?

b) Ricky won the Exponential Lottery! Congratulations! He has been given a special bank account with $100 in it. As long as he doesn't take any money out of it, each month it will increase by 75%.

   a. Function:

   b. How long will it take for Enrique to earn a million dollars?

   c. How much money will Enrique have after 2½ **years**?

c) Have you seen the TV show Lost? Well, there is a new show coming out this season. It starts with 42 people on an island. Well, they are never going to be rescued, so they might as well start repopulating. Assume that the number of people on the island grows by 10% each year.

a. Function:

b. How many people will there be on the island after 8 years? (Round to the nearest whole person!)

c. How many years will it take for the population to double?

d. How many years will it take for the population to triple?

e. How many years will it take for the population to quadruple?

f. How many years will it take for the population to reach 2000 people?

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Quiz #9-2 **Score: /30**

Exponentials and Logarithms Quiz

Simplify the following expressions (2 points each)

$\log_3 81$ $15^{\log_{15}(6x)}$ $3\log 100$

$\log_3 27^2$ $\log_6 2 + \log_6 3$ $\log_{36} 6$

$\log_{15} 45 - \log_{15} 3$

Solve the following equations for *x*. For the second equation, write your final answer in terms of log base 10. (3 points each)

$\log_4 16 = 4x$ $3^x - 8 = 20$

Determine if the following exponential functions model growth or decay. Remember to rewrite any negative exponents as positive before making a decision. (1 point each)

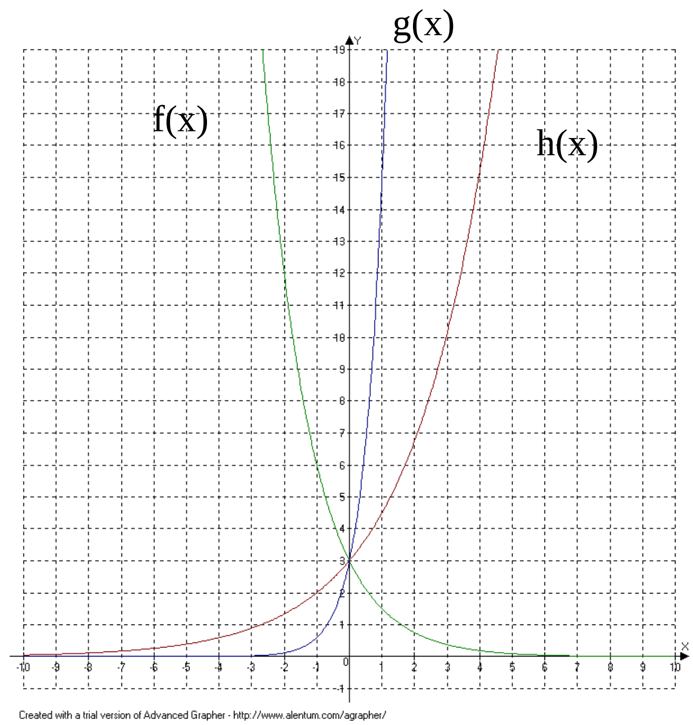
$y = 223(31)^{-x}$ $y = \frac{6}{7}(2)^x$ $y = 2(\frac{1}{3})^{-x}$ $y = \frac{1}{3}(\frac{4}{3})^x$

Match the following exponential functions:
(2 points each)

$\_\_\_\_\_ = 3(\frac{3}{2})^x$

$\_\_\_\_\_ = 3(\frac{1}{2})^x$

$\_\_\_\_\_ = 3(5)^x$



Take a breath, you're done. However, if you can't get enough and would like to earn a few bonus points try the following problems. (2 points each)

Solve for x

$$\log_{36} 6^x + \log_{36} 6^x = 8$$

Use the values on the following table to write an exponential function.

x	*f(x)*
0	24
1	12
2	6

f(x)=_____________

HW 9-5 – Answer Sheet

1) $x = -3$

2) $x = 12$

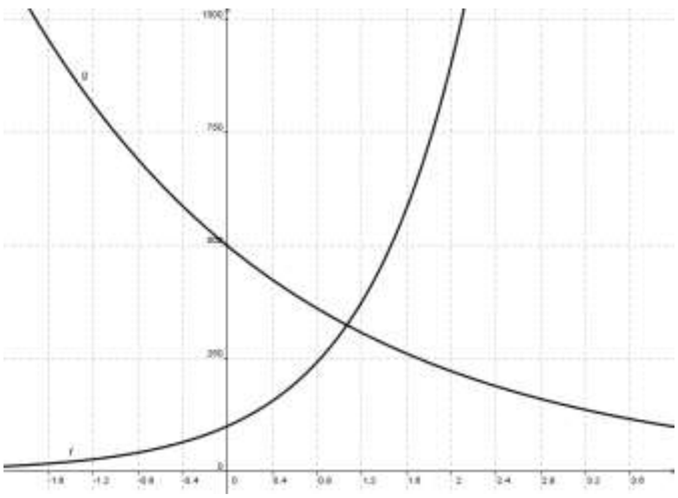
3) $x = 5$

4) $x = 10^{9/2}$

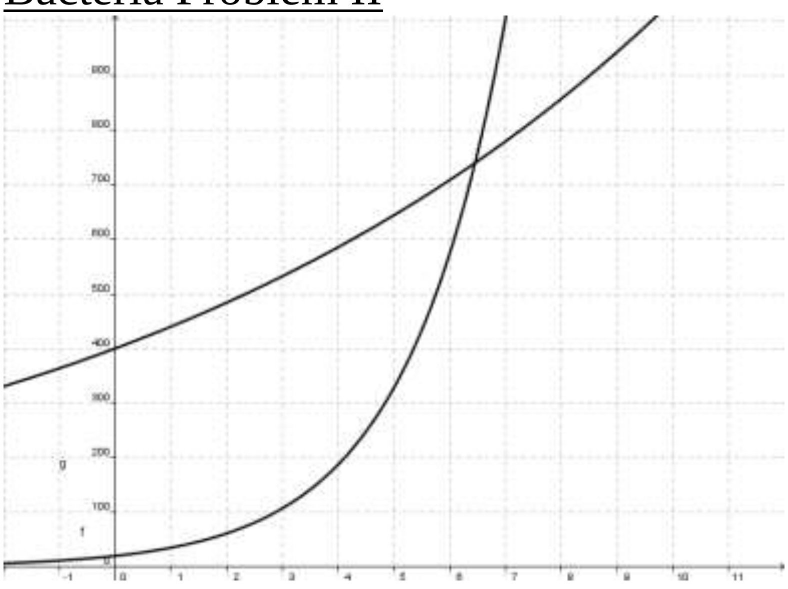
5) $x = 1$

6) $x = 9$

7 & 8



Bacteria Problem II



The population will be the same at 6.45 minutes and it will be approximately 740 bacteria of each.

HW 9-5 – Tally Sheet

1) 2)

3) 4)

5) 6)

7) Graph 8) Graph

Bacteria Problem II

Graph

Question 1

Question 2

HW 9-5 – Tally Sheet

1) 2)

3) 4)

5) 6)

7) Graph 8) Graph

Bacteria Problem II

Graph

Question 1

Question 2