Section 6
Using Averages, Weighted Averages, and Indices

Main Ideas
- Averages and Weighted Averages
- Weighted Averages in Sports
- Indices

What About Mode?
Historically, a mode of a data set often was considered to be a measure of center. A mode for a set of data is a peak within the data. A data set can have more than one mode.

Exercises 1–6 show that the most frequently occurring value within a data set is not always a good measure of center; in fact, the most frequent value sometimes can even be the maximum or minimum value of the data set.

Many Meanings of Mean
Within mathematics, the word mean is used in many ways: mean of a proportion, arithmetic mean, geometric mean, harmonic mean, and more. Within statistics, the mean (arithmetic mean, arithmetic average) of a set of data is the sum of the data values divided by the number of values.

A variation on the ordinary (arithmetic) mean is the weighted mean (weighted arithmetic mean, weighted average): In a weighted average of a set, we place a greater importance (weight) on some members of the set than on others, as illustrated over the next several pages.

Since middle school, you have worked with mean, median, and possibly midrange. Each of these is an average, or measure of center.

Example 1 Computing Mean, Median, and Midrange
If the scores of 10 students on a test are 100, 95, 95, 90, 88, 87, 86, 82, and 82, then
(a) the mean is the sum of the data values divided by the number of values: 900/10 = 90
(b) the median is the middle value of the sorted data, or the mean of the two middle values: (90 + 88)/2 = 89
(c) the midrange is the mean of the maximum and the minimum: (100 + 82)/2 = 91

Exploration 6.1 Mean, Median, and Midrange
A high school mathematics class has 10 students. Only one student scores a perfect 100. What are possible scores for the other nine students if
(a) the mean, median, and midrange are 90?
(b) the mean is 87, and the median and midrange are both 85?
(c) the mean is 88, the median is 85, and the midrange is 87?

Averages and Weighted Averages
The word average comes from the Arabic word for “goods damaged in shipping,” but over time came to mean the equal portion for the many parties who suffered due to the loss of a ship. The arithmetic mean treats each portion contributing to the mean equally. For example, if you have quiz grades of 42, 45, 36, and 49, the average of your quiz grades would be the arithmetic mean:

\[
\frac{42 + 45 + 36 + 49}{4} = \frac{172}{4} = 43
\]

If each quiz has a maximum score of 50, your average of 43 is 86%.

Example 2 Weighted Average Versus Regular Average
Brittney scored 15 out of 20 on Quiz 1, 27 out of 30 on Quiz 2, 24 out of 25 on Quiz 3, but only 26 out of 40 on Quiz 4. Brittney’s weighted quiz average is

\[
\frac{15 + 27 + 24}{20 + 30 + 25} = \frac{92}{115} = 80\%.
\]

Notice that Brittney’s individual quiz grades were 15/20 = 75%, 27/30 = 90%, 24/25 = 96%, and 26/40 = 65%, and that an ordinary average of these four percentages is

\[
\frac{75\% + 90\% + 96\% + 65\%}{4} = \frac{326\%}{4} = 81.5\%.
\]
Quick Question 6.2 Choosing an Average in Context

In Example 2, which of the two averages is a more reasonable and fair assessment of Brittney’s performance? Justify your answer.

Example 3 Batting Average

In baseball and softball, batting averages are ratios of hits to at bats, reported as 3-place decimal fractions. In a four-game series, Sam gets 2 hits in 4 at bats in Game 1, then goes 1 for 5, 2 for 3, and 5 for 6. Sam’s batting average for the series is

\[ \frac{2+1+2+5}{4+5+3+6} = \frac{10}{18} = .555. \]

Notice that, if Sam’s individual game batting averages were averaged, the result would be slightly lower:

\[ \frac{.500 + .200 + .667 + .833}{4} = .550 \]

In baseball and softball, weighted averages are always used to compute batting averages, where the weights are the number of at bats. In the following computation, Sam’s individual game batting averages are averaged using at bats as weights:

\[ \frac{.500 \cdot 4 + .200 \cdot 5 + .667 \cdot 3 + .833 \cdot 6}{4+5+3+6} = \frac{2+1+2+5}{18} = \frac{10}{18} = .555 \]

Notice this time we obtain the same result as the first computation in Example 3.

