



6.6: Use Proportionality Theorems

Objectives:

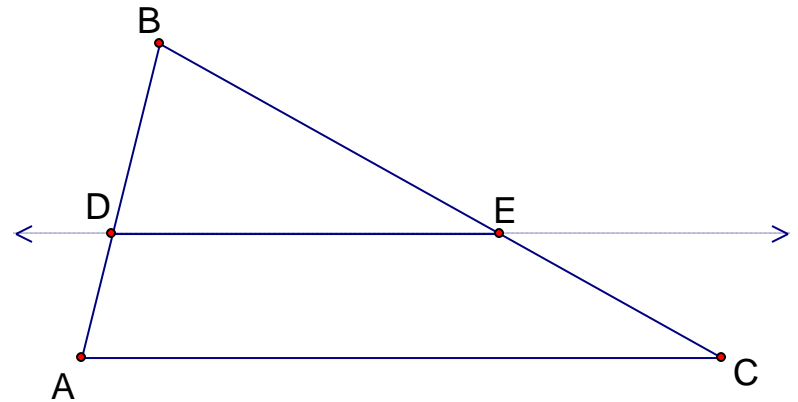
1. To discover, present, and use various theorems involving proportions with parallel lines and triangles



Investigation 1

In the diagram, \overleftrightarrow{DE} is parallel to \overline{AC} .

1. Name a pair of similar triangles and explain why they are similar.

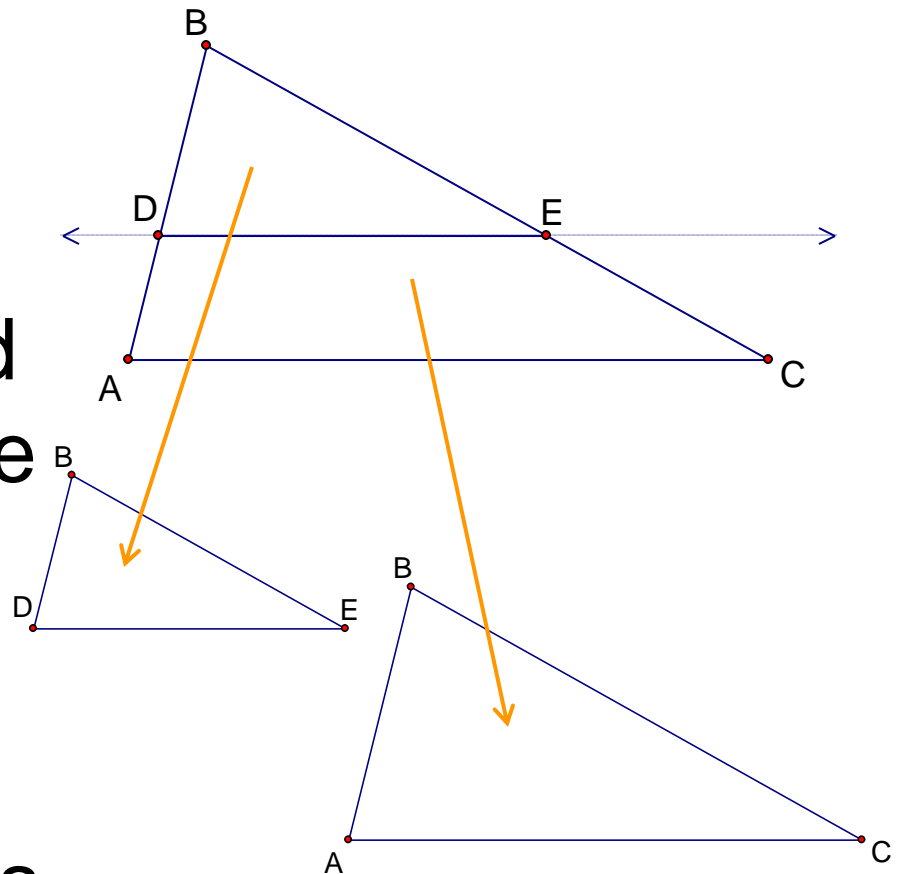




Investigation 1

In the diagram, \overleftrightarrow{DE} is parallel to \overline{AC} .

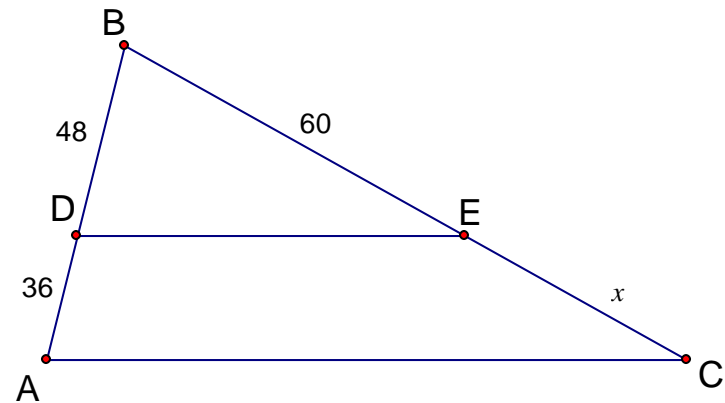
1. Name a pair of similar triangles and explain why they are similar.
2. Write three equal ratios involving the sides of the triangles.





Investigation 1

3. Write a proportion and solve for x .
4. What is the ratio $BD:DA$? Reduce your answer.
5. What is the ratio $BE:EC$? Reduce your answer.
6. What do you notice?

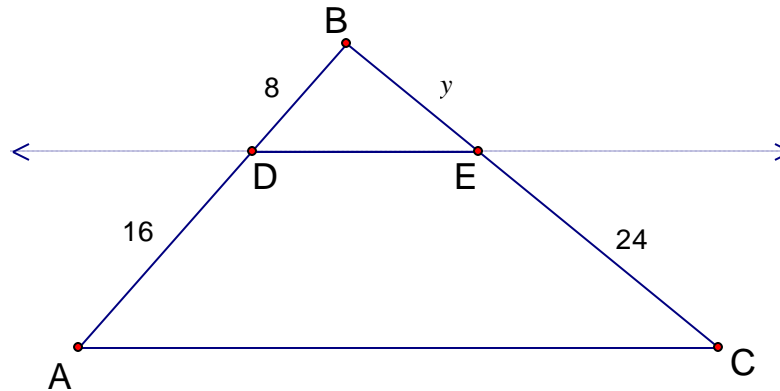


$$\frac{48}{48 + 36} = \frac{60}{60 + x}$$



Investigation 1

7. Find y .
8. What do you notice about the ratios $BD: AD$ and $BE: EC$?

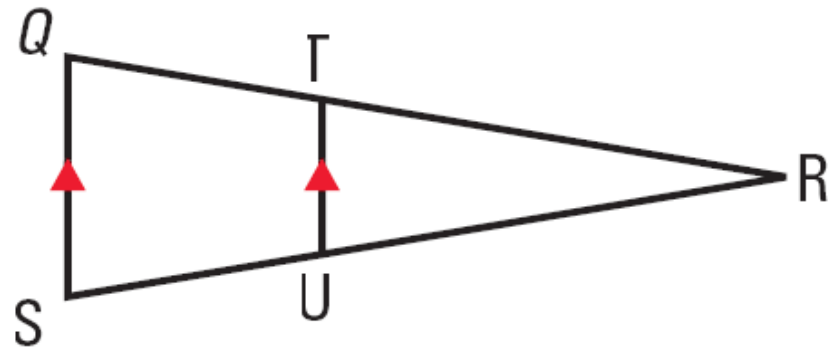




Proportionality Theorems!

