Unit Overview
In this unit you will extend your study of polygons as you investigate properties of triangles and quadrilaterals. You will study area, surface area, and volume of two- and three-dimensional figures.

Key Terms
As you study this unit, add these and other terms to your math notebook. Include in your notes your prior knowledge of each word, as well as your experiences in using the word in different mathematical examples. If needed, ask for help in pronouncing new words and add information on pronunciation to your math notebook. It is important that you learn new terms and use them correctly in your class discussions and in your problem solutions.

Academic Vocabulary
- composite

Math Terms
- theorem
- equiangular
- polygon
- quadrilateral
- consecutive angles
- perimeter
- area
- altitude
- net
- prism
- rectangular prism
- triangular prism
- volume

ESSENTIAL QUESTIONS
In what ways are geometric figures used in real life?
Why is it important to understand the characteristics of two- and three-dimensional figures?

EMBEDDED ASSESSMENTS
These assessments, following activities 24 and 26, will give you an opportunity to demonstrate how to find areas and perimeters of triangles and quadrilaterals as well as find the surface area and volume of prisms to solve mathematical and real-world problems.

Embedded Assessment 1:
Geometric Concepts p. 315

Embedded Assessment 2:
Surface Area and Volume of Prisms p. 343
1. Name each of the following geometric figures.

2. Name the geometric figures in the diagram below.

3. Find the perimeter of the figures pictured.

4. Give three characteristics of the following figures.
   a. square
   b. rectangle
   c. right triangle
   d. cube

5. Plot each point on the coordinate plane.
   \( A(-2, 3) \) \( B(4, 5) \) \( C(6, -1) \) \( D(-5, -3) \) \( E(2, 0) \)

6. State the coordinates of each point.

7. Draw a rectangle on the grid below that has a perimeter of 20 units.

8. Roger is creating designs with pieces of wood. One piece is an equilateral triangle with a perimeter of 24 inches. Another piece is a rectangle with one side the same length as a side of the triangle. The other side of the rectangle is 3 inches shorter than a side of the triangle. Roger places the pieces together so that one side of the triangle touches a side of the rectangle that has the same length.
   a. What are the dimensions of the triangle?
   b. What are the dimensions of the rectangle?
   c. Draw a sketch of the composite figure.
   d. Find the perimeter of the composite figure.
### Learning Targets:
- Determine when three side lengths form a triangle.
- Use the Triangle Inequality Property.
- Classify triangles by side length.

**SUGGESTED LEARNING STRATEGIES:** Interactive Word Wall, Summarizing, Look for a Pattern, Graphic Organizer

Students in Mr. Mira’s math class made up some geometry games. Here are the rules for the game Matt and Allie created.

<table>
<thead>
<tr>
<th>Players:</th>
<th>Three to four students</th>
</tr>
</thead>
<tbody>
<tr>
<td>Materials:</td>
<td>Three number cubes and a “segment pieces” set of three each of the following lengths: 1 inch, 2 inches, 3 inches, 4 inches, 5 inches, and 6 inches.</td>
</tr>
<tr>
<td>Directions:</td>
<td>Take turns. Roll the three number cubes. Find a segment piece to match each number rolled. See whether a triangle can be formed from those segment pieces. The value of the perimeter of any triangle that can be formed is added to that player’s score. The first player to reach 50 points wins.</td>
</tr>
</tbody>
</table>

Amir wonders what the game has to do with triangles.

1. Play the game above to see how it relates to triangles. Follow the rules. Record your results in the table.

<table>
<thead>
<tr>
<th>Player 1</th>
<th>Player 2</th>
<th>Player 3</th>
<th>Player 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Numbers</td>
<td>Score</td>
<td>Numbers</td>
<td>Score</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
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<td></td>
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</tr>
</tbody>
</table>
2. There is more to the game than just adding numbers. How does the game relate to triangles?

Amir noticed that he could tell whether the lengths would form a triangle even without the segment pieces.

3. Explain how Amir can determine whether a triangle can be formed from three given lengths.

Matt and Allie’s game illustrates the following property that relates the side lengths of a triangle.

**Triangle Inequality Property**

For any triangle, the sum of any two sides must be greater than the length of the third side.

Before students play another game, Mr. Mira wants to review the vocabulary terms *scalene*, *isosceles*, and *equilateral* with the class. He draws the following examples of triangles.
Lesson 22-1
Properties of Triangles and Side Lengths

4. Based on Mr. Mira’s examples, describe each type of triangle.
   a. scalene triangle
   b. isosceles triangle
   c. equilateral triangle

Amir creates a variation of Matt and Allie’s game. Here are the rules for Amir’s game.

<table>
<thead>
<tr>
<th>Triangle Trivia Rules - Name the Triangle</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Players:</strong> Three to four students</td>
</tr>
<tr>
<td><strong>Materials:</strong> Three number cubes</td>
</tr>
<tr>
<td><strong>Directions:</strong> Take turns rolling three number cubes.</td>
</tr>
<tr>
<td>• If you can, form</td>
</tr>
<tr>
<td>a scalene triangle ................add 5 points</td>
</tr>
<tr>
<td>an isosceles triangle ........add 10 points</td>
</tr>
<tr>
<td>an equilateral triangle .....add 15 points</td>
</tr>
<tr>
<td>no triangle .............................add 0 points</td>
</tr>
<tr>
<td>• If you make a mistake, deduct 10 points from</td>
</tr>
<tr>
<td>your last correct score.</td>
</tr>
<tr>
<td>• The first player to reach 25 points wins.</td>
</tr>
</tbody>
</table>

5. Make use of structure. When playing Amir’s variation of Triangle Trivia, suppose that the cubes landed on the following numbers. Tell how many points you would add to your score and why.
   a. 5, 5, 5
   b. 1, 6, 4
   c. 3, 2, 4
   d. 6, 6, 4
   e. 1, 4, 1

Share your responses with your group members. Make notes as you listen to other members of your group. Ask and answer questions clearly to aid comprehension and to ensure understanding of all group members’ ideas.
Lesson 22-1
Properties of Triangles and Side Lengths

LESSON 22-1 PRACTICE

For Items 8–14, use the Triangle Inequality Property to determine whether a triangle can be formed with the given side lengths in inches. If a triangle can be formed, classify the triangle by the lengths of its sides. Explain your thinking.

8. $a = 5, b = 5, c = 5$
9. $a = 3, b = 3, c = 7$
10. $a = 7, b = 4, c = 4$
11. $a = 8, b = 4, c = 5$
12. $a = 1, b = 2, c = 8$
13. $a = 8, b = 12, c = 4$
14. $a = 12, b = 5, c = 13$

15. Which of the following are possible side lengths of a triangle?
   A. 12, 20, 15
   B. 33, 20, 12
   C. 12, 20, 11

16. **Reason abstractly.** Is it necessary to find the sum of all three possible pairs of side lengths to use the Triangle Inequality Property when deciding if the sides form a triangle? Include an example in your explanation.

17. **Construct viable arguments.** Two sides of a triangle are 9 and 11 centimeters long.
   a. What is the shortest possible length in whole centimeters for the third side?
   b. What is the longest possible length in whole centimeters for the third side?
Learning Targets:
- Classify angles by their measures.
- Classify triangles by their angles.
- Recognize the relationship between the lengths of sides and measures of angles in a triangle.
- Recognize the sum of angles in a triangle.

SUGGESTED LEARNING STRATEGIES: Interactive Word Wall, Summarizing, Visualization, Graphic Organizer

Another way to classify triangles is by their angles. A right angle has a measure of 90°. An acute angle has a measure of less than 90°. An obtuse angle is greater than 90° and less than 180°.

1. Use the angles shown.

   a. Estimate the measure of each angle.
   \[ \angle A \approx \quad \angle B \approx \quad \angle C \approx \]
   \[ \angle D \approx \quad \angle E \approx \quad \angle F \approx \]

   b. Use appropriate tools strategically. Use a protractor to find the measure of each angle to the nearest degree. Then classify each angle as acute, obtuse, or right by its measure.
   \[ \angle A = \quad \angle B = \quad \angle C = \]
   \[ \angle D = \quad \angle E = \quad \angle F = \]
Now Mr. Mira draws the following examples of triangles.

2. Based on Mr. Mira’s examples, describe each type of triangle.

   a. acute triangle

   b. obtuse triangle

   c. right triangle

3. A triangle can be labeled using both its angle measures and the lengths of its sides.
   a. Label the triangles that Mr. Mira drew by side length.

   b. Choose one of the triangles and give the two labels that describe it.

   c. Explain how the two labels together provide a better description of the triangle than either one alone. Share your ideas with our group and be sure to explain your thoughts using precise language and specific details to help group members understand your ideas and reasoning.
Mr. Mira has his class investigate the sum of the measures of a triangle. Students measured the angles of some scalene, isosceles, and equilateral triangles. They recorded their results as shown.

### 4. a. Find the sum of the angle measures for each triangle.

The Triangle Sum **Theorem** states that the sum of the three angle measures in any triangle is always equal to a certain number.

### b. What is the sum of the angle measures in any triangle?

A **theorem** is a statement or conjecture that has been proven to be true.
The Triangle Sum Theorem allows you to find the measure of the third angle in a triangle when you are given the other two angle measures.

