Geometry

4

Unit Overview

In this unit you will extend your knowledge of two- and three-dimensional figures as you solve real-world problems involving angle measures, area, and volume. You will also study composite figures.

Key Terms

As you study this unit, add these and other terms to your math notebook. Include in your notes your prior knowledge of each word, as well as your experiences in using the word in different mathematical examples. If needed, ask for help in pronouncing new words and add information on pronunciation to your math notebook. It is important that you learn new terms and use them correctly in your class discussions and in your problem solutions.

Academic Vocabulary

- unique
- orientation
- decompose

Math Terms

- angle
- complementary angles
- adjacent angles
- vertical angles
- included angle
- similar figures
- corresponding parts
- plane
- circumference
- radius
- semicircle
- prism
- pyramid
- lateral face
- lateral area
- slant height
- complex solid
- vertex
- supplementary angles
- conjecture
- included side
- congruent
- circle
- center
- diameter
- composite figure
- inscribed figure
- net
- cross section
- right prism
- surface area
- regular polygon
- volume

ESSENTIAL QUESTIONS

? Why is it important to understand properties of angles and figures to solve problems?

? Why is it important to be able to relate two-dimensional drawings with three-dimensional figures?

EMBEDDED ASSESSMENTS

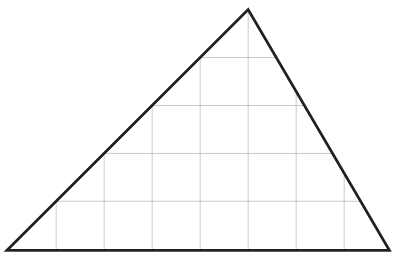
These assessments, following Activities 14, 17, and 19, will give you an opportunity to demonstrate how you can use your understanding of two- and three-dimensional figures to solve mathematical and real-world problems involving area and volume.

UNIT 4

Getting Ready

Write your answers on notebook paper. Show your work.

1. Write three ratios that are equivalent to $\frac{4}{6}$.
2. Solve each of the following equations.
   a. $3x + 4 = 21$
   b. $2x - 13 = 3x + 18$
   c. $\frac{6}{51} = \frac{3}{x}$
3. Sketch each of the following figures.
   a. square
   b. triangle
   c. parallelogram
   d. trapezoid
   e. right triangle
   f. $50°$ angle
4. Write an expression that can be used to determine the area of each figure.
   a. circle
   b. trapezoid
   c. parallelogram
   d. triangle
5. Determine the area of each plane figure described or pictured below.
   a. Circle with radius 5 inches. Round your answer to the nearest tenth.
   b. Right triangle with leg lengths 4 inches and 7 inches.
   c. Rectangle with length 6 inches and width 10 inches.
   d. Trapezoid with base lengths 3 inches and 7 inches and height 12 inches.
   e.



6. Compare and contrast the terms *complementary* and *supplementary* when referring to angles.
7. Think about triangles.
   a. List three ways to classify triangles by side length.
   b. List four ways to classify triangles by angle measure.
8. Polygons are named by the number of sides they have. Give the names of four different polygons and tell the number of sides each has.

Angle Pairs

Some of the Angles

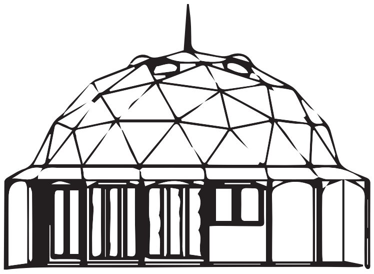
Lesson 13-1 Complementary, Supplementary, and Adjacent Angles

Learning Targets:

- Use facts about complementary, supplementary, and adjacent angles to write equations.
- Solve simple equations for an unknown angle in a figure.

SUGGESTED LEARNING STRATEGIES: Close Reading, Think Aloud, Create Representations, Marking the Text, Critique Reasoning, Sharing and Responding, Look for a Pattern

Architects think about angles, their measure, and special angle relationships when designing a building.



Two rays with a common endpoint form an ***angle***. The common endpoint is called the ***vertex***.