Triangle Proportionality Theorem

If a line parallel to one side of a triangle intersects the other two sides, then it divides the two sides proportionally.

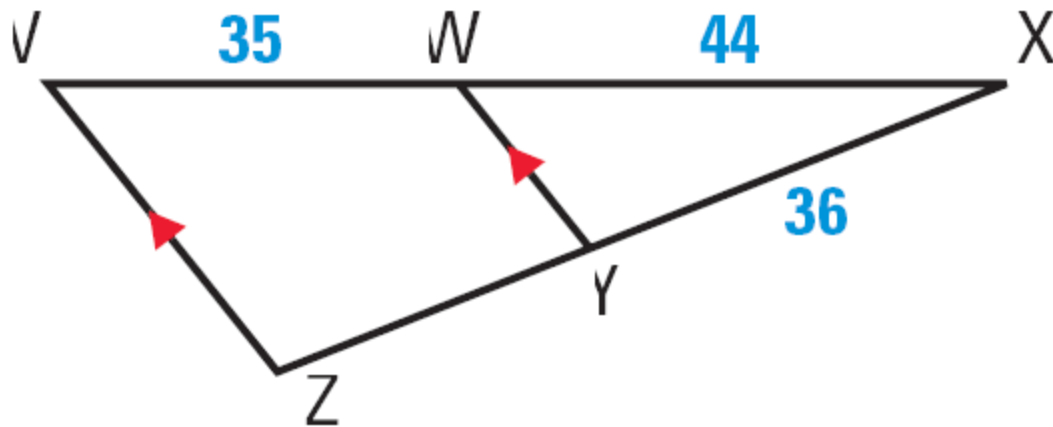


$$\text{If } \overline{TU} \parallel \overline{QS}, \text{ then } \frac{RT}{TQ} = \frac{RU}{US}.$$



Example 1

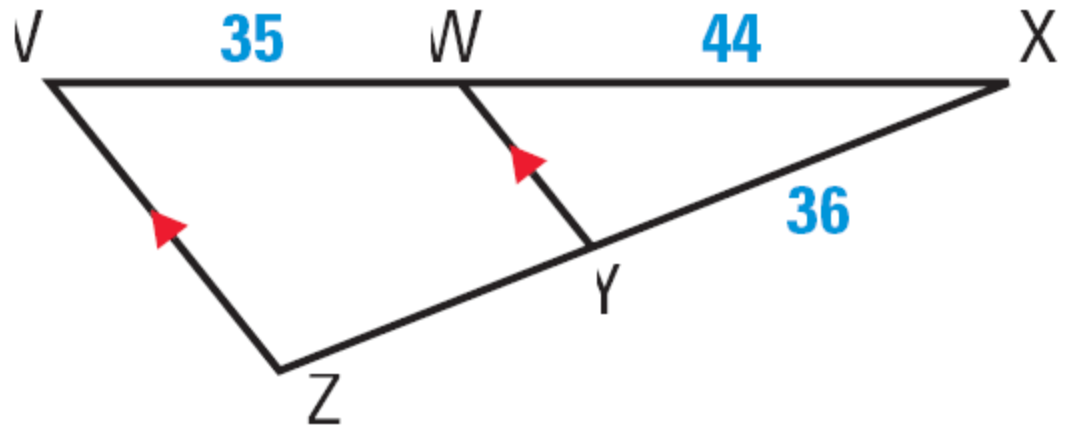
Find the length of \overline{YZ} .





Example 1

Find the length of \overline{YZ} .



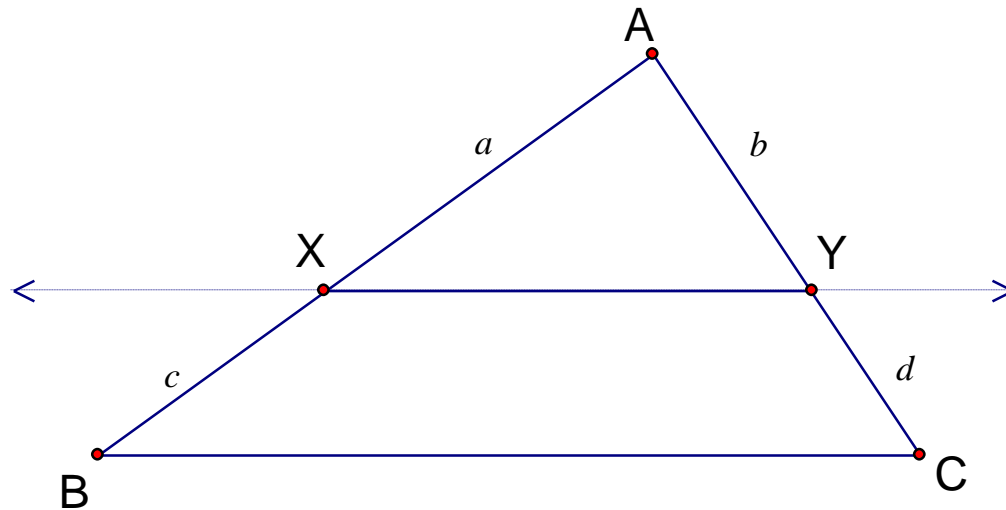
$$\frac{44}{36} = \frac{35}{x}$$

$$YZ = x = 28.64$$



Example 2

Given $\triangle ABC$ with $\overline{XY} \parallel \overline{BC}$, use algebra to show that $\frac{a}{c} = \frac{b}{d}$.