5. Students played a game in which they chose two angle measures of a triangle and then determined the third angle measure. What must be true about the two angle measures the students choose?

6. Some of the angle measures students created for triangles are shown. For each pair of angle measures, find the measure of the third angle in the triangle.
   a. $43^\circ, 94^\circ$
   b. $38^\circ, 52^\circ$
   c. $57^\circ, 39^\circ$
   d. $140^\circ, 12^\circ$
   e. $60^\circ, 60^\circ$
Lesson 22-2
Properties of Triangles and Angle Measures

The angle measures of a triangle can be used to determine if the triangle is scalene, isosceles, or equilateral. Look back at the triangles Mr. Mira drew.

7. Compare the angle measures of the triangles. Look for patterns in Mr. Mira’s examples to help you determine if the triangles described below are scalene, isosceles, or equilateral.
   a. a triangle with three different angle measures
   b. a triangle with exactly two congruent angle measures
   c. an equiangular triangle

8. Look back at Item 6. Classify each triangle by its side lengths and by its angle measures.

Another relationship exists between the angles and the sides of a triangle. In a triangle, the side opposite the angle with the greatest measure is the longest side.

9. Compare the angle measure to the side opposite the angle in a scalene triangle. What is true about the side opposite the angle with the least measure?
Lesson 22-2
Properties of Triangles and Angle Measures

Check Your Understanding

For Items 10–12, sketch a triangle described by each pair of words below or state that it is not possible. Use tick marks and right angle symbols where appropriate. If it is not possible to sketch a triangle, explain why not.

10. scalene, obtuse
11. isosceles, acute
12. equilateral, right
13. Two angles in a triangle measure 35° and 50°. Explain how to find the measure of the third angle.

LESSON 22-2 PRACTICE

For Items 14–19, sketch a triangle described by each pair of words below or state that it is not possible. If it is not possible to sketch a triangle, explain why not.

14. scalene, right
15. isosceles, obtuse
16. equilateral, acute
17. isosceles, right
18. scalene, acute
19. equilateral, obtuse
20. Use appropriate tools strategically. Use a ruler and a protractor to sketch a triangle that is scalene and has an angle that measures 30°. Is the triangle acute, right, or obtuse? Explain.
21. Two angles in a triangle measure 65° each. What is the measure of the third angle?
22. Reason quantitatively and abstractly. Find the missing angle measure or measures in each triangle below. Then classify the triangle by both its angle measures and its side lengths.
   a. The three angles in a triangle have the same measure.
   b. Two angles in a triangle measure 45° each.
   c. Two angles in a triangle measure 25° and 50°.
23. Construct viable arguments. Determine whether each statement below is always true, sometimes true, or never true. Explain your reasoning.
   a. The acute angles of an isosceles triangle add up to 90°.
   b. An isosceles triangle has two equal angles.
   c. An equilateral triangle has a right angle.
   d. The largest angle of a scalene triangle can be opposite the shortest side.
Lesson 22-1

1. The side lengths of a triangle are 16 cm, 9 cm, and 16 cm. Classify the triangle using its side lengths.

2. The side lengths of a triangle are 23, 14, and 30 yards. Classify the triangle using these side lengths.

3. Classify the triangles below using their side lengths.
   a. 7 ft, 24 ft, 21 ft
   b. 9 cm, 9 cm, 9 cm
   c. 35 mm, 25 mm, 35 mm

For Items 4–13, use the Triangle Inequality Property to determine whether a triangle can be formed with the given side lengths in centimeters. If a triangle can be formed, classify the triangle using the side lengths.

4. \(a = 2, b = 2, c = 5\)

5. \(a = 6, b = 3, c = 8\)

6. \(a = 3, b = 5, c = 5\)

7. \(a = 4, b = 5, c = 9\)

8. \(a = 11, b = 7, c = 6\)

9. \(a = 16, b = 16, c = 16\)

10. \(a = 21, b = 9, c = 21\)

11. \(a = 32, b = 5, c = 25\)

12. \(a = 13, b = 13, c = 30\)

13. \(a = 14, b = 25, c = 19\)

14. Which of the following are possible side lengths of a triangle?
   A. 5, 8, 13
   B. 8, 21, 16
   C. 11, 60, 61

15. Two sides of a triangle are 18 and 20 feet long. What is the shortest the third side of the triangle could be? What is the longest the triangle’s third side could be? Explain your reasoning.

16. Two sides of a triangle are 4 meters and 12 meters long. Which of the following is not a possible length of the third side?
   A. 8 meters   B. 12 meters
   C. 9 meters   D. 15 meters

Lesson 22-2

17. Use a protractor to determine the measure of the angle.

18. Identify Triangle \(ABC\) by side length and angle measure. Use appropriate measuring tools to justify your answer.

19. Sketch a triangle described by each pair of words below or state that it is not possible and explain why not. Use tick marks and right angle symbols where appropriate.
   a. scalene, acute
   b. isosceles, obtuse
   c. equilateral, right

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For Items 20–25, find the measure of the numbered angle in each triangle.

20. 

21. 

22. 

23. 

24. 

25. 

For Items 26–30, two angle measures of a triangle are given. Find the missing angle measure in each triangle. Then classify the triangle by both angle measures and side lengths.

26. 32°; 58°
27. 162°; 9°
28. 60°; 60°
29. 43°, 74°
30. 27°; 63°

31. One of the angle measures of a right triangle is 36°. Which of the following is the measure of one of the other two angles of the triangle?
   A. 36°
   B. 44°
   C. 54°
   D. 144°

32. Two of the angles of a triangle measure 42° and 67°. The side opposite which angle of the triangle is the longest side? Explain your reasoning.

33. Explain why it is not possible to draw an isosceles triangle with one angle that measures 52° and a second angle that measures 74°.

34. Remi says that every triangle must have at least two acute angles. Is Remi correct? Explain why or why not.

**Mathematical Practices**

**Express Regularity in Repeated Reasoning**

35. Create a graphic organizer showing how triangles are classified using angle measures and side lengths. Include three labeled examples for angle measure and three examples for side length.
Area and Perimeter of Polygons

Play Area
Lesson 23-1 Recalling Quadrilaterals

Learning Targets:
• Define and classify quadrilaterals based on their properties.
• Use properties of quadrilaterals to determine missing side lengths and angle measures.

SUGGESTED LEARNING STRATEGIES: Interactive Word Wall, Marking the Text, Visualization, Think-Pair-Share, Create Representations, Use Manipulatives

The student council at QUAD Middle School is helping to plan a new playground. The three polygons shown are quadrilaterals, structures that will be included on the playground.

The student council members want to understand the special properties of each quadrilateral to help them in their design. They did an Internet search and found quadrilaterals are grouped into three categories as shown in the table below.

Types of Quadrilaterals

<table>
<thead>
<tr>
<th>Quadrilateral with no special name</th>
<th>Trapezoid</th>
<th>Parallelogram</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="quadrilateral-no-special-name.png" alt="Image" /></td>
<td><img src="trapezoid.png" alt="Image" /></td>
<td><img src="parallelogram.png" alt="Image" /></td>
</tr>
</tbody>
</table>

1. Based on the categories in the table, describe the sides and angles in each type of quadrilateral.
   a. quadrilateral with no special name
   b. trapezoid
   c. parallelogram

MATH TERMS
A **polygon** is a closed figure formed by three or more line segments that intersect only at their endpoints.
A **quadrilateral** is a polygon with four sides.

MATH TIP
The arrowheads along opposite sides of the quadrilaterals indicate that those opposite sides are parallel.
A parallelogram can sometimes be classified as a rectangle, a rhombus, or a square.

2. **Model with mathematics.** Using the given definitions, mark each diagram to appropriately illustrate the properties.

   **Rectangle:** Parallelogram with four right angles.

   ![Rectangle Diagram]

   **Rhombus:** Parallelogram with four congruent sides.

   ![Rhombus Diagram]

   **Square:** Parallelogram with four right angles and four congruent sides.

   ![Square Diagram]

3. The playground designer investigates the following quadrilaterals.

   ![Quadrilateral 1 Diagram] ![Quadrilateral 2 Diagram]

   **Quadrilateral 1**  
   **Quadrilateral 2**

   a. Use a ruler to measure the length of the sides of each quadrilateral to the nearest quarter of an inch. Label the measures of each side on the diagram.

   b. List any patterns you notice about the side lengths in the quadrilaterals.

   c. Use a protractor to determine the measure of each of the angles of the quadrilaterals to the nearest degree. Label the measures of each angle on the diagram.

   d. List any patterns you notice about the angle measures in the quadrilaterals.

   e. **Make use of structure.** Select the best name for each quadrilateral. Justify your answers.
Lesson 23-1
Recalling Quadrilaterals

The opposite sides of a parallelogram have the same length. The opposite angles of a parallelogram have the same measure, and any pair of consecutive angles add up to 180°.