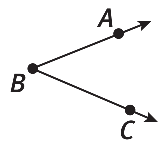
1. Angles are measured in degrees and can be classified by their relationship to the angle measures of 0°, 90°, and 180°. What angle measures characterize an acute angle, a right angle, an obtuse angle, and a straight angle?

Some angle relationships have special names. Two angles are ***complementary*** if the sum of their measures is 90°. Two angles are ***supplementary*** if the sum of their measures is 180°.

2. Compare and contrast the definitions of complementary and supplementary angles.

My Notes

READING MATH



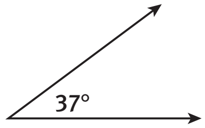
To read this angle, say "angle *ABC*," "angle *CBA*," or "angle *B*."

READING MATH

A small square at the vertex of an angle denotes a right angle.

My Notes

3. Name pairs of angles that form complementary or supplementary angles. Justify your choices.

a.


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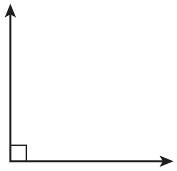

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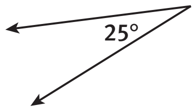

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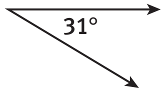

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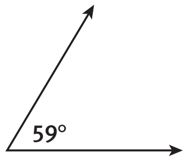

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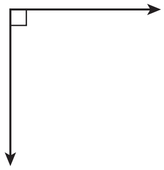

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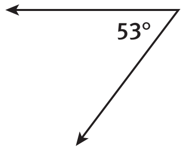

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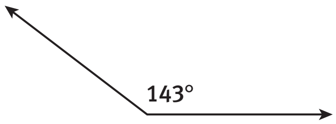


4. Find the ***complement*** and/or ***supplement*** of each angle or explain why it is not possible.

a. 32°

b. 113°

c. 68.9°

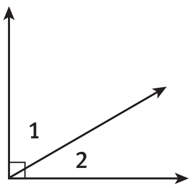
MATH TERMS

In a pair of complementary angles, each angle is the **complement** of the other.

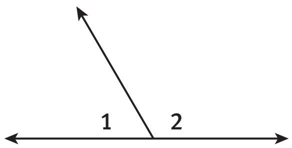
In a pair of supplementary angles, each angle is the **supplement** of the other.

My Notes

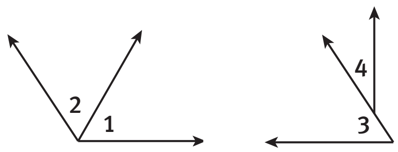
5. Why are angles 1 and 2 in this diagram complementary?



6. Why are angles 1 and 2 in this diagram supplementary?



7. Which of the following is a pair of ***adjacent angles***? Justify your answer.



MATH TERMS

Adjacent angles have a common side and vertex but no common interior points.

8. Angle A measures 32°.

a. Angle A and $\angle B$ are complementary. Find $m\angle B$.

b. Write an equation that illustrates the relationship between the measures of $\angle A$ and $\angle B$.

c. Solve your equation from Part b to verify your answer in Part a.

READING MATH

Read $m\angle B$ as "the measure of $\angle B$." This form indicates the size of the angle.

My Notes

9. **Model with mathematics.** Two angles are complementary. One measures $(2x)^\circ$ and the other measures 48°.
   a. Draw a pair of adjacent, complementary angles and label them using the given information.

   b. Write an equation to show the relationship between the two angles and solve for the value of x.

   c. Find the measure of both angles. Show your work.

10. Angle C measures 32°.
   a. Angle C and $\angle D$ are supplementary. Find $m\angle D$.

   b. Write an equation you could use to find the measure of $\angle D$.

   c. Solve your equation from Part b to verify your answer in Part a.

11. **Make use of structure.** Two angles are supplementary. One angle measures $(3x)^\circ$ and the other measures 123°.
   a. Draw a pair of adjacent supplementary angles and label them using the given information.

   b. Write an equation to show the relationship between the two angles. Solve the equation for x.

   c. Find the measure of both angles. Show your work.

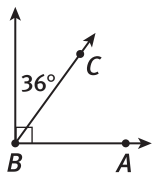
Check Your Understanding

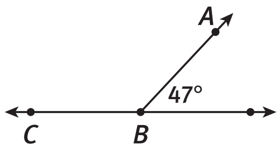
12. Explain how to find the complement and the supplement of an angle that measures 42°.