Example 2

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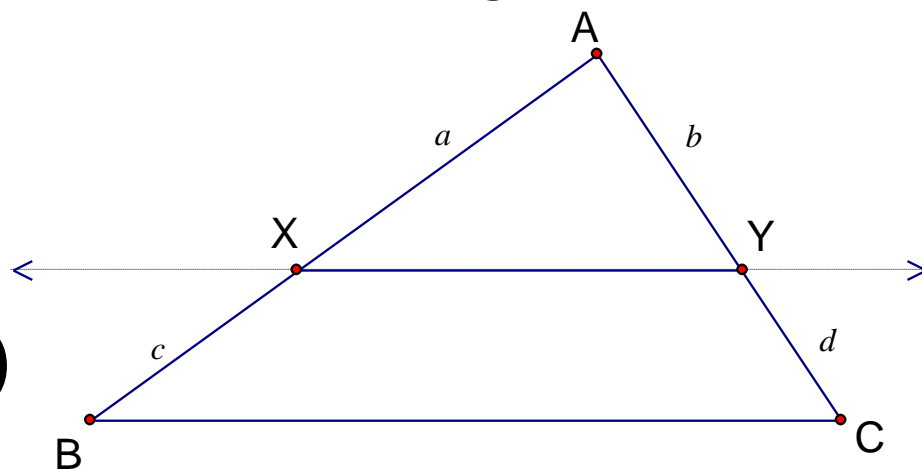
$$\frac{a}{a+c} = \frac{b}{b+d}$$

$$a(b+d) = b(a+c)$$

$$ab + ad = ab + bc$$

$$ad = bc$$

$$\frac{ad}{cd} = \frac{bc}{cd} \rightarrow \frac{a}{c} = \frac{b}{d} \quad \square$$

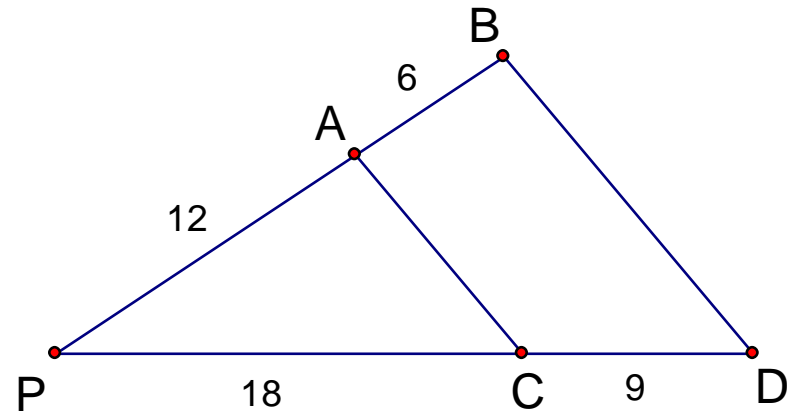




Investigation 2

In the diagram, notice that \overline{AC} divides the sides of the $\triangle PBD$ proportionally. In other words, $\frac{PA}{AB} = \frac{PC}{CD}$.

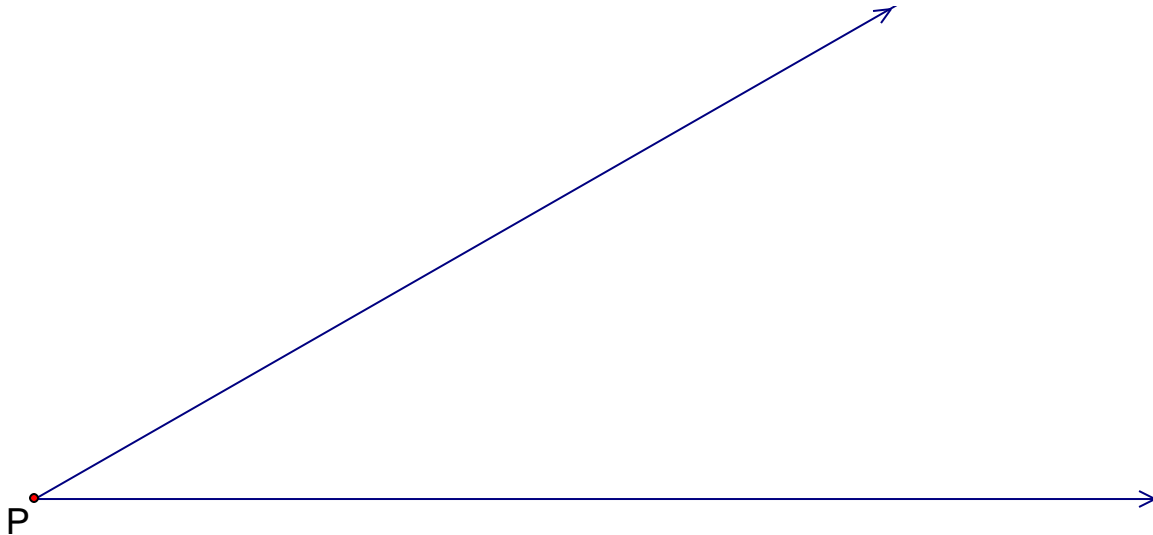
What relationship exists between \overline{AC} and \overline{BD} ? Are they parallel?





Investigation 2

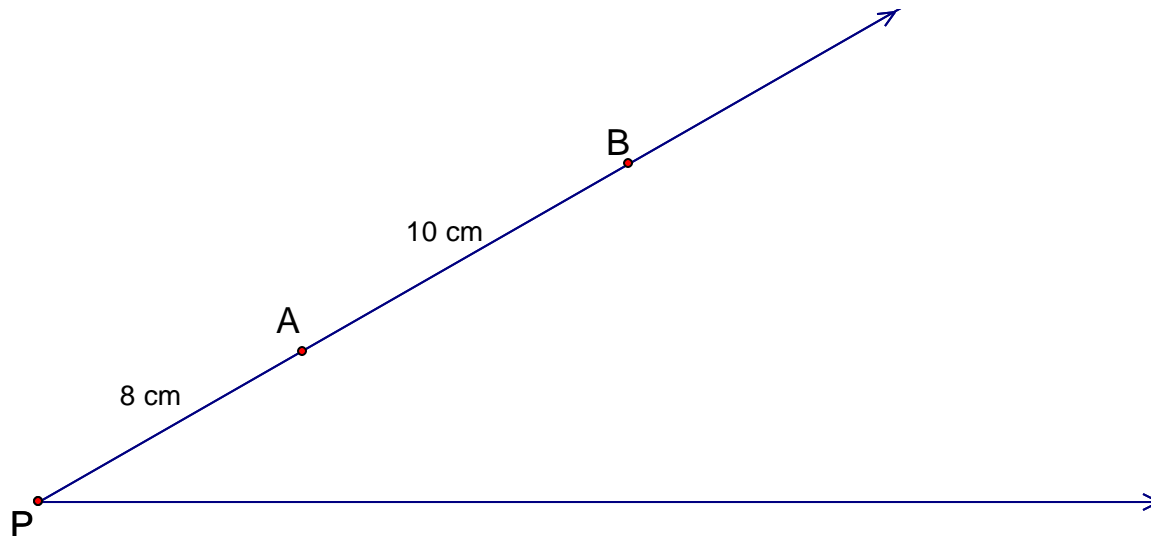
1. Draw an acute angle and label the vertex P .