Quick Question 6.3 Weighted Averages

What weighted average could be used to get the same result as the first computation in Example 2? What would you use as the weights?

Weighted averages can be tricky. They occur in many contexts, and their computations use many representations. For example, grades are often written as percentages, but batting averages are written as 3-place decimals. In school and college, grades are expressed as percentages, as letters (A, B, C, …), or even on a 4-point scale as grade point averages. The arithmetic of weighted averages is often basic, but you have to think to decide which operations to use and in which ways.

Investigation 6.4 Weighted Grading Systems

Mr. Alejandro and Ms. Banach-Tarski teach Advanced Quantitative Reasoning. When grading, Mr. A uses System A, and Ms. B uses System B—shown in Table 6.1.

(a) Your grades for Unit 1. Numerical Reasoning are as follows: class participation, 90; homework, 82; quiz average, 73; unit test, 85; unit project, 95. Which teacher’s grading system do you prefer? Explain.

(b) If your quiz average were 20 points higher, how much would your unit average improve in each system? Which teacher’s grading system would yield the better grade?

(c) Keeping the other grades the same as in part (a), what quiz average would yield the same overall unit average in both grading systems? What is the common unit average?

(d) Suppose your grades are class participation, 80; homework, 85; quiz grades, 70, 78, 83. Under each system, what grades would you need on your unit test and unit project to obtain an average of 90 or above for the unit? Explain.
Suppose your grades so far are class participation, 90; homework, 85; quiz grades, 80, 88, 84. Under each system, what grades would you need on your unit test and unit project to obtain an average of 90 or above for the unit? You may wish to express your answers in the form of a table or a chart.

Table 6.1

<table>
<thead>
<tr>
<th>Two Unit Grading Systems</th>
</tr>
</thead>
<tbody>
<tr>
<td>System A</td>
</tr>
<tr>
<td>Class participation: 10%</td>
</tr>
<tr>
<td>Homework: 20%</td>
</tr>
<tr>
<td>Quiz average: 30%</td>
</tr>
<tr>
<td>Unit test: 15%</td>
</tr>
<tr>
<td>Unit project: 25%</td>
</tr>
</tbody>
</table>

Weighted Averages in Sports

The title of Alan Schwarz’s 2004 book The Numbers Game: Baseball’s Lifelong Fascination With Statistics suggests that numerical reasoning has been applied to baseball for many years. The baseball box score has been reported in newspapers as far back as 1858—11 years before baseball became a professional sport. The Society for American Baseball Research (SABR) has given rise to the term sabermetrics for the numerical analysis of baseball. Sabermetricians analyze batting averages, runs batted in, on-base percentages, earned run averages, and many other statistics as objective measures of baseball performance.

Example 3 considered batting average, the ratio of hits to at bats. In baseball there are one-base hits (singles), two-base hits (doubles), three-base hits (triples), and four-base hits (home runs). The slugging average (also called slugging percentage even though it is reported as a decimal fraction) of a player not only considers hits per at bats but also weights the hits using the number of bases per hit:

$$\text{slugging average} = \frac{\text{singles} \cdot 1 + \text{doubles} \cdot 2 + \text{triples} \cdot 3 + \text{home runs} \cdot 4}{\text{at bats}}$$

Because the number of singles for a player is seldom reported, the following alternative formula is often used:

$$\text{slugging average} = \frac{\text{hits} \cdot 1 + \text{doubles} \cdot 2 + \text{triples} \cdot 3 + \text{home runs} \cdot 3}{\text{at bats}}$$

Quick Question 6.5 Verifying Equality

Why are these two formulas given above for slugging average equivalent? Justify your answer.