13. Reason quantitatively. What angle is its own complement? What angle is its own supplement? Explain.

LESSON 13-1 PRACTICE

Find $m\angle ABC$ in each diagram.

14. 

15. 

Find the complement and/or supplement of each angle. If it is not possible, explain.

	Angle	Complement	Supplement
16.	14°		
17.	98°		
18.	53.4°		

19. $\angle P$ and $\angle Q$ are complementary. $m\angle P = 52°$ and $m\angle Q = (3x + 2)°$. Find the value of x. Show your work.

20. $\angle TUV$ and $\angle MNO$ are supplementary. $m\angle TUV = 75°$ and $m\angle MNO = (5x)°$. Find the value of x and the measure of $\angle MNO$. Show your work.

21. $\angle ABC$ and $\angle TMI$ are complementary. $m\angle ABC = 32°$ and $m\angle TMI = (29x)°$. Find the measure of $\angle TMI$.

22. Make use of structure. $\angle ZTS$ and $\angle NRQ$ are supplementary. $m\angle ZTS = (5x - 3)°$ and $m\angle NRQ = (2x + 1)°$. Find the measure of each angle.

23. Model with mathematics. The supplement of an angle has a measure that is three times the angle. Write and solve an equation to find the measure of the angle and the measure of its supplement.

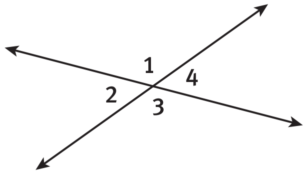
My Notes

Learning Targets:

- Write and solve equations using geometry concepts.
- Solve problems involving the sum of the angles in a triangle.
- Solve equations involving angle relationships.

SUGGESTED LEARNING STRATEGIES: Predict and Confirm, Think-Pair-Share, Create Representations, Identify a Subtask

Vertical angles are pairs of nonadjacent angles formed when two lines intersect. They share a vertex but have no common rays. $\angle 1$ and $\angle 3$ are vertical angles, as are $\angle 2$ and $\angle 4$.



1. Visually inspect the diagram above. Then make a ***conjecture*** about the measures of a pair of vertical angles.

MATH TERMS

A **conjecture** is a statement that seems to be true but has not been proven to be either true or false.

2. Find $m\angle 1$ and $m\angle 3$ above using your protractor.

3. Find the measure of $\angle 2$ and $\angle 4$ using geometry concepts.

4. **Construct viable arguments.** Do your answers to Items 2 and 3 support your conjecture? Support your reasoning.

5. **Reason quantitatively.** Two angles are vertical angles. One angle measures $(3x)^\circ$ and the other measures 63°.

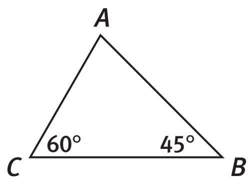
a. Draw the pair of vertical angles and label them using the given information.

b. Write an equation to show the relationship between the two angles. Then solve the equation for x.

My Notes

A *triangle* is a closed figure made of three line segments that meet only at their endpoints. The sum of the angle measures of a triangle is 180°.

6. Triangle *ABC* is shown.

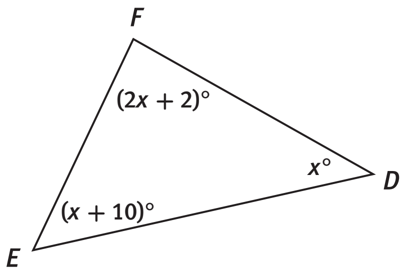


a. Find the measure of ∠*A*. Explain your answer.

b. Write an equation that illustrates the relationship between the measures of ∠*A*, ∠*B*, and ∠*C*.

c. Solve your equation from Part b to verify your answer in Part a.

7. Reason quantitatively. Triangle *DEF* is shown.



a. Write an equation that illustrates the relationship between the measures of ∠*D*, ∠*E*, and ∠*F*.

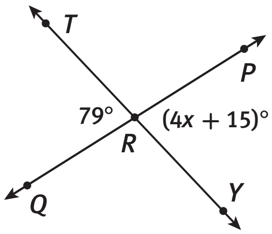
b. Solve the equation to find the value of *x*.

c. Find the measure of all three angles of △*DEF*.