Investigation 2

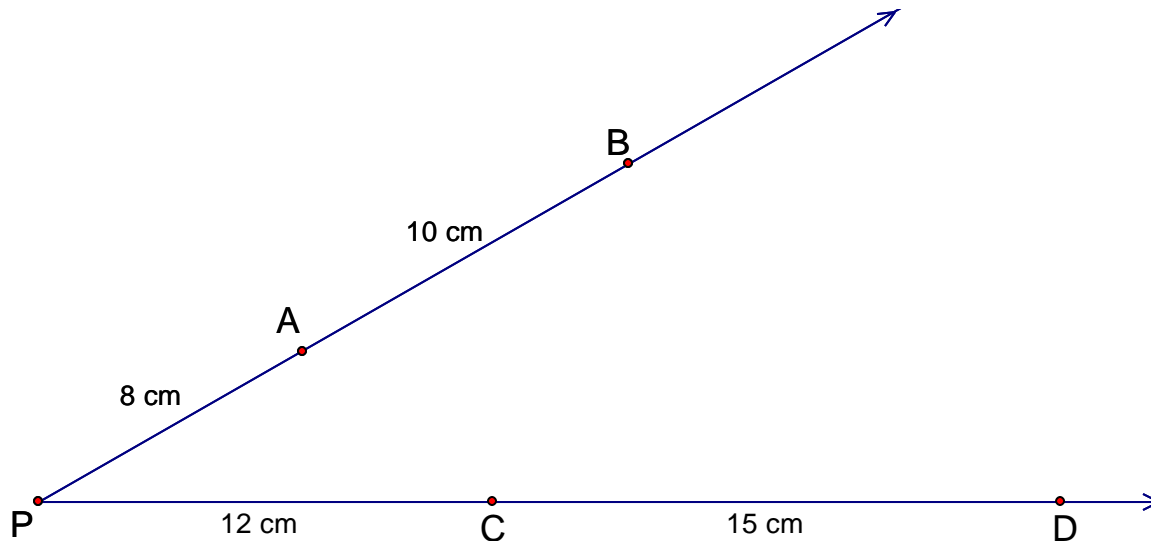
2. Beginning at point P , use your ruler to mark off lengths of 8 cm and 10 cm on one ray. Label the points A and B .





Investigation 2

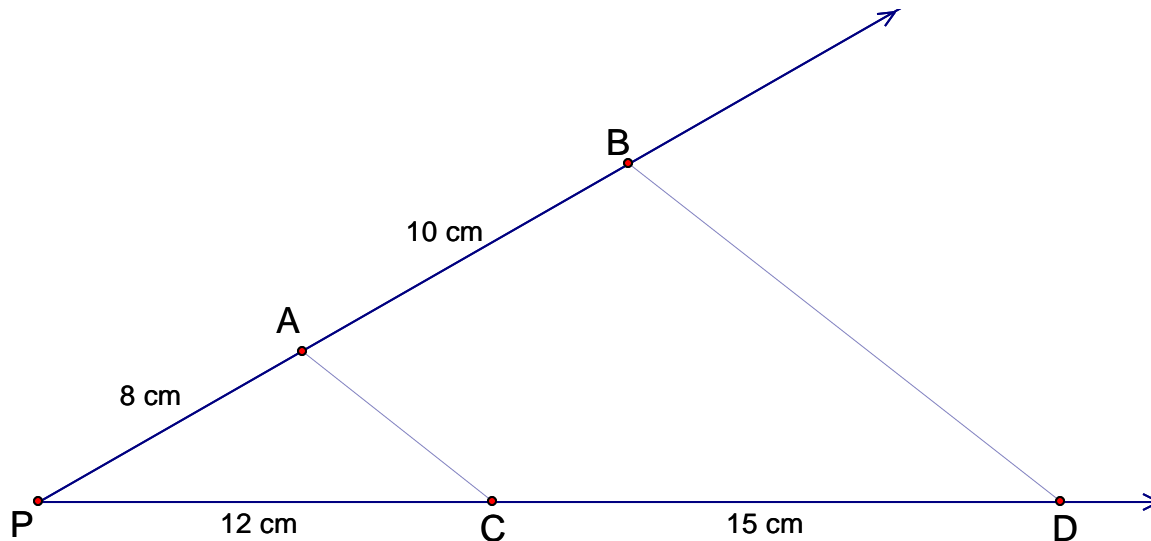
3. Mark off lengths of 12 cm and 15 cm on the other ray. Label the points C and D .
4. Notice that $\frac{8}{10} = \frac{12}{15}$.





Investigation 2

5. Draw \overline{AC} and \overline{BD} .
6. With a protractor, measure $\angle PAC$ and $\angle PBD$. Are \overline{AC} and \overline{BD} parallel?

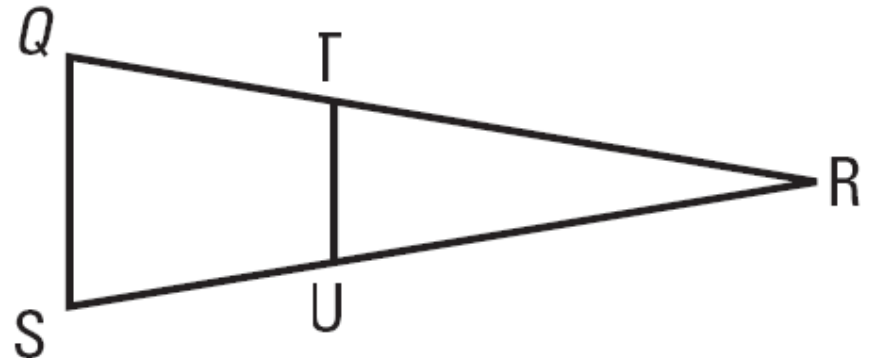




Proportionality Theorems!

Converse of the Triangle Proportionality Theorem

If a line divides two sides of a triangle proportionally, then it is parallel to the third side.

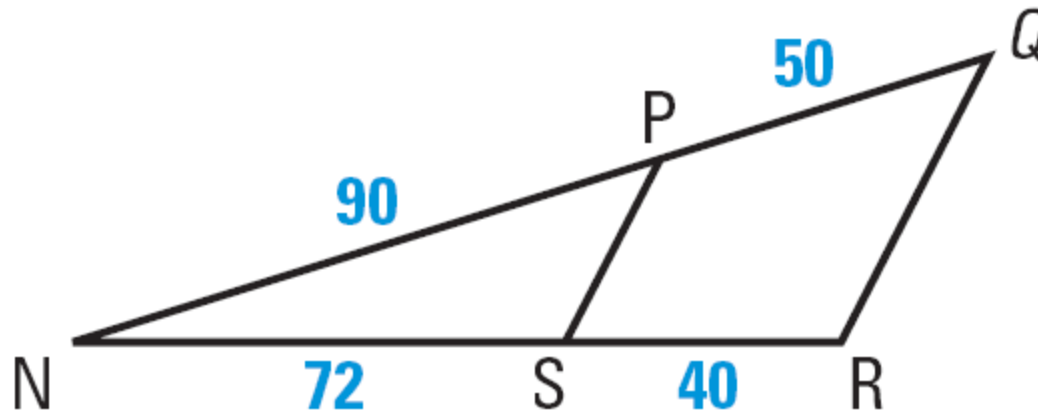


If $\frac{RT}{TQ} = \frac{RU}{US}$, then $\overline{TU} \parallel \overline{QS}$.



Example 3

Determine whether $\overline{PS} \parallel \overline{QR}$.