By introducing another piece of baseball terminology, total bases, the formula for slugging average becomes a simple ratio:

$$\text{slugging average} = \frac{\text{total bases}}{\text{at bats}}$$

According to the official rules of Major League Baseball (mlb.com), at bats are the “number of times batted, except … when a player (i) hits a sacrifice bunt or sacrifice fly; (ii) is awarded first base on four called balls; (iii) is hit by a pitched ball; or (iv) is awarded first base because of interference or obstruction.” [10.02]
Quick Question 6.6 Total Bases Formula

Write a definition for **total bases** that will guarantee that this last formula for slugging average will yield the same values as the original formula for slugging average.

During the 2009 Major League Baseball season, the top five sluggers in the National League all were first basemen shown in Table 6.2. They were Prince Fielder of the Milwaukee Brewers, Ryan Howard of the Philadelphia Phillies, Derek Lee of the Chicago Cubs, Albert Pujols of the St. Louis Cardinals, and Joey Votto of the Cincinnati Reds. Sabermetricians use various ways to judge the performance of such power hitters, including slugging average.

Investigation 6.7 Sabermetrics for Sluggers

Use the data in Table 6.2 and spreadsheet operations to compute and rank the five players’ statistics in the following categories: (a) at bats per game played; (b) batting average; (c) total bases; (d) slugging average; (e) extra base hits (doubles, triples, or home runs); and (f) home runs per at bat. For part (f) write the statistic as a ratio of 1: \( x \), where \( x \) is a number of at bats.

(g) Based on this information, who would you say was the top National League power hitter during the 2009 season?


**Example 4 Combining Batting Averages: Simpson’s Paradox**

Explain why David Justice had a better batting average than Derek Jeter in both 1995 and 1996, but Derek Jeter had the better combined batting average (Table 6.3).

**Solution** Notice that Jeter’s .250 batting average in 1995 was based on a small sample size, that is, on a small number of at bats. In addition, Justice’s batting average .321 was based on a relatively small sample size compared to his average of .253. As a result, the weighted average for Jeter is close to his better average .314, and the weighted average for Justice is close his worse batting average of .253. Known as **Simpson’s paradox**, this surprising phenomenon can occur whenever rates are combined.
Since 1960, the National Football League (NFL) has rated the passing ability of its quarterbacks on a statistic known as the quarterback rating (QR). The current QR system, adopted in 1973, is a weighted average of four ratios, with some adjustments. The ratios used are as follows:

- Percentage of completions per attempt ($\%\text{Comp} = 100 \cdot \frac{\text{Comp}}{\text{Att}}$)
- Average yards gained per attempt ($\text{Yd}/\text{Att}$)
- Percentage of touchdown passes per attempt ($\%\text{TD} = 100 \cdot \frac{\text{TD}}{\text{Att}}$)
- Percentage of interceptions per attempt ($\%\text{Int} = 100 \cdot \frac{\text{Int}}{\text{Att}}$)

For a quarterback with a sufficient number of attempted passes, the QR formula is

$$QR = \frac{10\%\text{Comp} + 50(\text{Yd}/\text{Att}) + 40(\%\text{TD}) - 50(\%\text{Int}) + 25}{12}.$$ 

### Table 6.3

**A Case of Simpson’s Paradox**

<table>
<thead>
<tr>
<th>Player</th>
<th>1995</th>
<th>1996</th>
<th>Combined</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jeter</td>
<td>12 for 48 .250</td>
<td>183 for 582 .314</td>
<td>195 for 630 .310</td>
</tr>
<tr>
<td>Justice</td>
<td>104 for 411 .253</td>
<td>45 for 140 .321</td>
<td>149 for 551 .270</td>
</tr>
</tbody>
</table>