4. Write the name of each figure in the Venn diagram below.

![Venn Diagram]

5. Use what you have learned about rhombi to find the missing length and angle measure.

   \[ BT = \]

\[ m\angle T = \]

6. a. Use the definition of a square or a rectangle. Find the sum of the measures of the four angles of a square or a rectangle.

   b. Use what you know about triangles to determine and justify that the sum of the measures of any quadrilateral is 360°.

7. What is the measure of the fourth angle in a quadrilateral with angles measuring 90°, 70°, and 120°? Explain how you found it.
Check Your Understanding

For Items 8–9, write the best name for each quadrilateral.

8. A parallelogram with four congruent sides.
9. Two parallel sides and two nonparallel sides.

10. Use the diagram of the parallelogram to find the length of side PL and the measure of angle R.

LESSON 23-1 PRACTICE

For Items 11–12, write the best name for each quadrilateral.

11. A parallelogram with four congruent sides.
12. Two parallel sides and two nonparallel sides.

13. Use the diagram of the rectangle to find the length of side AC and the measure of angle A.

For Items 14 and 15, find the missing angle measure in the quadrilaterals shown.

14. 108°
15. 78°

16. Jordan notices that the baseball infield is both equilateral and equiangular. What is the best quadrilateral name for the baseball infield? Explain your reasoning.

17. Construct viable arguments. Jordan claims that all squares can also be classified as either a rectangle or a rhombus, and Reyna claims that all rectangles and rhombuses can be classified as squares. Who is correct? Justify your answer.
Learning Targets:
- Model the area of a parallelogram by decomposing into triangles.
- Find the area of a special quadrilateral by decomposing into triangles.
- Write equations that represent problems related to the area of parallelograms and rectangles.
- Solve problems involving the area of parallelograms and rectangles.
- Find the area of special quadrilaterals and polygons by composing into rectangles or decomposing into triangles and other shapes.

SUGGESTED LEARNING STRATEGIES: Identify a Subtask, Use Manipulatives, Create Representations, Think-Pair-Share, Discussion Group, Sharing and Responding, Interactive Word Wall

Pictured is an aerial view of one of the possible playground designs. An aerial view is the view from above something.

1. Look at the shape of each figure. What piece of playground equipment do you think each figure represents?

2. List all the geometric shapes you can identify in each figure in the playground to complete the table.

<table>
<thead>
<tr>
<th>Figure</th>
<th>Geometric Shape(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td></td>
</tr>
<tr>
<td>E</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td></td>
</tr>
</tbody>
</table>
3. The diagram shows the dimensions of Figure E.

What is the perimeter of Figure E? Explain how you found the perimeter.

![Diagram of Figure E with dimensions: 2 feet by 10 feet.]

4. What is the area of Figure E? Explain how you found the area.

5. There is also a parallelogram in the playground design. List some characteristics of a parallelogram.

6. Use appropriate tools strategically
   a. Cut out the parallelogram on page 303. Then cut a right triangle from one side of the parallelogram so that you can form a rectangle with the two pieces. Put the two pieces together to form a rectangle.
   b. Use a ruler to measure the rectangle you cut out and find its area.
   c. How do the lengths of the base and the height of the rectangle formed from the parallelogram relate to those of the original parallelogram?

7. What is the relationship between the area of a parallelogram and its base and height? Describe the relationship using words, symbols, or both.
Lesson 23-2
Perimeter and Area of Composite Figures

The area, $A$, of a rectangle or a parallelogram is equal to the length of the base, $b$, times the height, $h$: $A = b \times h$.

8. The diagram shows the dimensions of Figure $B$, the ball pit in the playground. What is the area of Figure $B$?

A composite figure is a figure that can be decomposed into two or more figures. You can find the area of a figure that can be decomposed, or divided, into rectangles and parallelograms.


a. Fill in missing dimensions on the playground. Then find the perimeter of the playground.

b. The playground can be decomposed into a parallelogram and two rectangles. Use a colored pencil to draw lines on the diagram to decompose the figure. Explain how to use the shapes you just drew to find the area of the playground.

c. Use the rules or equations you wrote in Items 4 and 7 to find the area of the playground. Justify your answer.

ACADEMIC VOCABULARY

Composite means made up of various separate parts or pieces.
LESSON 23-2 PRACTICE

For Items 14–17, find each perimeter and use the rules or equations you wrote in Items 4 and 7 to find the area of each figure.

14. \( \text{16 ft} \)  
15. \( \text{15 m} \)  
16. \( \text{4 cm} \)  
17. \( \text{20 yd} \)

18. **Reason quantitatively.** A square tablecloth has a perimeter of 12 feet. What is the area of the tablecloth?

19. A square field has an area 121 square meters. What is the perimeter of the field?

20. The area of a parallelogram with a height of 6 meters is 126 square meters. What is the base length of the parallelogram?

21. A rectangular pool is 9 feet wide. The pool has an area of 117 square feet. What is the perimeter of the pool?

22. A rectangular floor is 12 feet wide and 18 feet long. How much will it cost to carpet the floor if the carpet costs $1.39 per square foot?

23. **Make sense of problems.** Jamie has to put 2 coats of paint on 6 rectangular walls. Each wall is 9 feet by 15 feet. Each can of paint covers 500 square feet. How many cans of paint should Jamie buy? Explain your thinking.
Lesson 23-3
Area of Triangles, Trapezoids, and Polygons

Learning Targets:
- Model area formulas for parallelograms, trapezoids, and triangles.
- Write equations that represent problems related to the area of trapezoids and triangles.
- Solve problems involving the area of trapezoids and triangles.
- Find the area of triangles, special quadrilaterals, and polygons.
- Model area formulas by decomposing and rearranging parts.
- Find the area of special quadrilaterals and polygons.

SUGGESTED LEARNING STRATEGIES: Identify a Subtask, Look for a Pattern, Discussion Groups, Sharing and Responding, Interactive Word Wall

The diagram shows the aerial view of climbing bars to be included in the playground. To find the area of the figure, decompose the polygon into other shapes. One of these shapes is a triangle.

1. Use the congruent triangles on page 303.

- Cut out one of the triangles.
- Label one of its sides \(b\).
- Draw the altitude of the triangle by drawing a line segment from a vertex perpendicular to side \(b\). Label the segment \(h\).
- Cut out the second triangle.
- Place the two triangles together to form a parallelogram whose base is the side labeled \(b\).

2. How does the area of each triangle compare to the area of the parallelogram? Explain your thinking.

3. Using words, symbols, or both, describe a method for finding the area of a triangle.

MATH TERMS
The altitude of a triangle is a perpendicular line segment from a vertex to the line containing the opposite side. The measure of an altitude is the height.
If you know the area of a triangle and the length of its base or its height, you can find the missing measure since the area, $A$, of a triangle is one-half the length of the base, $b$, times the height, $h$: $A = \frac{1}{2} \times b \times h$.

4. The area of a triangular garden near the playground is 12 square feet. The height of the garden is 4 feet. How long is the base of the garden? Explain your thinking.

5. Another shape seen in the aerial view of the playground looks like a trapezoid. The parallel sides of a trapezoid are called the bases. The two sides that are not parallel are called the legs. Label the bases and legs on the trapezoid shown.

6. Use the congruent trapezoids on page 303.
   - Cut out both the trapezoids.
   - On the inside of each figure, label the bases $b_1$ and $b_2$.
   - Draw the height of each trapezoid and label it $h$.
   - Form a parallelogram by turning one of the trapezoids so that its short base lines up with the long base of the other trapezoid. The long legs of the trapezoids will be adjacent.

7. How does the height of one of the trapezoids compare to the height of the parallelogram?

8. How does the base of one of the trapezoids compare to the base of the parallelogram?

9. What is the area of one of the trapezoids? Explain your thinking.
Lesson 23-3
Area of Triangles, Trapezoids, and Polygons

The area, $A$, of a trapezoid is equal to one-half the height, $h$, times the sum of the bases, $b_1$ and $b_2$: $A = \frac{1}{2} \times h \times (b_1 + b_2)$.  

10. A planter near the playground has the dimensions shown in the diagram to the right. What is the area of the planter?

You can find the area of a composite figure that can be decomposed, or divided, into rectangles, parallelograms, triangles, and trapezoids.

11. A pentagon is another polygon in the aerial view of the playground. Describe how to find the area of the pentagon using the figure shown.

12. The diagram shows the dimensions of Figure $A$ in the aerial view of the playground. Find the area of Figure $A$ using the formulas you have learned in this activity. Show your work in the My Notes column.

13. Attend to precision. The diagram shows the dimensions of Figure $F$ from an aerial view. Find the area of Figure $F$. Explain your thinking.
Lesson 23-3
Area of Triangles, Trapezoids, and Polygons

Check Your Understanding

For Items 14 and 15, find the area of each figure.

14.  
\[ \text{Triangle: } \begin{array}{c}
9 \text{ cm} \\
12 \text{ cm}
\end{array} \]

15.  
\[ \text{Trapezoid: } \begin{array}{c}
6 \text{ cm} \\
16 \text{ cm}
\end{array} \]

16. The area of a trapezoid is 40 square inches. The bases of the trapezoid are 9 inches and 11 inches long. Explain how to find the height of the trapezoid.

LESSON 23-3 PRACTICE

For Items 17–20, find the area of each figure.