My Notes

Check Your Understanding

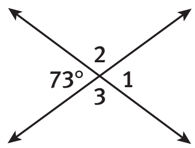
8. Explain how to find the value of x in the diagram shown.



9. **Construct viable arguments.** If you know the measure of one non-right angle of a right triangle, can you always find the measure of the third angle? Explain.

LESSON 13-2 PRACTICE

Use the diagram for Items 10–12.

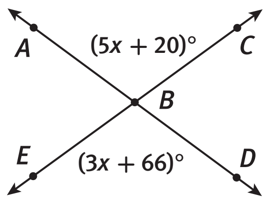


10. Find the measure of $\angle 1$.

11. Find the measure of $\angle 2$.

12. Find the measure of $\angle 3$.

Use the diagram for Items 13–15.

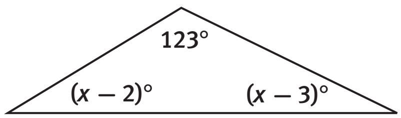


13. Find x.

14. Find the measure of $\angle ABC$.

15. Find the measure of $\angle ABE$.

16. **Reason quantitatively.** Find the measure of each of the angles in the triangle shown.



17. In triangle GKJ, $m\angle G = 72°$ and $m\angle J = 28°$. Write and solve an equation to find the measure of $\angle K$.

18. In a right triangle, one of the angles measures 29°. What are the measures of the other angles in the triangle? Explain.

19. **Reason quantitatively.** In an isosceles triangle, the two base angles are congruent. One of the base angles measures 36°. What are the measures of the other two angles in the triangle? Support your answer with words and equations.

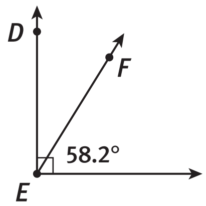
ACTIVITY 13 PRACTICE

Write your answers on notebook paper. Show your work.

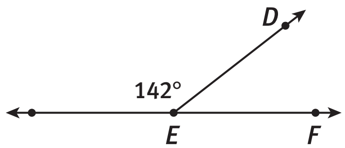
Lesson 13-1

Find the measure of angle DEF in each diagram.

1.



2.



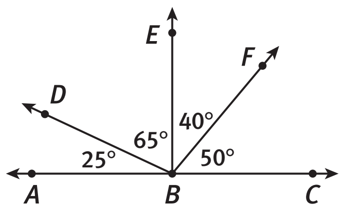
3. $\angle JKL$ and $\angle RST$ are complementary. $m\angle JKL = 36°$ and $m\angle RST = (x + 15)°$. Find the value of x and the measure of $\angle RST$.

4. $\angle SUN$ and $\angle CAT$ are supplementary. $m\angle SUN = (2x)°$ and $m\angle CAT = 142°$. Find the value of x and the measure of $\angle SUN$.

5. $\angle P$ and $\angle Q$ are supplementary. $m\angle P = (5x + 3)°$ and $m\angle Q = (x + 3)°$. What is the measure of $\angle Q$?

A. 17° **B.** 26°
C. 29° **D.** 32°

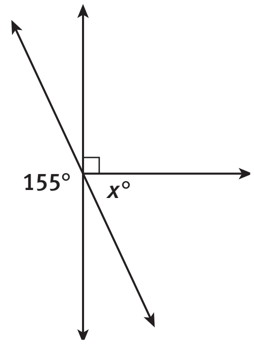
6. In the diagram shown, which angle pairs form complementary angles?



A. $\angle ABD$ and $\angle CBF$ **B.** $\angle DBE$ and $\angle FBE$
C. $\angle ABD$ and $\angle DBC$ **D.** $\angle EBF$ and $\angle FBC$

Lesson 13-2

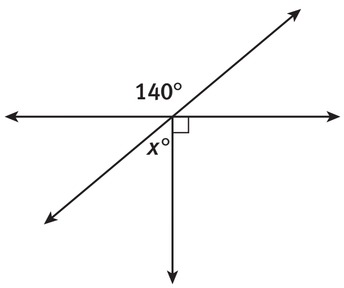
7. Find the measures of the vertical angles in the diagram. Then find x in the diagram.