### Table 6.4

**Selected NFL Quarterback Ratings**

<table>
<thead>
<tr>
<th>Player</th>
<th>Team</th>
<th>Comp</th>
<th>Att</th>
<th>%Comp</th>
<th>Yd</th>
<th>Yd/Att</th>
<th>TD</th>
<th>Int</th>
<th>QR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peyton Manning</td>
<td>IND</td>
<td>133</td>
<td>181</td>
<td>73.5</td>
<td>1,645</td>
<td>9.1</td>
<td>12</td>
<td>4</td>
<td>114.1</td>
</tr>
<tr>
<td>Eli Manning</td>
<td>NYG</td>
<td>87</td>
<td>135</td>
<td>64.4</td>
<td>1,212</td>
<td>9.0</td>
<td>10</td>
<td>2</td>
<td>111.7</td>
</tr>
<tr>
<td>Drew Brees</td>
<td>NO</td>
<td>87</td>
<td>129</td>
<td>67.4</td>
<td>1,031</td>
<td>8.0</td>
<td>9</td>
<td>2</td>
<td>108.4</td>
</tr>
<tr>
<td>Brett Favre</td>
<td>MIN</td>
<td>103</td>
<td>149</td>
<td>69.1</td>
<td>1,069</td>
<td>7.2</td>
<td>9</td>
<td>2</td>
<td>104.1</td>
</tr>
<tr>
<td>Matt Hasselbeck</td>
<td>SEA</td>
<td>53</td>
<td>84</td>
<td>63.1</td>
<td>617</td>
<td>7.3</td>
<td>7</td>
<td>2</td>
<td>103.1</td>
</tr>
</tbody>
</table>


### Investigation 6.8 Quarterback Ratings

Use the data in Table 6.4 and spreadsheet operations to check the quarterback ratings (QRs) listed. Are all five ratings correct based on the formula given above?

### Indicators

Weighted averages are used as indirect measurements or assessments of achievement in school and sports. The NFL quarterback rating really goes beyond a weighted average in that it arbitrarily adds 25/12 to a weighted average of four performance ratios. The QR formula given above is actually a simplification of the index used:

$$QR = \frac{[5(\%\text{Comp} - 30)]}{6} + \frac{[25(\text{Yd}/\text{Att} - 3)]}{6} + \frac{[10(\%\text{TD})]}{3} + \frac{[25(19 - 2(\%\text{Int}))]}{12},$$

where each bracketed item is truncated to be between 0 and 475/12, inclusive, and the overall rating is rounded to the nearest tenth.

### Quick Question 6.9 The Two Shall Be One?

Are the two formulas given for quarterback rating equivalent if we do not truncate or round? Justify your answer.
You may have noticed that we called the more complicated formula for QR an index. The word index is Latin for “pointer,” as in the index finger on the human hand. Related words are indicate and indicator. An index is a number on a defined scale used to make comparisons among values, to see trends over time, or to compare a value to some fixed reference point or reference number. When the NFL quarterback rating was developed, it was weighted and adjusted to be a 100-point scale, so that a top performing quarterback would have a rating of 100. Table 6.4 shows that some quarterbacks have had ratings above 100, but most have QRs below 100.

Another index is the consumer price index (CPI), which is an indicator of the cost of goods (food, clothing,…) and services (transportation, health care,….) for an average consumer. In the U.S., the Bureau of Labor Statistics has used the CPI since 1919 and now publishes it monthly. The CPI is on a 100-point scale; since 1988, the years 1982–1984 have been used as the reference period. The 2007 CPI for medical care was 351.1, meaning that medical care in 2007 cost 3.511 times what it cost in 1982–1984.

Each month, the Bureau of Labor Statistics publishes an overall CPI as well as individual consumer price indices for food, shelter, gasoline, and other goods and services. The Bureau also provides CPIs by region and for major cities.

**Example 5 Interpreting Consumer Price Indices**

In the first half of 2008, the CPI for the Dallas–Fort Worth metro area was 204.5 and for Houston–Galveston was 189.4. What does this mean?

**Solution** The total cost for goods and services for both metro areas was about double the national average for the 1982–1984 reference period. In the first half of 2008, Dallas–Fort Worth was the more expensive place to live. Because $204.5/189.4 = 1.0797…$, Dallas–Fort Worth was about 8% more expensive than Houston–Galveston. Put another way, on average, you would have to spend $1.08 in Dallas–Fort Worth for every dollar you would spend in Houston–Galveston. The current situation is likely different as CPIs change from month to month.