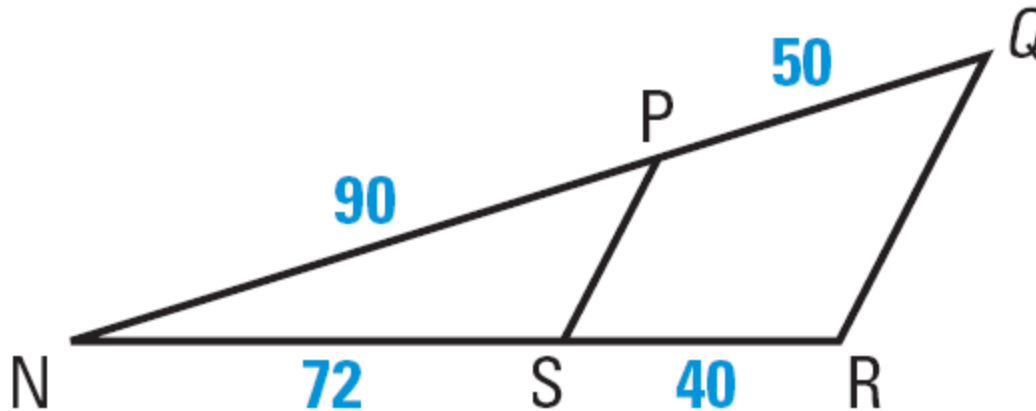




Example 3

Determine whether $\overline{PS} \parallel \overline{QR}$.

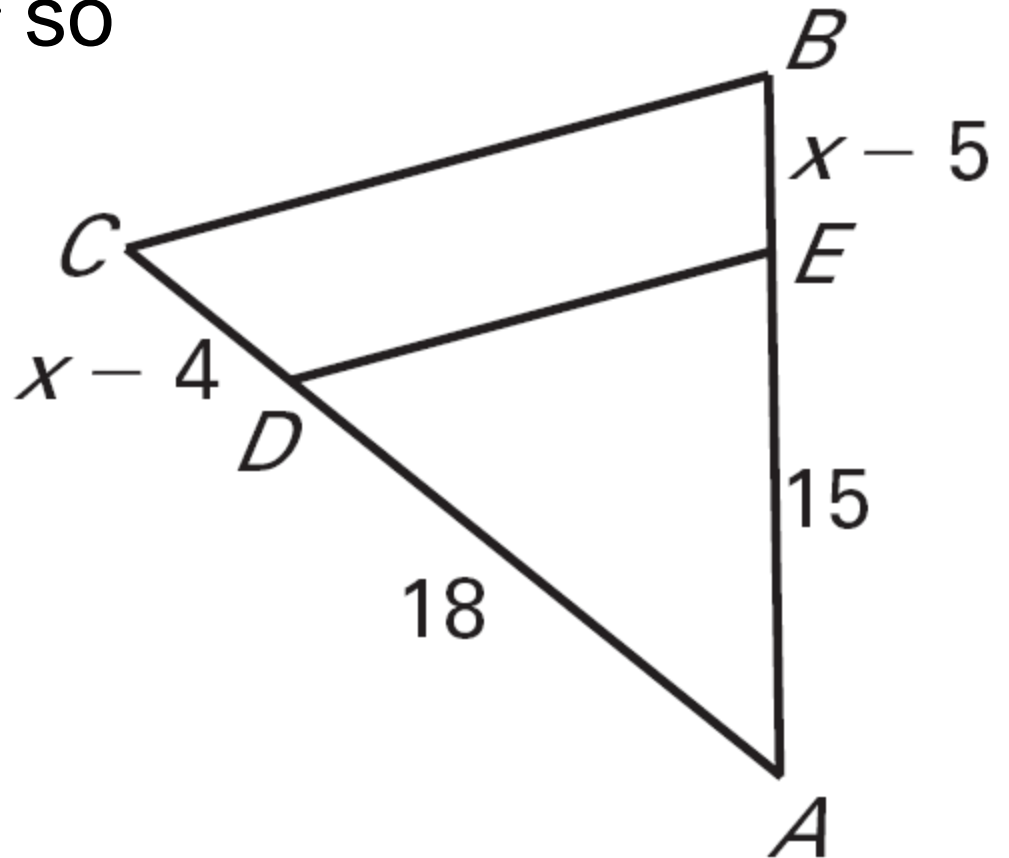
$$\frac{90}{72} = \frac{50}{40} \rightarrow \frac{5}{4} = \frac{5}{4} \text{ YES!}$$





Example 4

Find the value of x so
that $\overline{BC} \parallel \overline{ED}$.





Example 4

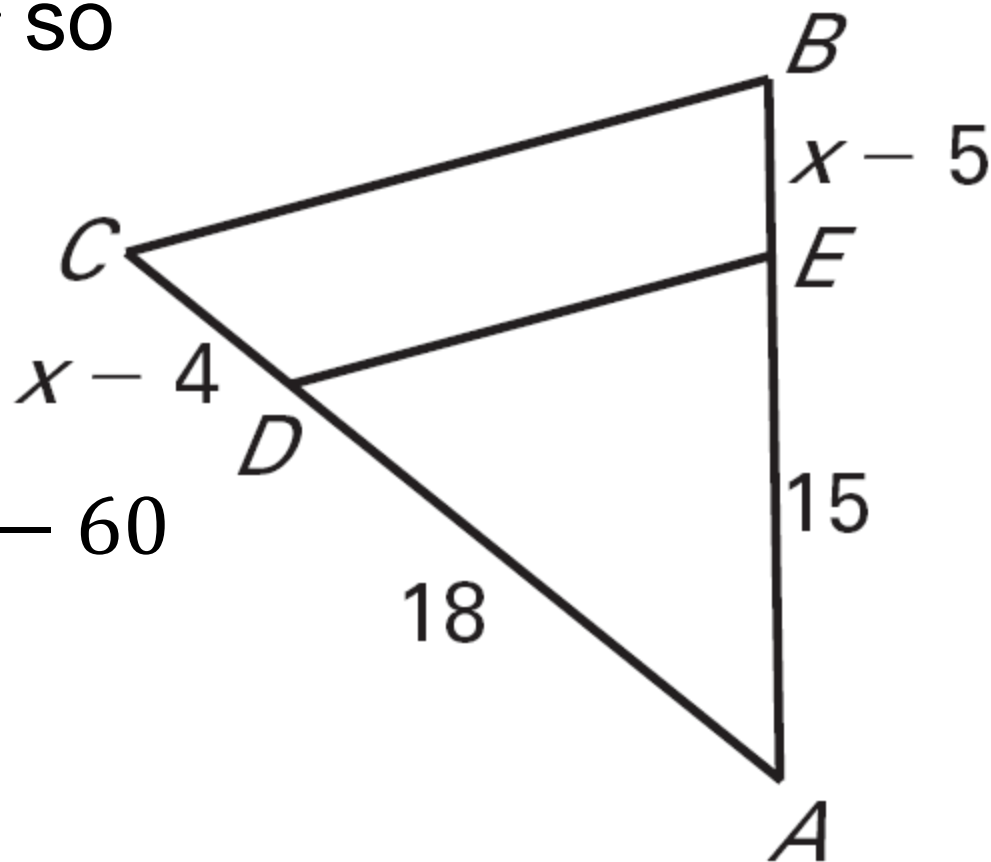
Find the value of x so that $\overline{BC} \parallel \overline{ED}$.