17.  
\[ \text{Triangle: } \begin{array}{c}
4 \text{ ft} \\
5 \text{ ft} \\
11 \text{ ft}
\end{array} \]

18.  
\[ \text{Trapezoid: } \begin{array}{c}
5 \text{ ft} \\
8 \text{ ft} \\
3 \text{ ft}
\end{array} \]

19.  
\[ \text{Triangle: } \begin{array}{c}
12 \text{ cm} \\
5 \text{ cm} \\
13 \text{ cm}
\end{array} \]

20.  
\[ \text{Trapezoid: } \begin{array}{c}
28 \text{ cm} \\
25 \text{ cm}
\end{array} \]

21. A triangular sail with a height of 6 feet has a base that is 9 feet long. What is the area of the sail?

22. A triangle with a height of 12 square inches has an area of 36 square inches. How long is the base of the triangle?

23. A trapezoidal window has a height of 18 centimeters. The bases of the window are 34 and 28 centimeters long. What is the area of the window?

24. Reason abstractly. A trapezoid with a height of 6 meters has an area of 36 square meters. One of the bases is twice as long as the other base. How long are the bases of the trapezoid?

25. Make sense of problems. The diagram shows an aerial view of a parking lot that needs new concrete. The concrete costs $75 per square yard. How much will the concrete for the parking lot cost? Explain your thinking.
ACTIVITY 23 PRACTICE
Write your answers on a separate piece of paper. Show your work.

Lesson 23-1

1. Write the best name for each quadrilateral.
   a. 
   b. 

2. PONY is a parallelogram.
   a. Label these sides of PONY: \( PY = 5 \) inches and \( PO = 7 \) inches. What are the lengths of \( ON \) and \( YN \)?
   b. Label this angle in PONY: \( m\angle P = 112^\circ \). What are the measures of \( \angle O \), \( \angle N \), and \( \angle Y \)?

3. Find the length of each side of a rhombus with perimeter of 40 meters.

4. Complete each statement with always, sometimes, or never.
   a. A square is ______ a rectangle.
   b. A rectangle is ______ a rhombus.
   c. A parallelogram is ______ a square.
   d. A trapezoid is ______ a square.

5. Use the diagram of each parallelogram to find the missing side and angle measures.
   a. square
   
   b. rhombus

   If \( m\angle M = 78^\circ \) and \( HM = 6 \) cm, then \( m\angle H = \_\_\_\_\_\_ \) and \( HT = \_\_\_\_\_\_ \).

6. Explain how to determine whether a rectangle is a square. Use an example in your explanation.

7. The measures of three angles in a quadrilateral are 70°, 82°, and 120°. Find the measure of the fourth angle.

8. Identify the quadrilateral. Then find the value of \( x \).

Lesson 23-2

For Items 9 and 10, find the perimeter and area of each figure.

9. 

10. 

Activity 23 • Area and Perimeter of Polygons 301
For Items 11–13, find the area of each figure.

11. [Diagram of a trapezoid with dimensions: bases 1.5 in. and 7 in., height 1.5 in.]

12. [Diagram of a rectangle with dimensions: length 15 m, width 9 m]

13. [Diagram of a trapezoid with dimensions: bases 3 cm and 5 cm, height 5 cm, and base 11 cm]

14. The area of a parallelogram with a base of 26 meters is 494 square meters. What is the height of the parallelogram?

15. A square window has a perimeter of 60 inches. What is the area of the window?

16. A rectangular stage that is 9 feet wide has an area of 252 square feet. What is the perimeter of the stage?

Lesson 23-3

For Items 17–21, find the area of each figure.

17. [Diagram of a triangle with dimensions: base 15 cm, height 12 cm]

18. [Diagram of a trapezoid with dimensions: bases 12 cm and 8 cm, height 15.4 cm]

19. [Diagram of a trapezoid with dimensions: bases 28 in. and 16 in., height 21 in.]

20. [Diagram of a trapezoid with dimensions: bases 14.4 in. and 7.8 in., height 14.3 in.]

21. [Diagram of a trapezoid with dimensions: bases 6 cm and 4 cm, height 3 cm]

22. Mikel is building a doghouse for his puppy. The diagram shows the shape of the floor. Use the area formula for trapezoids to find the area of the doghouse floor. Confirm your answer by finding the sum of the areas of the rectangle and the right triangle.

23. Describe and model the area formula for trapezoids by breaking up the figure in Item 19 and rearranging the parts.

24. A triangular banner has a base length of 48 inches. The banner has an area of 1,632 square inches. What is the height of the banner?

25. How does knowing how to find the area of a rectangle help you determine the area of other polygons?

MATHEMATICAL PRACTICES

Modeling with Mathematics

26. Draw and label a triangle, a trapezoid, and a parallelogram that each has an area of 48 square inches. Show your work justifying that each figure has an area of 48 square inches.
Parallelogram Cut out this parallelogram for Item 6 of Activity 23-2.

Two Congruent Triangles Cut out these triangles for Item 1 of Activity 23-3.

Two Congruent Trapezoids Cut out these trapezoids for Item 6 of Activity 23-3.
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Learning Targets:
- Draw polygons in the coordinate plane given vertex coordinates.
- Find the length of a segment joining points with the same first coordinate or the same second coordinate.
- Use coordinate geometry to identify locations on a plane.
- Graph points in all four quadrants.
- Solve problems involving the area on the coordinate plane.

SUGGESTED LEARNING STRATEGIES: Visualization, Think-Pair-Share, Create Representations, Identify a Subtask

Zena is hired to paint a mural on the side of a large building. She creates a scale drawing of her mural, shown on the coordinate grid. She will use the model to plan the painting. Each block on the grid represents 1 foot by 1 foot.

1. The border of Zena’s design forms quadrilateral $WXYZ$.
   a. What are the coordinates of point $W$ and point $X$?

   b. What do these coordinates have in common?
c. What is the length of side $XW$ of quadrilateral $WXYZ$? Explain your answer.

d. What are the coordinates of point $Z$?

e. What do the coordinates of points $W$ and $Z$ have in common?

f. What is the length of side $WZ$ of quadrilateral $WXYZ$? Explain your answer.

g. What is the best name for quadrilateral $WXYZ$? Explain your reasoning.

2. Consider only the portion of Zena's design in the first quadrant. The vertices of the inner square are labeled $A$, $B$, $C$, and $D$.

   a. The coordinates of point $B$ are $(11, 5)$. What are the coordinates of point $A$?

   b. What is the length of $AB$?

   c. What is the area of square $ABCD$?

3. a. Make use of structure. Explain how to find the length of a vertical line segment using the coordinates of the endpoints. Include an example in your explanation.

   b. Explain how to find the length of a horizontal line segment using the coordinates of the endpoints. Include an example in your explanation.
4. The endpoints of a line segment are (−4.5, 6) and (−4.5, −2).
   a. Use the My Notes section to create a coordinate grid. Draw the line segment on the coordinate grid.
   b. What is the length of the line segment? Explain how you determined this length.

5. Look at parallelogram $ADPR$ in Zena’s first quadrant design.
   a. What is the length of the base of the parallelogram, $AD$?
   b. What is the height of the parallelogram? Explain how you determined this.
   c. What is the area of parallelogram $ADPR$? Explain your reasoning.

6. a. What is the total area of the parallelograms in Zena’s first quadrant design? Explain your reasoning.
   b. Each of the four quadrants has two of the light blue parallelograms in the mural design. What is the total area of the light blue parallelograms in the mural design?
   c. Make sense of problems. Each gallon of light blue paint costs $45 and will cover 75 square feet. How much will the light blue paint cost that is needed to paint the light blue parallelograms in all four quadrants of the mural design? Explain your reasoning.
Lesson 24-1
Defining Polygons on the Coordinate Plane

Check Your Understanding

Use the coordinate grid for Items 7–10.
7. The vertices of rectangle JKL M are represented by these coordinates: J(−2.5, 3), K(2, 3), L(2, −1), and M(−2.5, −1). Draw rectangle JKL M.
8. What is the length of side JK?
9. What is the length of side JM?
10. What is the area of rectangle JKL M?

LESSON 24-1 PRACTICE

Use the coordinate grid for Items 11–14.
11. The coordinates Q(2, 2), R(1, −3), S(−2, −3), and T(−1, 2) represent the vertices of parallelogram QRST. Draw parallelogram QRST.
12. What is the length of the base of parallelogram QRST?
13. What is the height of parallelogram QRST?
14. What is the area of parallelogram QRST?
15. What is the distance between the points (1, 3) and (1, 7)? Explain your reasoning.
16. A line segment has the endpoints (2.25, 5.75) and (−1, 5.75). What is the length of the line segment?

Use parallelogram ABCD for Items 17 and 18.
17. What are the coordinates of the vertices of the parallelogram?
18. A stained-glass designer will align eight different colored parallelograms of this same size to create a pattern. What is the total area of the designer’s pattern if each square on the grid represents 1 square centimeter?

19. Reason abstractly. A square has vertices (1, 7), (1, 2), (6, 7), and (6, 2). What is the area of the square? How can you find the area of the square without drawing the square in a coordinate plane? Explain.
Learning Targets:
- Use coordinate geometry to identify locations on a plane.
- Graph points in all four quadrants.
- Solve problems involving the area of parallelograms, trapezoids, and triangles.