**Investigation 6.10 Consumer Confidence Index**

What is the consumer confidence index (CCI)? How is it calculated? Who calculates it?

**Investigation 6.11 Dow Jones Industrial Average**

(a) What is the Dow Jones Industrial Average?
(b) When was it started, and by whom?
(c) How has it evolved over time?
(d) How is it calculated?
(e) What is its significance?
(f) Obtain a graph of the Dow since 1900 and since 2000. Interpret these graphs.

The Gunning fog index (GFI), developed in 1952 by American businessman Robert Gunning, is a measure of the readability of written English. It is intended to correspond roughly to grade level. If a book has a fog index of 10, it is at the 10th grade reading level. The fog index is calculated using all of the sentences in a passage of about 100 words, and the following steps:

- Compute the number of words and the number of sentences in the passage.
- Count the number of complex words, that is, words of three or more syllables, unless they are proper nouns (e.g., Mississippi), familiar jargon (e.g., Internet),...
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compound words (e.g., bookkeeper), or three-syllable words ending in common suffixes, such as -ed, -er, -es, or -ing.

• Then use the following formula:

\[
GFI = 0.4 \left( \frac{\text{words}}{\text{sentences}} + 100 \cdot \frac{\text{complex words}}{\text{words}} \right)
\]

An alternative to the GFI is the Flesch-Kincaid grade level index (FKGLI), based on the total number of sentences, words, and syllables in a written passage:

\[
FKGLI = 0.39 \left( \frac{\text{words}}{\text{sentences}} \right) + 11.8 \left( \frac{\text{syllables}}{\text{words}} \right) - 15.59
\]

Investigation 6.12  Readability Indices

(a) Compute the GFI and the FKGLI for the following passage:

Weighted averages can be tricky because they occur in many contexts and because the computations involve a variety of representations. As we have already seen, grades are often expressed as percentages, but batting averages are expressed as 3-place decimal fractions. In school and college, grades can be expressed not only as percentages but also as letters (A, B, C, …), sometimes with pluses and minuses, and even on a 4-point scale using grade point averages. So even though the arithmetic of weighted averages is often fairly basic, you have to think before you compute to determine which operations to use and in which ways.

[Hint: Count “3-place” as one two-syllable word. Count A, B, and C as words.]

(b) Compute the GFI and the FKGLI for the following revised passage:

Weighted averages can be tricky. They occur in many contexts, and their computations use many representations. For example, grades are often written as percentages, but batting averages are written as 3-place decimals. In school and college, grades are expressed as percentages, as letters (A, B, C, …), or even on a 4-point scale as grade point averages. The arithmetic of weighted averages is often basic, but you have to think to decide which operations to use and in which ways.

[Hint: Count “3-place” as one two-syllable word. Count A, B, and C as words.]

(c) Which paragraph is easier to read and understand? How much content has been lost in going to the shorter version? Think about your answers and discuss them with at least one of your classmates. Then share your thinking as part of a whole-class discussion.

Quick Review for Section 6

Determine the mean, median, and midrange for the data without using a calculator. [Hint: Sort the numbers if they are not already sorted.]

1. 17, 18, 19, 20, 21
2. 500, 600, 700, 800, 900
3. 25, 15, 55, 45, 60
4. 6/7, 2/7, 5/7, 3/7, 9/7
5. 2, 2, 2, 10, 10, 10, 18
6. 800, 800, 500, 500, 500, 200
7. 1, 2, 5, 3, 8, 9, 5, 7
8. 1, 1, 1, 15, 11, 13, 9, 5, 7
9. 15, 5, 25, 15, 17, 13, 10, 10, 10
10. 15, 5, 25, 15, 17, 13, 10, 15, 10
Exercises for Section 6

A measure of center? Determine the mean, median, and the most frequently occurring value of the data set. Is the most frequently occurring value a good measure of center for the data? How would knowing the context help you decide?