$$\frac{15}{18} = \frac{x-5}{x-4}$$

$$18x - 90 = 15x - 60$$

$$3x = 30$$

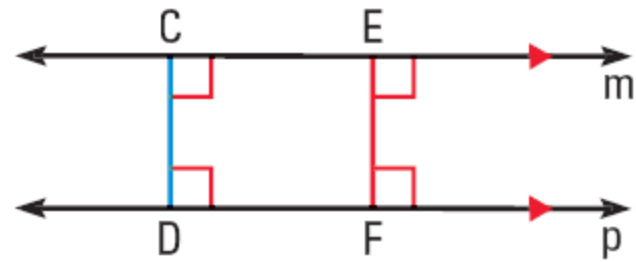
$$x = 10$$





Investigation 3

Recall that the distance between two parallel lines is always equal. This distance, however, must be measured along a perpendicular segment.

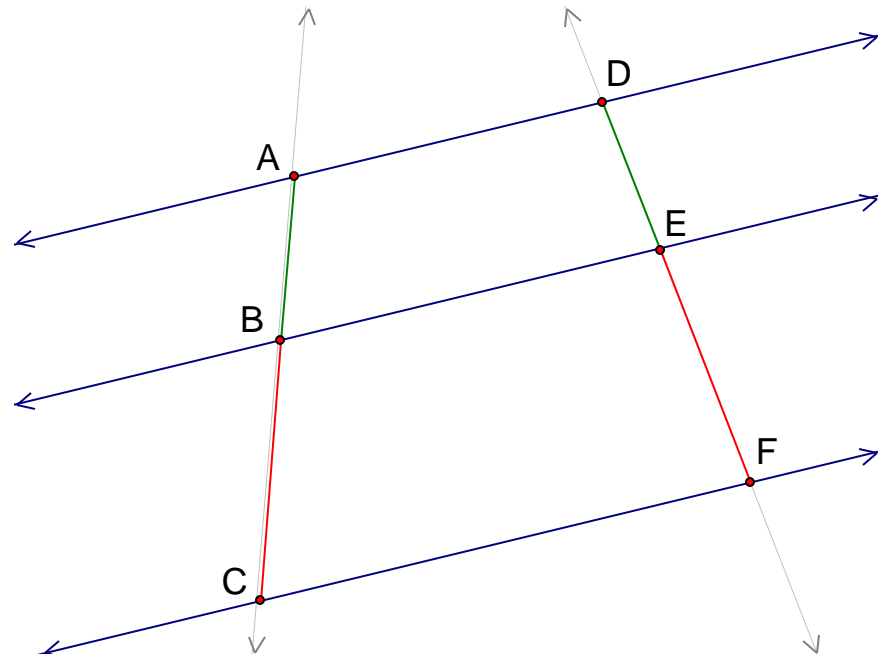


$$CD = EF$$



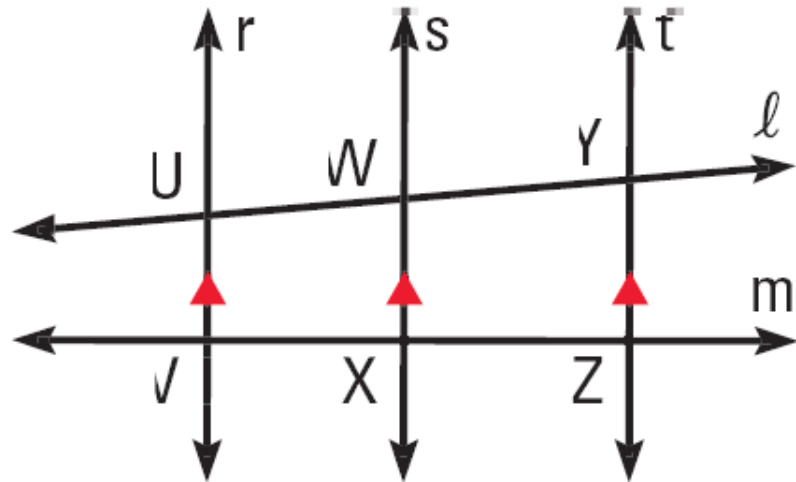
Investigation 3

But what if the distance is not perpendicular? Are these lengths still equal? Or does some other relationship exist?





Proportionality Theorems!



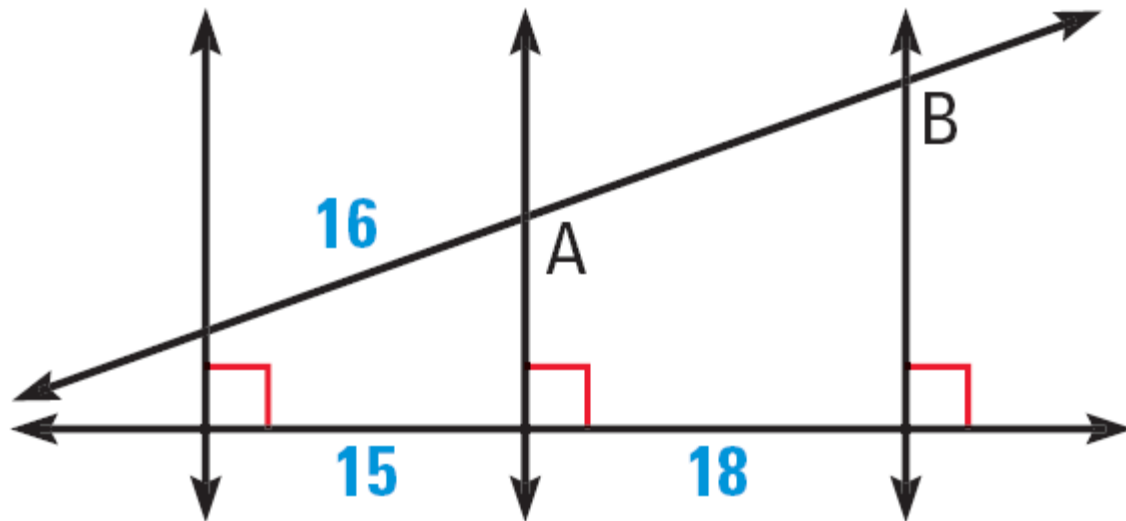
$$\frac{UW}{WY} = \frac{VX}{XZ}$$

If three parallel lines intersect two transversals, then they divide the transversals proportionally.



Example 5

Find the length of \overline{AB} .