8. $\angle B$ and $\angle D$ are vertical angles. $m\angle B = (2x + 1)°$ and $m\angle D = (x + 36)°$.

Find the measure of each angle.

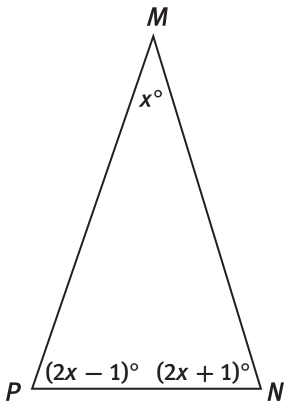
9. Find x in the diagram shown.



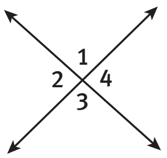
10. In right triangle TWZ, $\angle W$ is a right angle and $m\angle Z = 41°$. Find $m\angle T$.

11. In triangle QRS, $\angle Q$ and $\angle S$ have the same measure. If $m\angle R = 58°$, find the measures of $\angle Q$ and $\angle S$.

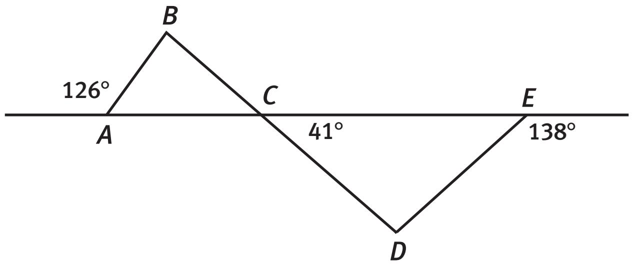
12. For triangle MNP shown, find the value of x and the measure of each of the three angles.



13. In the diagram, $m\angle 2 = 86°$. Erica says that $m\angle 3 = 86°$ because the diagram shows vertical angles and all vertical angles are congruent. Is her statement reasonable? Explain.



14. Use the diagram shown to find the measures of the each of the angles of $\triangle ABC$ and $\triangle CDE$.



15. The angles of an equilateral triangle are congruent. What are the measures of the angles?

16. In isosceles triangle ABC, $\angle B$ is a right angle. What are the measures of angles A, B, and C? Justify your answer.

MATHEMATICAL PRACTICES
Reason Abstractly

17. Consider the angle pair classifications from this activity: adjacent, complementary, supplementary, and vertical angles. Can two angles fit all four categories? Explain.

Triangle Measurements

Rigid Bridges

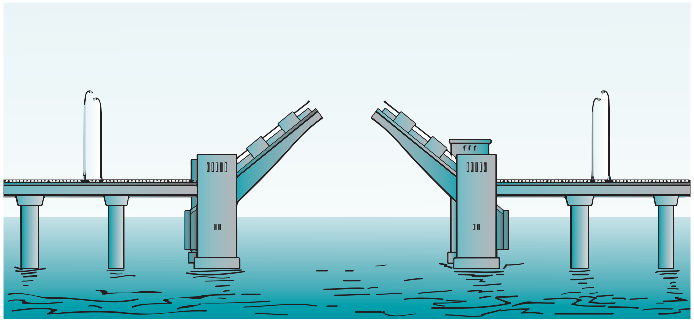
Lesson 14-1 Draw Triangles from Side Lengths

Learning Targets:

- Decide if three side lengths determine a triangle.
- Draw a triangle given measures of sides.

SUGGESTED LEARNING STRATEGIES: Create Representations, Marking the Text, Use Manipulatives, Predict and Confirm, Shared Reading, Visualization

When the sides of a drawbridge are raised or lowered, the sides move at the same rate.



1. Look at the drawbridge.

a. What will happen if the sum of the lengths of the moving sides is greater than the opening? Draw an illustration to explain your answer.

b. What will happen if the sum of the lengths of the moving sides is equal to the length of the opening? Draw an illustration to explain your answer.

c. What will happen if the sum of the lengths of the moving portions of the bridge is less than the length of the opening? Draw an illustration to explain your answer.