1. 2, 2, 8, 8, 14
2. 1, 2, 3, 4, 5, 10, 10
3. 17, 18, 19, 19, 20, 21
4. 7, 8, 10, 10, 2, 3, 20, 10
5. 7, 7, 7, 7, 10, 20, 30, 40
6. 16, 18, 20, 22, 24, 26

Mean vs. median. Determine the mean and median of the data. In your opinion, which is the better measure of center for the data? How would knowing the context help you decide?

7. 8, 10, 10, 16, 20, 22, 327
8. 6, 500, 600, 700, 800, 900
9. 2, 8, 10, 222, 250, 276, 280
10. 1, 1, 1, 1, 1, 1, 1, 1, 21
11. 7, 8, 10, 10, 2, 3, 20, 10
12. 7, 8, 10, 10, 2, 3, 20, 100

Geometric mean. The geometric mean of $x_1, x_2, x_3, ..., x_n$ is $\sqrt[n]{x_1 \cdot x_2 \cdot x_3 \cdot ... \cdot x_n}$. Use this formula to compute the geometric mean of the data.

13. 4, 16
14. 10, 20
15. 2, 8, 32, 128
16. 3, 6, 12, 24, 48
17. 3, 6, 24, 48
18. 6, 18, 54, 162, 486

Writing to Learn. Geometric mean and geometric sequence. Solve Exercises 13–18, and then write a paragraph explaining the relationship between a finite geometric sequence and its geometric mean.

20. Writing to Learn. Mean and median. Consider the statement: “The median is the midpoint of a distribution; the mean is the balance point of a distribution.”

(a) What are the five key terms in the statement?

(b) Look up the meanings of distribution, midpoint, and balance point. In your own words, write a definition of each that fits the context of the statement.

(c) Based on part (b), do you agree with the statement? Why, or why not?

21. Slugging pitcher. Micah Owings was an unusually good hitting pitcher for the Cincinnati Reds. Table 6.5 shows some batting statistics for Owings and four Reds power hitters in 2009. Use this data and spreadsheet operations to compute and rank the five players’ statistics in the following categories:

(a) at bats per game played;

(b) batting average;

(c) total bases;

(d) slugging average; and

(e) home runs per at bat. (Write as a ratio of 1:x, where $x$ is a number of at bats.)

22. Averaging averages. Use the data in Table 6.5.

(a) Compute the batting averages of the five players.

(b) What is the mean of their five batting averages?

(c) What is the weighted mean of the five batting averages, using at bats as the weights? Is this the same value as the total hits for the five players divided by their total at bats? Why, or why not?

(d) Compute the slugging averages of the five players.

(e) What is the mean of their five slugging averages?

(f) What is the weighted mean of the five slugging averages, using at bats as the weights? Is this the same value as the total bases for the five players divided by their total at bats? Why, or why not?

(g) Which is the more appropriate way to combine rates, such as batting averages or slugging averages—using a mean, or a mean weighted by denominators? Why?

(h) Review parts (c) and (f). Which is the easier way to compute weighted means of rates—weighting by denominators, or dividing the total of the numerators by the total of the denominators? Why?

23. Grade point average (GPA). At Enormous State University, an A is worth 4 grade points; a B is worth 3; a C = 2; a D = 1; and an F = 0. Courses range from 1 to 5 credits each. The credits are used as weights to compute the grade point average (GPA). For example, an A in a 5-credit course and a B in a 2-credit course would be averaged as follows:

$$\text{GPA} = \frac{4 \cdot 5 + 3 \cdot 2}{5 + 2} = \frac{26}{7} = 3.714$$

(a) Rebecca makes an A in a 4-credit math course, a B in a 3-credit English course, a C in a 3-credit science course, a B in the associated 1-credit science lab, and an A in a 4-credit Spanish course. What is her GPA for the semester?

(b) What would Rebecca’s average be if you ignore the credits, or just average the five grades?

(c) Which of the two averages is a more reasonable and fair assessment of Rebecca’s performance? Justify your answer.