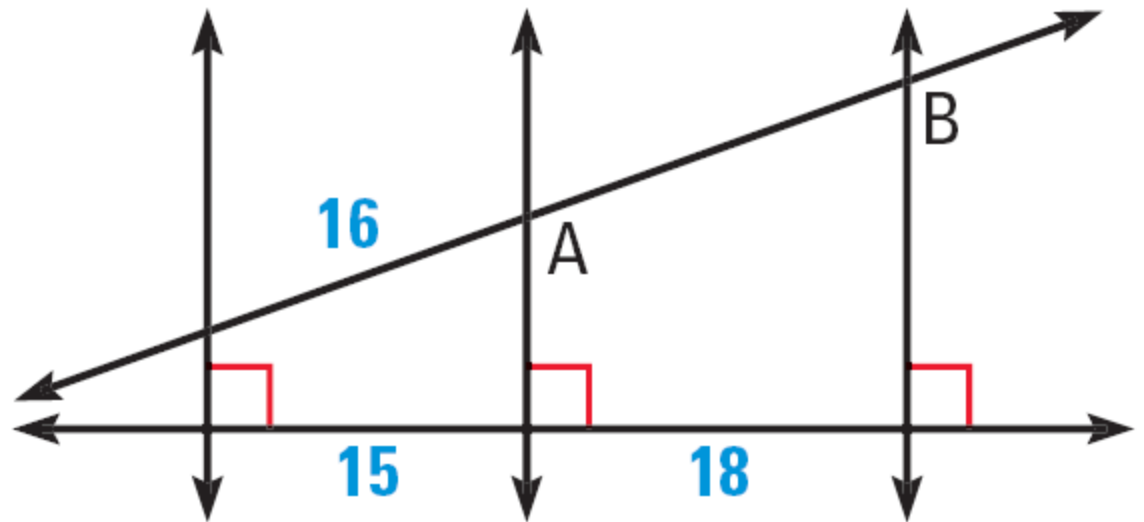




Example 5

Find the length of \overline{AB} .

$$\frac{15}{16} = \frac{18}{x}$$

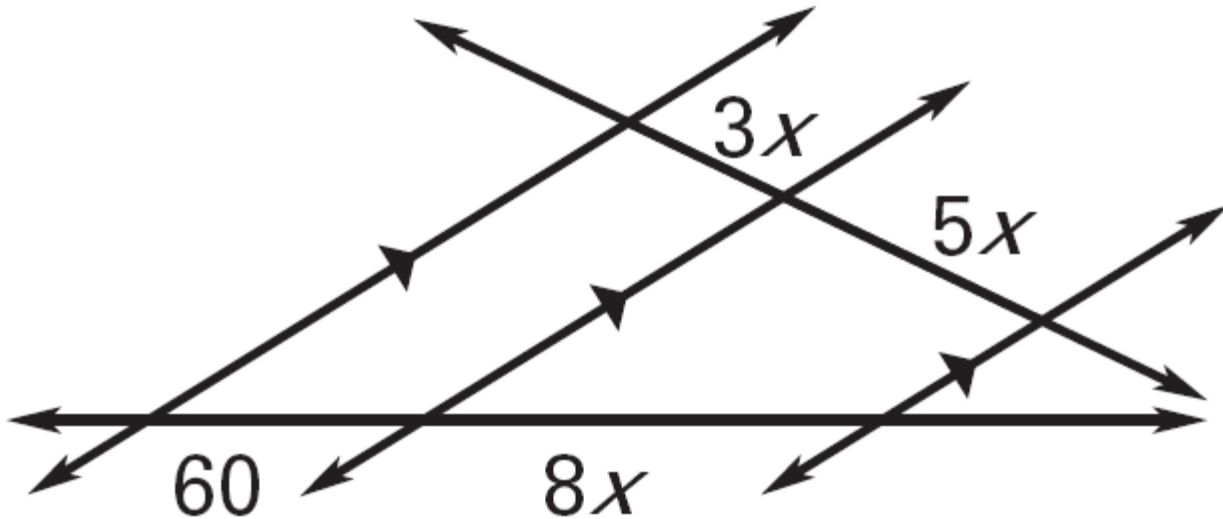


$$AB = x = 19.2$$



Example 6

Find the value of x .





Example 6

Find the value of x .