SUGGESTED LEARNING STRATEGIES: Visualization, Sharing and Responding, Create a Plan, Create Representations

There are three different types of triangles in the scale drawing of Zena’s mural design. They are shown in the first quadrant section of the mural. Remember, each square on the grid represents 1 square foot.

1. Find the area of each triangle. Justify your answers.
   a. gray triangle 1
   b. white triangle 2
   c. dark blue triangle 3
2. The endpoints of triangle ABC on a coordinate grid plane are
   \( A(-3.5, 4), B(-3.5, -1), \) and \( C(2.5, -1) \).
   a. Use the My Notes section to create a coordinate grid. Draw the
triangle on the coordinate grid.

   b. What is the area of triangle ABC? Explain.

3. There are four congruent white trapezoids in the original mural
design. One of the trapezoids is shown on the grid.

   a. What are the coordinates of the vertices of the trapezoid?

   b. What is the height of the trapezoid in the mural design? What are
   the lengths of the bases?

   c. What is the area of the trapezoid in the mural design? Explain
   your reasoning.

   d. How much of the mural area do the four white trapezoids make up?
The complete model of Zena's mural design is shown.

4. Look back at Items 1 and 3.
   a. What is the total area of the four white triangles? Explain your reasoning.

   b. What is the total area of the white trapezoids and the white triangles? Explain your reasoning.

   c. Each can of white paint costs $42.75 and will cover 50 square feet. How much will the white paint cost for the mural? Explain your reasoning.

5. Construct viable arguments. There is an octagon in the center of the mural. Use what you know about the area of triangles and rectangles to find the area of the octagon. Explain your reasoning.
Lesson 24-2
Area of Polygons on a Coordinate Plane

Check Your Understanding

Use the coordinate grid for Items 6–9.

6. Trapezoid $ABCD$ has vertices at $A(1, 3)$, $B(-2, 3)$, $C(-3, -2)$, and $D(4, -2)$. Draw trapezoid $ABCD$.

7. What is the length of the bases of trapezoid $ABCD$?

8. What is the height of trapezoid $ABCD$?

9. Find the area of trapezoid $ABCD$.

LESSON 24-2 PRACTICE

Use the coordinate grid for Items 10–13.

10. Triangle $PQR$ has vertices at $P(-1, 1)$, $Q(-1, -3)$, and $R(2.5, -2)$. Draw triangle $PQR$.

11. What is the length of the base of triangle $PQR$?

12. What is the height of triangle $PQR$?

13. Find the area of triangle $PQR$.

Use trapezoid $CDEF$ for Items 14–18.

14. What are the coordinates of the vertices of this trapezoid?

15. What are the lengths of the bases of the trapezoid?

16. What is the height of the trapezoid?

17. Find the area of the trapezoid if each square on the grid represents 1 square meter.

18. Make sense of problems. The trapezoid in Items 14–17 is a scale model of a deck that Gayle is planning to build. The material for the deck costs $4.35 per square meter. What will be the cost of the deck?

19. Construct viable arguments. A right triangle has vertices at $(16.5, 12)$, $(16.5, -8)$, and $(-10, 12)$. Explain how to find the area of the triangle without drawing the triangle on a coordinate grid.
ACTIVITY 24 PRACTICE

Write your answers on a separate piece of paper. Show your work.

Lesson 24-1

1. Name another point on the same horizontal line as the point (2, 8).
2. Name another point on the same vertical line as the point (−1, 4).
3. Name another point on the same horizontal line as the point (−6, 3.5).
4. Name another point on the same vertical line as the point (−2.5, −4.25).
5. Find the length of the line segment connecting each pair of points.
   a. (1, 3), (6, 3)
   b. (−4, 2), (1, 2)
   c. (7, −5), (7, −3)
   d. (−8, −4), (8, −4)
6. The length of the line segment connecting the points (2, 8) and (2, y) is 7. What are the two possible values of y?

For Items 7 and 8, use parallelogram JKLM.

7. What are the coordinates of the vertices of parallelogram JKLM?
8. What is the area of parallelogram JKLM?

For Items 9–11, find the area of each figure.

9.

10.

11.

12. On a coordinate grid, draw a rectangle with vertices at (0, 4), (0, 6), (3, 4), and (3, 6). What is the area of the rectangle?
13. On a coordinate grid, draw a square with vertices at (3, 2), (−1, 2), (−1, −2), and (3, −2). What is the area of the square?
14. On a coordinate grid, draw a parallelogram with vertices at (2, 4), (0, −1), (−3, −1), and (−1, 4). What is the area of the parallelogram?
15. On a coordinate grid, draw a rectangle with vertices at (3.5, 8), (−2.5, 8), (−2.5, −6), and (3.5, −6). What is the area of the rectangle?
16. On a coordinate grid, a scale drawing of a banner is shaped like a parallelogram with vertices at (−15, 10), (0, −5), (30, −5), and (15, 10). Each square on the grid represents 1 square inch. What is the area of the banner?
Lesson 24-2
For Items 17 and 18, write the missing coordinates for the vertices of each figure. Then find the area of the figure.

17. 

18. 

For Items 19–24, find the area of each figure.

19. 

20. 

21. 

22. 

23. 

24. 

25. What is the area of a triangle with vertices at \((-8, 4), (12, 16), \) and \((12, -4)\)?

MATHEMATICAL PRACTICES
Use Appropriate Tools Strategically

26. On a coordinate grid, a trapezoid has vertices at \((-6, 9), (-6, 6), (3, -9), \) and \((3, 12)\). Each square on the grid represents 1 square foot. Explain how to find the area of the trapezoid.
Write your answers on a separate sheet of paper. Show your work.

Students at STAR Middle School are designing a logo for their astronomy club. They are considering different designs for the logo. The students have decided to use different polygons to create their logo. Before they begin designing, they review some of the properties and characteristics of polygons.

1. Some of the shapes they may use in the logo are shown. Identify the best name for each triangle or quadrilateral and explain your reasoning.

   a. 
   b. 
   c. 
   d. 

2. Find the missing angle measure in each polygon. Explain your reasoning.

   a. 
   b. 

3. Determine whether each statement below is always true, sometimes true, or never true. Explain why you chose each answer.

   a. A rectangle is a rhombus.
   b. Two angles of a scalene triangle are congruent.
   c. A rhombus is equilateral.
   d. An equilateral triangle is equiangular.
   e. A parallelogram is a rectangle.
   f. An obtuse triangle contains at least two obtuse angles.
   g. A square is a rhombus.
   h. A triangle has side lengths 3 inches, 4 inches, and 8 inches.
After each student designed a logo, the club members voted on their favorite. The two designs that got the greatest number of votes are shown below.

4. Determine the areas and perimeters of both designs. Explain your reasoning.

5. Which design will take up the greatest amount of space on the students’ T-shirts? Explain how you made your decision.

6. Which design would you recommend the students use for the T-shirt? Use mathematical reasons to support your decision.

<table>
<thead>
<tr>
<th>Scoring Guide</th>
<th>Exemplary</th>
<th>Proficient</th>
<th>Emerging</th>
<th>Incomplete</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematics Knowledge and Thinking</td>
<td>Clear and accurate understanding of finding angle measures, perimeter, and area of triangles and quadrilaterals.</td>
<td>An understanding of finding angle measures, perimeter, and area of triangles and quadrilaterals.</td>
<td>Partial understanding of finding angle measures, perimeter, and area of triangles and quadrilaterals.</td>
<td>Incorrect or incomplete understanding of finding angle measures, perimeter, and area.</td>
</tr>
<tr>
<td>Problem Solving</td>
<td>Interpreting a problem accurately in order to find angle measures, perimeter, or area.</td>
<td>Interpreting a problem to find angle measures, perimeter, or area.</td>
<td>Difficulty interpreting a problem to find angle measures, perimeter, or area.</td>
<td>Incorrect or incomplete interpretation of a problem.</td>
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<tr>
<td>Mathematical Modeling / Representations</td>
<td>Clear and accurate understanding of the characteristics of triangles and quadrilaterals.</td>
<td>Identifying and naming triangles and quadrilaterals correctly.</td>
<td>Difficulty identifying triangles and quadrilaterals.</td>
<td>Incorrect or incomplete identification of triangles and quadrilaterals.</td>
</tr>
<tr>
<td>Reasoning and Communication</td>
<td>Precise use of appropriate terms to explain reasoning in geometry concepts.</td>
<td>Competent use of appropriate terms to explain reasoning in geometry concepts.</td>
<td>Partially correct use of terms to explain reasoning in geometry concepts.</td>
<td>An incomplete or inaccurate use of terms to explain reasoning in geometry concepts.</td>
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ACTIVITY 25

Nets and Surface Area

All Boxed Up

Lesson 25-1 Nets and Surface Area of Cubes

Learning Targets:
- Represent three-dimensional figures using nets.
- Use nets to find the surface area of figures.
- Write equations that represent problems related to the area of rectangles.
- Determine solutions for problems involving the area of rectangles.