My Notes

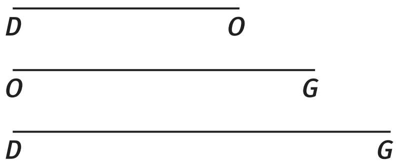
CONNECT TO ENGINEERING

A drawbridge is a bridge that can be drawn up, let down, or drawn aside, to permit or prevent ships and other watercraft from passing below it.

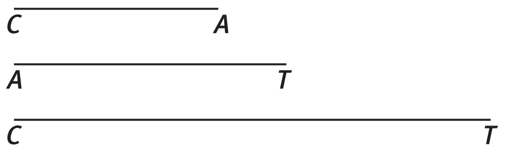
My Notes

2. Work with a partner or with your group. Use paper or string to cut out segments that are the same length as the segments shown. If possible, connect the segments at the endpoints to create a triangle and draw the triangle. If it is not possible, explain.

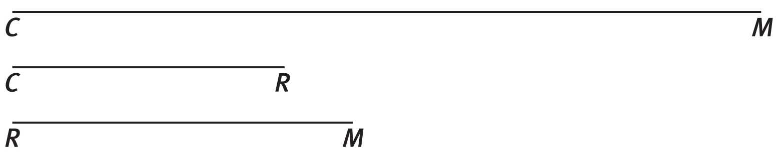
a. $\triangle DOG$



b. $\triangle CAT$



c. $\triangle RCM$



3. Use appropriate tools strategically. Measure the lengths in centimeters of the segments in Item 2. Then write an equation or inequality to compare the sum of two side lengths to the longest side length.

4. Based on your results for Items 2 and 3, make a conjecture about what side lengths can form a triangle. As you share your ideas with your group about making the conjecture, be sure to explain your thoughts using precise language and details to help the group understand your ideas and reasoning.

My Notes

5. Use a ruler to draw each triangle described below. You may want to cut out segments and use the segments to help form the triangle.

a. Draw a triangle with side lengths that each measure 3 centimeters. Can you form more than one triangle with the given side lengths? Explain.

b. Draw a triangle with side lengths that are 3 centimeters, 3 centimeters, and 5 centimeters long. Can you form more than one triangle with the given side lengths? Explain.

c. Draw a triangle with side lengths that are 3 centimeters, 4 centimeters, and 5 centimeters long. Can you form more than one triangle with the given side lengths? Explain.

6. Construct viable arguments. When a triangle is formed from three given side lengths, is the triangle a ***unique*** triangle or can more than one triangle be formed using those same side lengths? Explain.

ACADEMIC VOCABULARY

The term ***unique*** means "only" or "single." In geometry, a unique triangle is a triangle that can be drawn in only one way.

My Notes

Check Your Understanding

7. Is it possible to draw a triangle with sides that are 4 inches, 5 inches, and 8 inches long? Justify your answer.

8. Draw a triangle with sides that are 2 inches, 2 inches, and 3 inches long. Can you form more than one triangle with the given side lengths? Explain.

Lesson 14-1 PRACTICE

Determine whether it is possible to draw a triangle with the given side lengths. Justify your answers.

9. 7 feet, 5 feet, and 2 feet

10. 3 meters, 4 meters, and 6 meters

11. 5 inches, 7 inches, and 15 inches

12. 6 yards, 12 yards, and 6 yards

13. 8 millimeters, 11 millimeters, and 4 millimeters

14. 30 feet, 9 feet, and 16 feet

15. **Express regularity in repeated reasoning.** To check that three side lengths can form a triangle, you only have to check the sum of the two shortest lengths. Explain why.

16. **Look for and make use of structure.** Two sides of a triangle measure 3 inches and 6 inches. What is one possible length for the third side of the triangle? Explain.

17. Draw a triangle with side lengths that are 6 centimeters, 8 centimeters, and 10 centimeters long.

18. Draw a triangle with side lengths that are 2 centimeters, 6 centimeters, and 7 centimeters long.

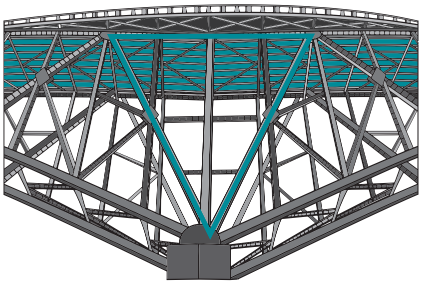
19. **Express regularity in repeated reasoning.** A triangle is formed using three given side lengths. Do these side lengths always form a unique triangle? Explain.

