Unit 2 – Functions
Continuity and End Behavior
(Unit 2.3)

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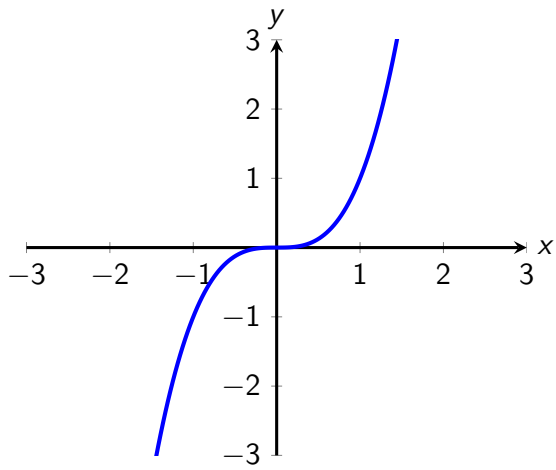
Lesson Goals

When you have completed this lesson you will:

- Identify whether a function is continuous.
- Identify 3 types of discontinuities.
- Describe the end-behavior of a function.

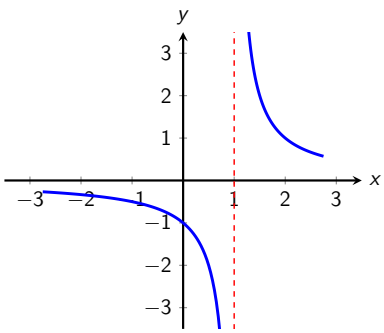
Continuity

The graph of a function is **continuous** if it has no breaks or gaps.

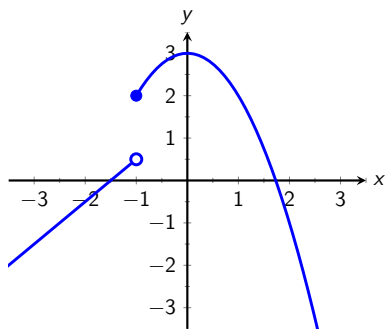


Discontinuities

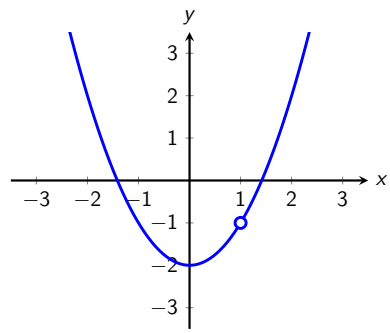
Three types of **discontinuities**



Infinite discontinuity at $x = c$



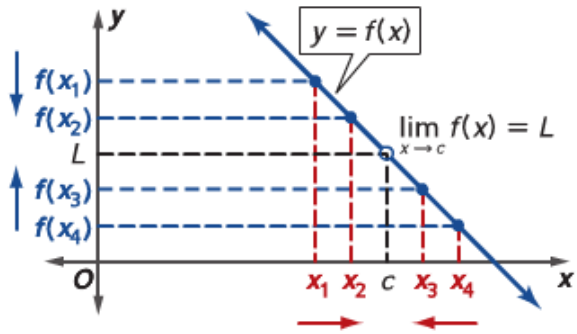
Jump discontinuity at $x = c$



Removable discontinuity at $x = c$

Limits

As $f(x)$ approaches some output L as the input x approaches some input c, then the **limit** of $f(x)$ as x approaches c is L.



$$\lim_{x \to c} f(x) = L$$

Continuity Test

A function $f(x)$ is continuous at $x = c$ if:

- $f(c)$ exists
- $f(x)$ approaches the same output value from either side of c (this means $\lim_{x \to c} f(x)$ exists)
- The output value the function approaches from either side of c is $f(c)$ (this means $\lim_{x \to c} f(x) = f(c)$)

Example 1

Determine whether $f(x) = \dfrac{1}{2x+1}$ is continuous at $x = 0.5$ using the continuity test. If discontinuous, identify the type of discontinuity.

Example 2

Determine whether $f(x) = \dfrac{1}{x-1}$ is continuous at $x = 1$ using the continuity test. If discontinuous, identify the type of discontinuity.

Example 3

Determine whether $f(x) = \dfrac{x-2}{x^2-4}$ is continuous at $x = 2$ using the continuity test. If discontinuous, identify the type of discontinuity.

End Behavior of a Function

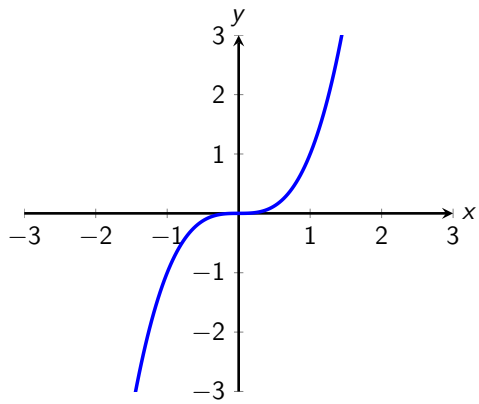
The **end behavior** of a function is a description of the output of the function as x goes left and right towards negative and positive infinity.

Left-End Behavior

$$\lim_{x \to -\infty} f(x) = \_\_\_\_$$

Right-End Behavior

$$\lim_{x \to +\infty} f(x) = \_\_\_\_$$



Example 4

Use the graph of $f(x) = x^3 - x^2 - 4x + 4$ to describe its end behavior.

Example 5

Use the graph of $g(x) = \dfrac{x+2}{x^2 - x - 2}$ to describe its end behavior.

Example 6

In physics, the symmetric energy function is

$$E = \frac{x^2 + y^2}{2}$$

If the y-value is held constant, what happens to the value of symmetric energy when the x-value approaches $-\infty$?

What You Learned

You can now:

- Identify whether a function is continuous.
- Identify 3 types of discontinuities.
- Describe the end-behavior of a function.
- Do problems Chap 1.3 #1-11 odd, 23-31 odd