SUGGESTED LEARNING STRATEGIES: Visualization, Create Representations, Identify a Subtask, Use Manipulatives

A net is a two-dimensional drawing used to represent or form a three-dimensional figure. Nets can be used to form different types of boxes.

The shape in Figure 1 is a net.

1. Use the net of Figure 1 on page 327.
   Cut out the net. Fold it along the dotted lines to form a box.
   The figure formed is a cube. What are the characteristics that make the three-dimensional figure a cube?

2. The net shown will also form a cube. If face 1 is the bottom of the cube, which numbered face is the top of the cube?

3. **Model with mathematics.** Many other nets can be used to represent a cube.
   a. Use graph paper to draw as many of these other nets as you can find. Cut out each net and fold it to verify that a cube can be formed.
   b. Sketch the nets you found that form a cube in the My Notes space.
   c. Below, sketch two nets made up of six squares that do not form a cube.
Elaine sells shipping materials, including boxes and packing peanuts, at her business All Boxed Up. Her box supplier charges her $\frac{1}{2}$ cent per square inch of surface area for each box. Elaine can find the surface area of a box by adding the areas of the six faces of the box (front, back, top, bottom, left, and right).

Elaine needs to find the surface areas of boxes of many different sizes. She wants to find a pattern that will make it faster to find the surface area.

4. One type of box that Elaine keeps in stock is a cube. For one of the cube-shaped boxes, the length of each edge is 5 inches.
   a. Find the area of each face of the cube.
   b. Find the total surface area of the cube.

5. Elaine makes a table to record the surface areas of the cube-shaped boxes.
   a. Complete the table.

<table>
<thead>
<tr>
<th>Length of Edge (in.)</th>
<th>Number of Faces</th>
<th>Area of One Face (in.²)</th>
<th>Surface Area (in.²)</th>
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<tr>
<td>5</td>
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<tr>
<td>10</td>
<td>100</td>
<td>600</td>
<td>1,350</td>
</tr>
</tbody>
</table>

   b. Describe any patterns you see in the table.

   c. For each box, how does the area of one face relate to the surface area?
Lesson 25-1
Nets and Surface Area of Cubes

6. You can use a variable to represent the length of the edge of a cube.
   a. What variable is used to represent the length of the edge of the cube in the diagram?

   b. What is the area of one face of a cube in terms of the length of an edge, e?

   c. **Make use of structure.** Write a rule for finding the surface area, \( SA \), of a cube in terms of the length of an edge, \( e \).

7. Cube-shaped boxes with 12-inch edges are kept in stock at All Boxed Up.
   a. Determine the surface area of a box with 12-inch edges.

   b. **Attend to precision.** The supplier charges Elaine \( \frac{1}{2} \) cent per square inch of surface area for each box. Determine how much profit Elaine will make on a 12-inch cube-shaped box if she sells the box for \$4.95. Explain your reasoning.
LESSON 25-1 PRACTICE

Use the net for Items 12 and 13.

12. What is the area of each face of the cube?
13. What is the surface area of the cube?
14. Draw a net and use it to find the surface area of a cube with edges that are 11 meters long.
15. Draw a net and use it to find the surface area of a cube with edges that are 14 feet long.
16. The edges of a cube are 20 millimeters long. Draw a net and use it to find the surface area of the cube.
17. The edges of a cube are 5.2 centimeters long. Draw a net and use it to find the surface area of the cube.
18. What is the surface area of a cube with edges that are $2\frac{1}{2}$ feet long?
19. **Reason quantitatively.** A cube has a surface area of 384 square inches. What is the edge length of the cube? Explain your reasoning.
20. **Make sense of problems.** A cube-shaped box has edges that are 18 centimeters long. The box does not have a top. What is the surface area of the box? Justify your answer.
21. **Critique the reasoning of others.** The edge of a cube-shaped display case is 30 inches long. Drake says that the surface area of the display case is 3,600 square inches. Is he correct? Explain why or why not.
Learning Targets:
• Represent three-dimensional figures using nets.
• Use nets to find the surface area of figures.
• Write equations that represent area problems.
• Solve problems involving the area of rectangles and triangles.

SUGGESTED LEARNING STRATEGIES: Note Taking, Visualization, RAFT, Graphic Organizer, Create Representations

Elaine has boxes in stock at All Boxed Up that are rectangular prisms and triangular prisms.

A **prism** is a three-dimensional figure with parallel congruent bases that are both polygons. The faces (sides) of a prism are rectangles. A prism is named for the shape of its bases. A **rectangular prism** has bases that are rectangles. Its faces are also rectangles.

The net of one of Elaine's boxes is shown. The box is a rectangular prism.

1. **a.** Show the calculation needed to find the area of each face of the rectangular prism.

   Face 1: 
   Face 2: 
   Face 3: 
   Face 4: 
   Face 5: 
   Face 6: 

   **b.** Find the surface area of the rectangular prism and explain your process.
You can use a net to find the surface area of a rectangular prism. You can also use congruence to help you find the surface area of a rectangular prism.

2. **a.** Cut out the net (Figure 2) on page 327. Fold it to form a rectangular prism with the measurements on the outside.

   **b.** Label the length, width, and height of the rectangular prism you formed on this diagram.

   ![Diagram of a rectangular prism with labels](image)

   - Face 1
   - Face 4
   - Face 6

   - \( l = \)
   - \( w = \)
   - \( h = \)

   **c.** Faces 2, 3, and 5 cannot be seen in the diagram. Describe the location of each of the hidden faces.

   - Face 2:
   - Face 3:
   - Face 5:

   **d.** Which pairs of faces have the same area?

   **e.** How can you use this observation to find the surface area of a rectangular prism?

3. One of the boxes in Elaine’s shop is 18 inches long, 6 inches wide, and 9 inches tall. Explain how to find the surface area of the box.
Lesson 25-2
Nets and Surface Area of Prisms

Write a rule to determine the surface area, \( SA \), of a rectangular prism with length \( l \), width \( w \), and height \( h \). Explain your thinking.

Some of the boxes in stock at All Boxed Up are triangular prisms. A triangular prism has two parallel bases that are congruent triangles. The three faces are rectangles.

5. a. Cut out the net of Figure 3 on page 329. Identify and label which sides of the prism are the bases and which sides are the faces.

b. Explain how to find the area of each face of the triangular prism.

c. Explain how to find the surface area of the triangular prism.

6. Explain how to use a net to find the surface area of the triangular prism shown.
For Items 11 and 12, use the nets to find the surface area of the prisms.

11. 12.

For Items 13 and 14, draw and use nets to find the surface area of each prism.


15. A battery shaped like a rectangular prism is 8 inches long, 5 inches wide, and 4 inches tall. What is the surface area of the battery?

16. Make sense of problems. Elaine has a new box that is 24 inches long, 12 inches wide, and 10 inches high. Write a proposal to Elaine recommending a price for this size box. Remember that her box supplier charges her $\frac{1}{2}$ cent per square inch of surface area for each box. Be certain to explain how you arrived at your recommendation.
Nets and Surface Area
All Boxed Up

ACTIVITY 25 PRACTICE
Write your answers on a separate piece of paper. Show your work.

Lesson 25-1

1. Determine whether this net can form a cube.

2. Use the net below to find the surface area of the cube.

3. Use the net below to find the surface area of the cube.

For Items 4–6, draw a net and use it to find the surface area of each cube.

4. 6 ft

5. 16 in.

6. 4.5 cm

7. Find the surface area of a cube with edges 7 centimeters long.

8. Find the surface area of a cube with edges 19 inches long.

9. Find the surface area of a cube with edges 21 millimeters long.

10. What is the surface area of a cube with edges 6.3 centimeters long?

11. What is the surface area of a cube with edges 40 inches long?

12. A cube has a surface area of 486 square centimeters. How long is each edge of the cube?

13. A cube-shaped block has edges that are 21 millimeters long. All of the faces of the block are painted except for the bottom face. What is the surface area of the painted faces of the block?

Lesson 25-2
Use the rectangular prism for Items 14 and 15.

14. Draw and label a net that represents the rectangular prism.

15. What is the surface area of the rectangular prism?
Use the triangular prism for Items 16 and 17.

16. Draw and label a net that represents the triangular prism.

17. What is the surface area of the triangular prism?

18. Use the net of the triangular prism to find the surface area of the prism.

For Items 19–25, draw a net and use it to find the surface area of each prism.

19.

20.

21.

22.

23.

24. A trunk is shaped like a rectangular prism and is 4 feet long, 1\(\frac{1}{2}\) feet wide, and 2 feet tall. What is the surface area of the trunk?

25. A cereal box is 8 inches long, 3 inches wide, and 12 inches tall. A pasta box is 12 inches long, 1 inch wide, and 2 inches tall. How much greater is the surface area of the cereal box than the pasta box?

MATHEMATICAL PRACTICES
Reason Abstractly

26. Create a graphic organizer comparing and contrasting the methods used to find the surface area of cubes, rectangular prisms, and triangular prisms.
Nets and Surface Area
All Boxed Up

Cut out this net for Item 1 of Lesson 25-1.

Figure 1

Cut out this net for Item 2 of Lesson 25-2.

