Hypothesis Test for Sample Means 

\[ t = \frac{\bar{x} - \mu}{s / \sqrt{n}} \]

1. Based on a SRS of size n drawn from a normal population with unknown mean \( \mu \) and unknown \( \sigma \).
2. The t test statistic has a standard normal distribution when \( H_0 \) is true with degrees of freedom \( df = n - 1 \).
3. In theory, using the t-distribution requires that the data be from a large sample (n > 30) OR you have an approximately normal distribution (check that there are no outliers in the data).
4. The sample must be less than 10% of the population.

#1 The mean systolic blood pressure for white males ages 35-44 in the United States is 127.3. The paper “Blood Pressure in a Population of Diabetic Persons Diagnosed After 30 Years of Age” (Amer. Journal of Public Health) reports that the mean blood pressure and standard deviation of a sample of 101 diabetic males 35-44 are 130 and 11, respectively. The researchers wanted to know if the blood pressure of diabetic males 35-44 differed significantly from 35-44 year old males in the general population. Based on the data, what conclusions would the researchers reach? Let \( \alpha = .02 \). What is the observed level of significance?

\[ \bar{x} = 130, \quad s = 11 \]

\[ n = 101 \]

\[ H_0: \mu = 127.3 \]
\[ H_a: \mu \neq 127.3 \]

\[ t = \frac{\bar{x} - \mu}{s / \sqrt{n}} \]

\[ t = 2.467 \quad df = 100 \]

\[ p-value = .015 \]

Since the p-value of .015 is < \( \alpha \) of .02, I reject \( H_0 \). There is sufficient evidence to suggest that the blood pressure of diabetic males 35-44 years old is different from all males 35-44 years old.

Assume men selected randomly
- Men are independent
- 101 men are less than 10% of diabetic 35-44 year old males

#2 The manufacturer of a certain foreign car claims that it will average 36 mpg. To test this claim a sample of 10 cars is taken and driven under normal conditions. These cars average 31 mpg with a standard deviation of 8.3 mpg. Should we reject the claim at the 5% level of significance? At the 1% level?

\[ \bar{x} = 31 \text{ mpg}, \quad s = 8.3 \text{ mpg} \]

\[ n = 10 \]

\[ H_0: \mu = 36 \]
\[ H_a: \mu < 36 \]

\[ t = \frac{\bar{x} - \mu}{s / \sqrt{n}} \]

\[ t = -1.905 \quad df = 9 \]

\[ p-value = .045 \]

Since the p-value of .045 < \( \alpha \) of .05, I reject \( H_0 \). There is sufficient evidence to suggest that the mean mpg is significantly less than 36 mpg.

Assume cars selected randomly
- Cars are independent
- 10 cars less than 10% of all these cars

\[ n = 10 < 30 \text{ But I don't believe there are any outliers, symmetry reasonable} \]
A new fuel mixture for missile propellant is being tested to determine if it is superior to the current formula. From past experience, the mean distance traveled has been 340 miles. Twelve missiles with the new propellant are fired into the Pacific Ocean and their distances measured. The data is given below. Test at the 5% level of significance. 

\[ \alpha = 0.05 \]

\[ \bar{x} = 356.17 \quad s = 28.78 \]

1. \( H_0: \mu = 340 \)
2. \( H_a: \mu > 340 \)
3. 1-sample test 
   \[ t = \frac{\bar{x} - \mu}{s / \sqrt{n}} \]
4. \[ t = 1.94 \quad df = 11 \]
5. Since the p-value of 0.039 ≤ \( \alpha \) of 0.05, I reject \( H_0 \). There is sufficient evidence to suggest that the new fuel propels missiles further than the old fuel.

A series of tests of fire prevention sprinkler systems that use a foaming agent to quell the fire were performed to determine how long it took (in seconds) for the sprinklers to be activated after the detection of a fire by the system. The system has been designed so that the true average activation time is supposed to be at most 25 seconds. Do the data strongly indicate that the design specifications have not been met? 

\[ H_0: \mu = 25 \quad (\text{really } \mu \leq 25) \]

\[ H_a: \mu > 25 \quad \text{spec not met} \]

1. 1-sample test 
   \[ t = \frac{\bar{x} - \mu}{s / \sqrt{n}} \]
2. \[ t = 1.876 \quad df = 12 \]
3. Since the p-value of 0.043 ≤ \( \alpha \) of 0.05, I reject \( H_0 \). There is sufficient evidence to suggest that the sprinklers take longer than 25 seconds to activate.

However, this cannot be trusted since we are unsure of the normality of distribution.