My Notes

Learning Targets:

- Draw a triangle given measures of angles and/or sides.
- Recognize when given conditions determine a unique triangle, more than one triangle, or no triangle.

SUGGESTED LEARNING STRATEGIES: Create Representations, Graphic Organizer, Think-Pair-Share, Visualization

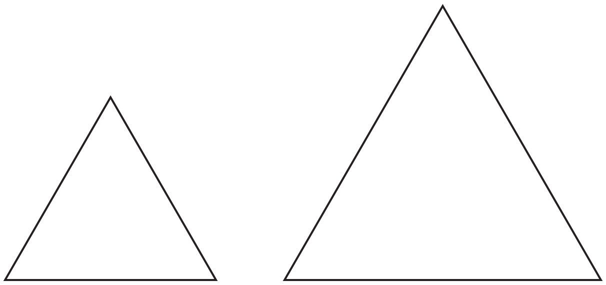


Triangles are rigid shapes. Structures, such as bridges and towers, are reinforced with triangles that give the structures added strength. Notice how triangles within triangles are used to support the bridge in the picture.

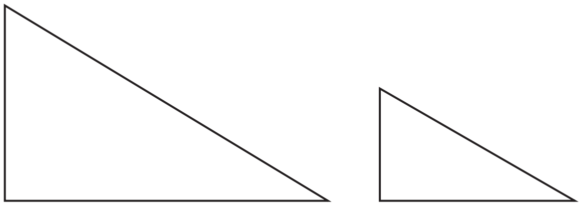
In the last lesson, you learned that three given side lengths determine a unique triangle. Other conditions can also determine a unique triangle.

1. Use a protractor to measure each angle in the triangles.

   a.



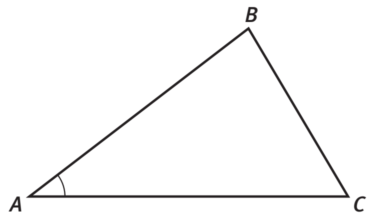
   b.



My Notes

2. Reason abstractly. When a triangle is formed from three given angle measures, is the triangle a unique triangle, or can more than one triangle be formed using those same angle measures? Explain.

READING MATH



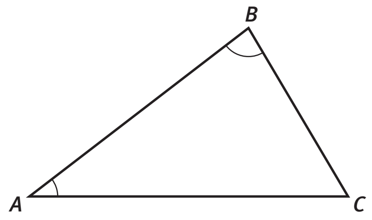
Angle *A* is the **included angle** between sides *AB* and *AC*.

Two sides of a triangle form an angle called an ***included angle***.

3. a. Use a ruler and protractor. Draw a triangle with two sides that measure 4 centimeters each and an included angle of 30°.

b. Is there only one triangle that fits the description given in Part a? Explain.

READING MATH



Side *AB* is the **included side** between angles *A* and *B*.

An ***included side*** is the side between two angles.

4. a. Use a ruler and protractor. Draw a triangle with two angles that each measure 30° and an included side that measures 5 centimeters.

b. Is there only one triangle that fits the description given in Part a? Explain.

5. **Construct viable arguments.** Decide if the given conditions create a unique triangle.
   a. When a triangle is formed from two side lengths and an included angle measure, is the triangle a unique triangle, or can more than one triangle be formed? Explain.

   b. When a triangle is formed using two given angle measures and an included side length, is the triangle a unique triangle, or can more than one triangle be formed? Explain.

Two known angle measures and the length of a non-included side also form a unique triangle. However, two given side lengths and the measure of a non-included angle may or may not form a unique triangle.

6. Two angles of a triangle measure 40° and 110°. The side opposite the 40° angle is 6 inches long. Can more than one triangle be drawn with these conditions? Explain.

7. Two sides of a triangle are 4 inches and 7 inches long. The included angle has a measure of 35°. Can more than one triangle be drawn with these conditions? Explain.

8. **Make use of structure.** Two angles of a triangle each measure 70° and 55°. The side adjacent to the 70° angle is 3 inches long. Can more than one triangle be drawn with these conditions? Justify your answer.