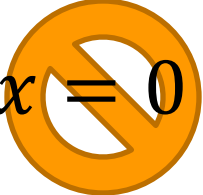
$$\frac{3x}{60} = \frac{5x}{8x}$$

$$24x^2 = 300x$$

$$24x^2 - 300x = 0$$

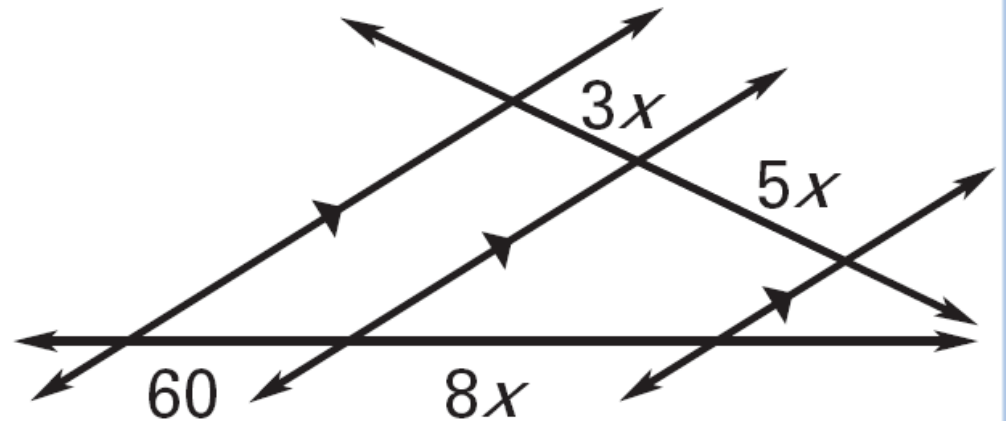
$$4x(6x - 75) = 0$$

$$4x = 0 \quad \text{or} \quad 6x - 75 = 0$$

$$x = 0$$


$$6x = 75$$

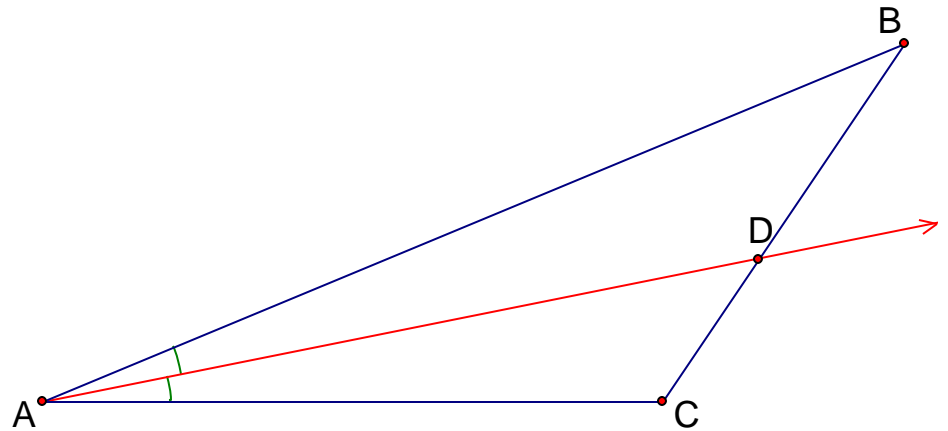
$$x = 12.5$$





Investigation 4

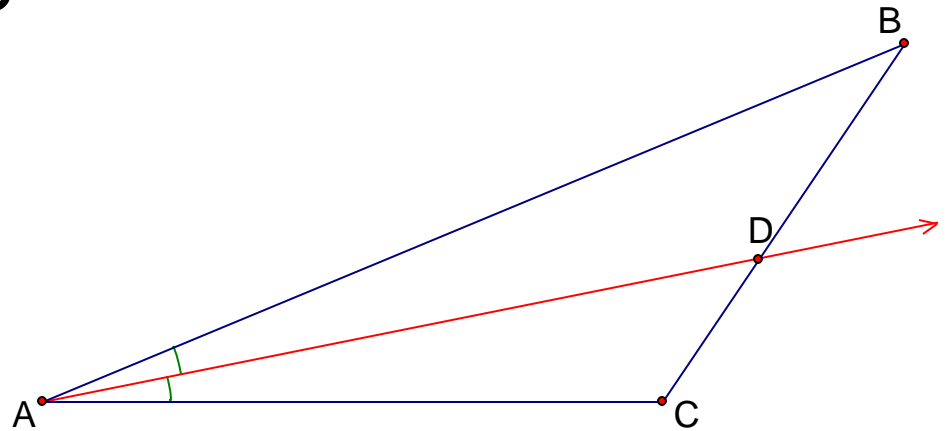
Recall that an angle bisector is a ray that divides an angle into two congruent parts.





Investigation 4

Notice that the angle bisector also divides the third side of the triangle into two parts. Are those parts congruent? Or is there some other relationship between them?

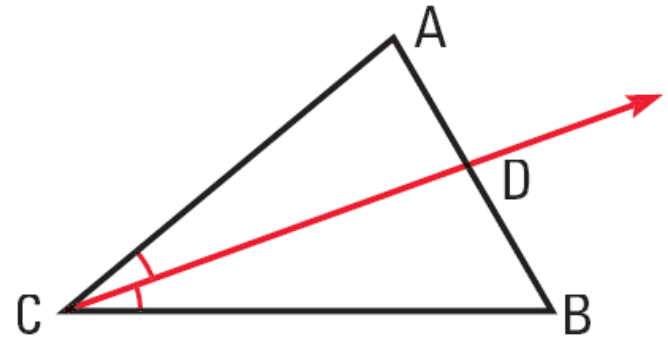




Proportionality Theorems!

Angle Bisector Proportionality Theorem

If a ray bisects an angle of a triangle, then it divides the opposite side into segments whose lengths are proportional to the other two sides.

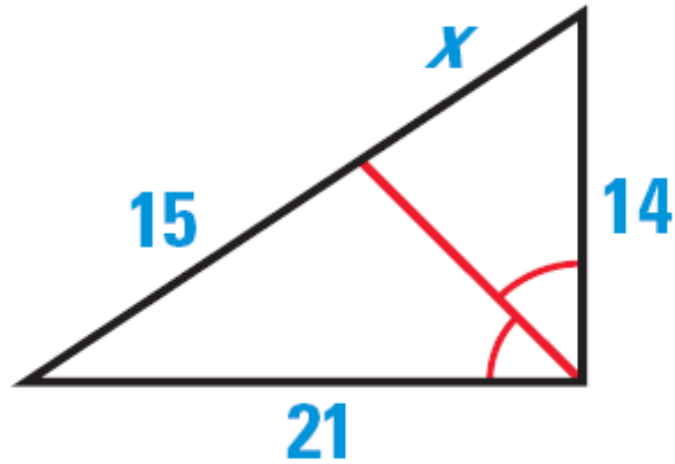


$$\frac{AD}{DB} = \frac{CA}{CB}$$



Example 7

Find the value of x .



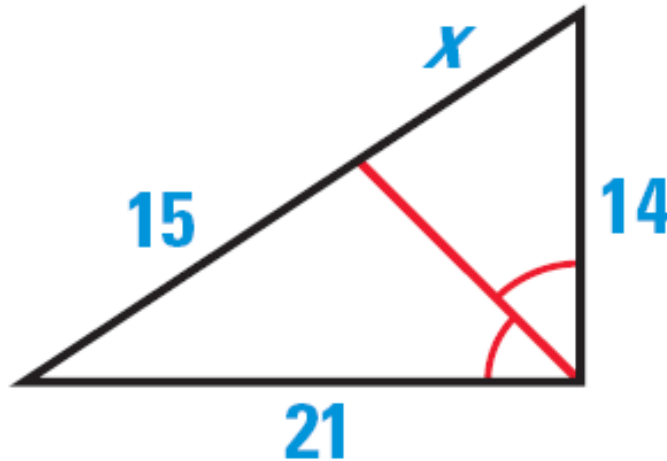


Example 7

Find the value of x .

$$\frac{21}{15} = \frac{14}{x}$$

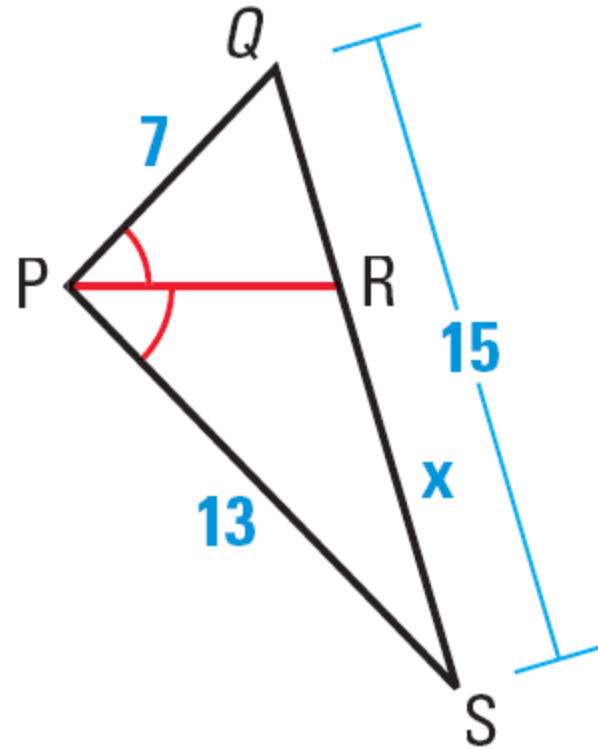
$$x = 10$$





Example 8

Find the value of x .





Example 8

Find the value of x .

$$\frac{13}{x} = \frac{7}{15 - x}$$

$$7x = 195 - 13x$$

$$20x = 195$$

$$x = 9.75$$