Figure 2
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Cut out this net for Item 5 of Lesson 25-2.

Figure 3
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Learning Targets:
- Find the volume of a right rectangular prism with fractional edge lengths.
- Write equations that represent problems related to the volume of right rectangular prisms.

SUGGESTED LEARNING STRATEGIES: Close Reading, Paraphrasing, Think Aloud, Visualization, Vocabulary Organizer, Construct Arguments, Create a Plan, Use Manipulatives, Look for a Pattern

Crystals are solids formed by a regular repeated pattern of molecules connecting together. They have a regular shape and flat sides. Some crystals form cubes while others grow into columns with three or more sides. The figures shown below are crystals.

The collected atoms that make up crystals are called unit cells. They are the simplest repeating unit in the crystal and are repeated in exactly the same arrangement throughout the solid. Opposite faces of a unit cell are parallel. A simple cubic unit cell is in the shape of a cube.

Cubes are named for the lengths of their edges:
- A 1-inch cube is a cube with edges that are 1 inch in length.
- A 2-inch cube is a cube with edges that are 2 inches in length.
- A \( \frac{1}{2} \)-inch cube is a cube with edges that are \( \frac{1}{2} \) inch in length.
- Any size cube can be used to build larger cubes.

**Volume** is a measure of the amount of space a solid occupies. It is measured in cubic units, such as cubic inches (in.\(^3\)), cubic feet (ft\(^3\)), cubic centimeters (cm\(^3\)), or cubic meters (m\(^3\)).

One way to find the volume of a solid is to fill the solid with cubes. The volume is the total number of cubes needed to fill the solid.

MATH TIP

The cube in the diagram is an \( a \)-unit cube. Each edge is \( a \) units long.
1. Halite, or table salt, is a mineral that is made up of cubic crystals. Use unit cubes, provided by your teacher, as models of 1-inch cubes.
   a. Use the unit cubes to build models of 2-inch and 3-inch cubes. Then complete the table.

<table>
<thead>
<tr>
<th>Length of Edge (in.)</th>
<th>Area of Face (in.²)</th>
<th>Volume of Cube (in.³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   b. Describe any patterns you see in the table.

2. Make use of structure. Describe how you can use the patterns you found in the table to determine the volume of a cube when you do not have enough unit cubes to build the cube.

3. Let the variable $e$ represent the length of the edge of a cube. Write a rule for finding the volume, $V$, of a cube in terms of the length of an edge, $e$.

4. Model with mathematics. Cut out the nets of cubes, Figure 1 and Figure 2, on page 341.
   a. Fold each figure to form a cube.

   b. How many of the smaller cubes made from Figure 1 will fit into the larger cube made from Figure 2?

   c. Look back at the table in Item 1. How many cubes with edge length 1 inch will fit into the cube with edge length 2 inches?
5. Consider a cube with edge length 4 inches. How many of these smaller cubes will fit into an 8-inch cube? Explain your thinking.

In Items 4 and 5, the ratio of the edge length of the smaller cube to the edge length of the larger cube is $\frac{1}{2}$.

The rule you determined earlier in the activity to find the volume of a cube can also be used to find the volume of a cube with fractional edge lengths.

6. Find the volume of each cube with the given edge length.
   a. $\frac{2}{3}$ foot
   b. $1\frac{1}{4}$ inches

7. A storage shed shaped like a cube has sides that are 3.5 meters long. What is the volume of the storage shed?
LESSON 26-1 PRACTICE
For Items 10–13, find the volume of each cube.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>10.</td>
<td>11.</td>
</tr>
<tr>
<td>7 in.</td>
<td>18 mm</td>
</tr>
<tr>
<td>12.</td>
<td>13.</td>
</tr>
<tr>
<td>2(\text{ft}1)</td>
<td>3.4 cm</td>
</tr>
</tbody>
</table>

14. A cube has an edge length of 4.3 meters. What is the volume of the cube?
15. What is the volume of a cube with an edge length of \(\frac{1}{4}\) yard?
16. A cube has a volume of 1,000 cubic feet. What is the edge length of the cube?
17. The area of one face of a cube is 36 cubic inches. What is the volume of the cube?
18. How much greater is the volume of a cube with edges that are 2\(\frac{1}{2}\) feet long than a cube with edges that are 2 feet long?
19. A fish tank shaped like a cube has sides that are 9 inches long. What is the volume of the fish tank?
20. **Reason quantitatively.** A cube has a surface area of 96 square inches. What is the volume of the cube? Explain your thinking.
21. **Make sense of problems.** A tower is made from three cubes stacked on top of each other. The edges of the cubes are 4 inches, 6 inches, and 8 inches. What is the total volume of the tower?
Lesson 26-2
Volume of Rectangular Prisms

Learning Targets:
• Write equations that represent problems related to the volume of right rectangular prisms.
• Apply the formulas \( V = lwh \) and \( V = bh \) to find volumes of right rectangular prisms.

SUGGESTED LEARNING STRATEGIES: Close Reading, Paraphrasing, Think Aloud, Visualization, Vocabulary Organizer, Construct an Argument, Create a Plan, Use Manipulatives, Look for a Pattern

Some crystals grow into columns in the shape of rectangular prisms. For example, zircon is a tetragonal crystal shaped like a rectangular prism.

1. Crystals can be stored in display cases. A museum has many different cases. Each display case is shaped like a rectangular prism. The table shows the dimensions of some different cases.

   a. Complete the table. Use unit cubes, provided by your teacher, as models of 1-inch cubes.

<table>
<thead>
<tr>
<th>Length (in.)</th>
<th>Width (in.)</th>
<th>Height (in.)</th>
<th>Volume (in.³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

   b. Describe any patterns you see in the table.
2. **Make use of structure.** Look back at the table in Item 1.
   
   a. Compare the product of the length, width, and height of each prism to its volume. Write a rule for finding the volume, \( V \), of a rectangular prism in terms of its length, \( l \), width, \( w \), and height, \( h \). Explain how this rule follows from the rule you wrote in part b.

   b. Compare the product of the area of the base (length times width) and the height of each prism to its volume. Write a rule for finding the volume, \( V \), of a rectangular prism in terms of the area of the base, \( B \), and the height, \( h \). Explain your thinking.

3. A crystal shaped like a rectangular prism has the dimensions shown.

   ![Crystal Diagram](image)

   a. What is the area of the base of the crystal?

   b. What is the volume of the crystal?

4. A crystal shaped like a rectangular prism has a volume of 480 cubic millimeters. The crystal is 10 millimeters long and 8 millimeters tall. How wide is the crystal? Explain your thinking.
Lesson 26-2
Volume of Rectangular Prisms

You can use either of the rules that you determined to find the volume of a rectangular prism with fractional edge lengths.

5. A crystal shaped like a rectangular prism has the dimensions shown.

[Diagram of a rectangular prism with dimensions 1 1/2 in., 1/2 in., and 3/4 in.]

a. What is the area of the base of the crystal?

b. Use the area of the base in finding the volume of the crystal.

c. Use the rule using the length, width, and height to find the volume of the crystal.

6. A crystal shaped like a rectangular prism has a volume of 2.88 cubic centimeters.

a. The crystal has a square base with an area of 1.44 square centimeters. How tall is the crystal? Explain your thinking.

b. What are the dimensions of the crystal? Explain your thinking.

7. A rectangular prism with a base area of 35 square inches is 12 inches tall. What is the volume of the prism?

8. A rectangular prism is 9 centimeters long, 6 centimeters wide, and 3.5 centimeters tall. What is the volume of the prism?

Check Your Understanding
**LESSON 26-2 PRACTICE**

For Items 9–12, find the volume of each rectangular prism.

9. 

10. 

11. 

12. 

13. A battery shaped like a rectangular prism is 11 inches long, 5 inches wide, and 4 inches tall. What is the volume of the battery?

14. A toy box is \(4 \frac{1}{2}\) feet long, 2 feet wide, and 2 feet tall. What is the volume of the toy box?

15. A rectangular prism with a volume of 288 cubic inches has a base area of 72 square inches. How tall is the prism?

16. A rectangular prism has a volume of 540 cubic centimeters. The prism is 5 centimeters tall and 18 centimeters wide. How long is the prism?

17. **Reason abstractly.** What is the maximum number of cubes with a side length of 2 inches that can fit inside the box shown?

18. **Make sense of problems.** Daniel needs to buy sand to fill the box shown almost to the top. He will leave 6 inches empty at the top of the sandbox. How much sand does Daniel need?
ACTIVITY 26 PRACTICE
Write your answers on a separate piece of paper.
Show your work.

Lesson 26-1
For Items 1–4, find the volume of each cube.

1. 
   \[ \text{6 ft} \]

2. 
   \[ \text{16 in.} \]

3. 
   \[ \text{4.5 cm} \]

4. 
   \[ \text{1} \frac{2}{3} \text{yd} \]

5. A crystal shaped like a cube has edges that are 8 inches long. What is the volume of the crystal?

6. A cube has a volume of 125 cubic centimeters. What is the edge length of the cube?

7. The area of one face of a cube is 144 square inches. What is the volume of the cube?

8. A cube has a surface area of 294 square inches. What is the volume of the cube?

9. A cube has an edge length of 9.2 centimeters. What is the volume of the cube?

10. What is the volume of a cube with an edge length of \( \frac{3}{8} \) foot?

11. What is the volume of a cube with an edge length of \( 2 \frac{1}{2} \) inches?

12. What is the volume of a cube with an edge length of \( 3 \frac{3}{4} \) feet?

13. How much greater is the volume of a cube with edges that are \( 1 \frac{1}{2} \) feet long than a cube with edges that are \( \frac{1}{2} \) foot long?