Table 6.5

<table>
<thead>
<tr>
<th>2009 Batting Statistics for Selected Cincinnati Reds Players</th>
<th>Player</th>
<th>Games</th>
<th>At Bats</th>
<th>Runs</th>
<th>Hits</th>
<th>Doubles</th>
<th>Triples</th>
<th>Home Runs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bruce, Jay</td>
<td>101</td>
<td>345</td>
<td>47</td>
<td>77</td>
<td>15</td>
<td>2</td>
<td>2</td>
<td>22</td>
</tr>
<tr>
<td>Gomes, Jonny</td>
<td>98</td>
<td>281</td>
<td>39</td>
<td>75</td>
<td>17</td>
<td>0</td>
<td>0</td>
<td>20</td>
</tr>
<tr>
<td>Nix, Laynce</td>
<td>116</td>
<td>309</td>
<td>42</td>
<td>74</td>
<td>26</td>
<td>1</td>
<td>1</td>
<td>15</td>
</tr>
<tr>
<td>Owings, Micah</td>
<td>41</td>
<td>54</td>
<td>7</td>
<td>14</td>
<td>4</td>
<td>1</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>Phillips, Brandon</td>
<td>153</td>
<td>584</td>
<td>78</td>
<td>161</td>
<td>30</td>
<td>5</td>
<td>5</td>
<td>20</td>
</tr>
</tbody>
</table>

Source: sports.espn.go.com/mlb/teams
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24. **Weighted grading systems.** Ms. Xavier and Mr. Yankovic teach Senior English. They coordinate all of their readings, writing assignments, tests, exams, and scoring rubrics. But when averaging grades, Ms. X uses System X, and Mr. Y uses System Y (Table 6.6).

(a) Daniel’s grades are CD = 80; WA = 85; TA = 88; TP = 95; FE = 92. Which teacher’s weighted averaging system would Daniel prefer? Why?

(b) Danielle’s grades are CD = 90; WA = 95; TA = 88; TP = 85; FE = 82. Which teacher’s weighted averaging system would Danielle prefer? Why?

(c) Guillermo is trying hard to stay in the top 10% of his graduating class. He figures that if he gets a 90 or above in Senior English, he will maintain his top 10% status. With just the term paper and final exam remaining, Guillermo’s grades are CD = 88; WA = 83; TA = 85. Under each system, what grades would Guillermo need on his term paper and final exam to reach his goal of 90 or above? Explain.

<table>
<thead>
<tr>
<th>Table 6.6 Two Weighted Grading Systems for Senior English</th>
</tr>
</thead>
<tbody>
<tr>
<td>System X</td>
</tr>
<tr>
<td>Class discussion: 5%</td>
</tr>
<tr>
<td>Writing assignments: 25%</td>
</tr>
<tr>
<td>Test average: 30%</td>
</tr>
<tr>
<td>Term paper: 15%</td>
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<tr>
<td>Final exam: 25%</td>
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25. **Writing to Learn. Simpson’s paradox.** Do some Internet or library research to learn more about Simpson’s paradox. Write a paragraph or more about the topic that includes examples from medicine, psychology, or other fields of study in which the paradox occurs.

26. **Writing to Learn. Quarterback ratings.** Do some Internet research to learn

(a) who among Brett Favre, Eli Manning, and Peyton Manning has the best career QR;

(b) who among active quarterbacks in the NFL has the best career QR; and

(c) who has the best all-time NFL QR. Report your findings as a paragraph with at least one supporting chart, graph, or table.

27. **Investigation. Readability indices.** Pick one of your favorite books. Compute the GFI and the FKGLI for three passages of roughly 100 words: one from a randomly selected page from the 1st, 2nd, and 3rd thirds of the book. (Use the `randInt` feature to select the page from each section of the book. Then on the page, use the `randInt` feature to select the passage.) Determine an appropriate way to average the results of both indices, and report to the class on your findings.

28. **Writing to Learn. Other indices.** Do some Internet or library research to learn about an index that has not been addressed in this section. Report to the class on your findings.