14. A cubical storage box has edges that are 2 feet 4 inches long. What is the volume of the storage box?
   - A. 4,704 in.\(^3\)
   - B. 13,824 in.\(^3\)
   - C. 17,576 in.\(^3\)
   - D. 21,952 in.\(^3\)

Lesson 26-2
For Items 15 and 16, find the volume of each rectangular prism.

15. 
   \[ \text{9 ft} \]
   \[ \text{6 ft} \]
   \[ \text{15 ft} \]

16. 
   \[ \text{7 in.} \]
   \[ \text{12 in.} \]
For Items 17–19, find the volume of each rectangular prism.

17. 

\[
\text{Volume} = \text{length} \times \text{width} \times \text{height} = 6 \text{ in.} \times 6 \text{ in.} \times 30 \text{ in.} = 1080 \text{ in.}^3
\]

18. 

\[
\text{Volume} = 10.4 \text{ m} \times 6 \text{ m} \times 8.5 \text{ m} = 544.8 \text{ m}^3
\]

19. 

\[
\text{Volume} = 2 \text{ in.} \times 3 \text{ in.} \times 4\frac{1}{2} \text{ in.} = 30 \text{ in.}^3
\]

20. A rectangular prism with a volume of 720 cubic inches has a base area of 45 square inches. How tall is the prism?

\[
\text{Volume} = \text{base area} \times \text{height} \\
720 \text{ in.}^3 = 45 \text{ in.}^2 \times \text{height} \\
\text{height} = \frac{720 \text{ in.}^3}{45 \text{ in.}^2} = 16 \text{ in.}
\]

21. A rectangular prism has a volume of 3,220 cubic centimeters. The prism is 20 centimeters tall and 23 centimeters long. How wide is the prism?

\[
\text{Volume} = \text{length} \times \text{width} \times \text{height} \\
3220 \text{ cm}^3 = 23 \text{ cm} \times \text{width} \times 20 \text{ cm} \\
\text{width} = \frac{3220 \text{ cm}^3}{23 \text{ cm} \times 20 \text{ cm}} = 7 \text{ cm}
\]

22. The dimensions of a brick shaped like a rectangular prism are 23 centimeters by 11 centimeters by 7.6 centimeters. What is the volume of the brick?

\[
\text{Volume} = 23 \text{ cm} \times 11 \text{ cm} \times 7.6 \text{ cm} = 2100.8 \text{ cm}^3
\]

23. A party favor box is in the shape of a rectangular prism. The box is 3 inches long, 1\(\frac{1}{2}\) inches wide, and \(\frac{3}{4}\) inch high. What is the volume of the box?

\[
\text{Volume} = 3 \text{ in.} \times 1\frac{1}{2} \text{ in.} \times \frac{3}{4} \text{ in.} = \frac{27}{4} \text{ in.}^3 = 6\frac{3}{4} \text{ in.}^3
\]

24. A chunk of cheese is cut into the shape of a rectangular prism. The piece is \(3\frac{1}{4}\) inches long, \(2\frac{1}{2}\) inches wide, and \(1\frac{3}{4}\) inches tall. What is the volume of the chunk of cheese?

\[
\text{Volume} = 3\frac{1}{4} \text{ in.} \times 2\frac{1}{2} \text{ in.} \times 1\frac{3}{4} \text{ in.} = 11\frac{3}{4} \text{ in.}^3
\]

25. A hole shaped like a rectangular prism is 3 feet wide, 5 feet long, and 3 feet deep. If the hole is made 2 feet deeper, how much will the volume of the hole increase?

\[
\text{Original volume} = 3 \text{ ft} \times 5 \text{ ft} \times 3 \text{ ft} = 45 \text{ ft}^3 \\
\text{New volume} = 3 \text{ ft} \times 5 \text{ ft} \times 5 \text{ ft} = 75 \text{ ft}^3 \\
\text{Increase} = 75 \text{ ft}^3 - 45 \text{ ft}^3 = 30 \text{ ft}^3
\]

26. A cereal box is 10 inches long, 2 inches wide, and 14 inches tall. A pasta box is 15 inches long, 1 inch wide, and 4 inches tall. How much greater is the volume of the cereal box than the pasta box?

\[
\text{Cereal box volume} = 10 \text{ in.} \times 2 \text{ in.} \times 14 \text{ in.} = 280 \text{ in.}^3 \\
\text{Pasta box volume} = 15 \text{ in.} \times 1 \text{ in.} \times 4 \text{ in.} = 60 \text{ in.}^3 \\
\text{Difference} = 280 \text{ in.}^3 - 60 \text{ in.}^3 = 220 \text{ in.}^3
\]

27. A trunk is 3 feet long, 1\(\frac{1}{2}\) feet wide, and 2 feet tall. What is the volume of the trunk?

\[
\text{Volume} = 3 \text{ ft} \times 1\frac{1}{2} \text{ ft} \times 2 \text{ ft} = 9 \frac{3}{4} \text{ ft}^3
\]

A. \(6\frac{1}{2} \text{ ft}^3\)  
B. \(6\frac{1}{8} \text{ ft}^3\)  
C. \(9 \text{ ft}^3\)  
D. \(9\frac{1}{8} \text{ ft}^3\)

MATHEMATICAL PRACTICES
Model with Mathematics

28. Design a box that will hold 24 2-inch cubes, with no empty space left in the box after it is filled with the cubes. Describe two possible designs for the box. Sketch each design and find the surface area and volume of each box.
Cut out these nets for Item 4 of Lesson 26-1.

Figure 1

Figure 2
This page is intentionally blank.
Write your answers on a separate sheet of paper. Show your work.

Artie is designing crayons and packages for restaurants to give children with their menus. One part of Artie’s marketing plan is to create crayons that will not roll off the tables. He decides to create crayons in the shape of triangular prisms. Each of the crayons that are shaped like triangular prisms will have a paper wrapper around the faces. The crayons have a base edge length of 10 millimeters and the height of each face is 50 millimeters, as shown in the diagram below.

1. Artie must create a paper label that will wrap around the crayon to cover all of the faces.
   a. Sketch each face and include the dimensions of each shape.
   b. Find the total area needed for the paper label. Explain your reasoning.

2. Artie also created a box in the shape of a triangular prism to hold four different color crayons, as shown in the diagram. Find the total surface area of the box.

3. Another crayon type the company will make is in the shape of a rectangular prism. The dimensions of each of these crayons is $10 \frac{1}{2}$ mm by $10 \frac{1}{2}$ mm by 50 mm.
   a. Sketch a net of one of these crayons.
   b. Find the volume and surface area of one of these crayons and explain what each piece of information tells you about the crayon.
4. A third type of crayon is being created for toddlers. They are referred to as “block” crayons. The dimensions of one of the block crayons is shown in the diagram.

Find the maximum number of these block crayons that can be stacked in a box that is 12 centimeters by 12 centimeters by 6 centimeters. Explain your reasoning.

<table>
<thead>
<tr>
<th>Scoring Guide</th>
<th>Exemplary</th>
<th>Proficient</th>
<th>Emerging</th>
<th>Incomplete</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematics Knowledge and Thinking</td>
<td>• Accurately and efficiently finding the surface area and volume of prisms.</td>
<td>• Finding the surface area and volume of prisms with few if any errors.</td>
<td>• Difficulty finding the surface area and volume of prisms.</td>
<td>• No understanding of finding the surface area and volume of prisms.</td>
</tr>
<tr>
<td>(Items 1a-b, 2, 3a-b, 4)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Problem Solving</td>
<td>• An appropriate and efficient strategy that results in a correct answer.</td>
<td>• A strategy that may include unnecessary steps but results in a correct answer.</td>
<td>• A strategy that results in some incorrect answers.</td>
<td>• No clear strategy when solving problems.</td>
</tr>
<tr>
<td>(Items 1b, 2, 3b, 4)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mathematical Modeling / Representations</td>
<td>• Clear and accurate understanding of how a net represents a three-dimensional figure.</td>
<td>• Relating a net to the surfaces of a three-dimensional figure.</td>
<td>• Difficulty recognizing how a net represents a three-dimensional figure.</td>
<td>• No understanding of how a net represents a three-dimensional figure.</td>
</tr>
<tr>
<td>(Items 1a-b, 3a)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reasoning and Communication</td>
<td>• Precise use of appropriate terms to explain the surface area and volume of solids.</td>
<td>• An adequate explanation of finding the surface area and volume of solids.</td>
<td>• A partially correct explanation of finding the surface area and volume of solids.</td>
<td>• An incomplete or inaccurate explanation of the surface area and volume of solids.</td>
</tr>
<tr>
<td>(Items 1b, 3b, 4)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>