My Notes

DISCUSSION GROUP TIPS

As your group explores and discusses the relationship between the sides and angles of triangles in the Check Your Understanding problems, demonstrate listening comprehension of what each group member says by taking notes on their contributions. Ask and answer questions to clearly aid understanding of all group members' ideas.

Check Your Understanding

9. Is it possible to draw a unique triangle with angle measures of 35°, 65°, and 100°? Explain.

10. Is it possible to draw a unique triangle with two sides that are each 5 centimeters long and an included angle that measures 40°? Explain.

LESSON 14-2 PRACTICE

Determine whether the given conditions determine a unique triangle or more than one triangle. Justify your answers.

11. Two angles of a triangle measure 40° and 75°. The side between the angles is 3 feet long.

12. Two angles of a triangle each measure 55°. The side opposite one of the 55° angles is 2 meters long.

13. The angles of a triangle measure 40°, 60°, and 80°.

14. The sides of a triangle are 5 inches, 12 inches, and 13 inches long.

15. Two sides of a triangle are 10 centimeters and 13 centimeters long. One of the nonincluded angles measures 95°.

16. Two angles of a triangle measure 61° and 48°. One of the sides formed by the 48° angle is 15 millimeters long.

17. The two sides that form the right angle of a right triangle are 9 centimeters and 12 centimeters long.

18. **Look for and make use of structure.** If the measures of the angles of a triangle are known, is the length of one side of the triangle sufficient to determine if the triangle formed is a unique triangle? Explain.

ACTIVITY 14 PRACTICE

Write your answers on a separate piece of paper. Show your work.

Lesson 14-1

For 1–6, determine whether it is possible to draw a triangle with the given side lengths. Justify your answers.

1. 8 feet, 5 feet, and 9 feet
2. 3 centimeters, 2 centimeters, and 7 centimeters
3. 14 inches, 6 inches, and 10 inches
4. 3 yards, 2 yards, and 5 yards
5. 1.5 meters, 1.1 meters, and 2 meters
6. 42 feet, 18 feet, and 23 feet
7. Draw a triangle with side lengths that are 3 inches, 5 inches, and 6 inches long. Is this the only triangle that you can draw using these side lengths? Explain.
8. Multiple Choice: Which of the following cannot be the side lengths of a triangle?
   A. 4 inches, 4 inches, and 4 inches
   B. 3 inches, 3 inches, and 5 inches
   C. 15 centimeters, 16 centimeters, and 17 centimeters
   D. 2 centimeters, 10 centimeters, and 20 centimeters
9. Multiple Choice: Which of the following could be the length of the third side of a triangle with side lengths 2 feet and 10 feet?
   A. 12 feet
   B. 20 feet
   C. 11 feet
   D. 8 feet
10. **Express regularity in repeated reasoning.** Explain how to determine whether a triangle can be formed from three given segment lengths.

Lesson 14-2

Determine whether the given conditions determine a unique triangle or more than one triangle. Justify your answers.

11. Two angles of a triangle measure 36° and 102°. One of the sides formed by the 36° angle is 9 inches long.
12. The angles of a triangle measure 25°, 73°, and 82°.
13. Two angles of a triangle measure 86° and 67°. The side between the angles is 2.5 meters long.
14. Two sides of a triangle are 3.6 meters and 5.2 meters long. One of the non-included angles measures 48°.
15. The two sides that form the right angle of a right triangle are 6 inches and 4 inches long.
16. The sides of a triangle are 10 centimeters, 12 centimeters, and 14 centimeters long.
17. Two angles of a triangle each measure 62°. The side opposite one of the 62° angles is 34 inches long.

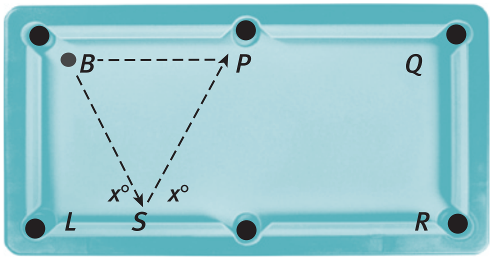
MATHEMATICAL PRACTICES

Look for and Make Use of Structure

18. **a.** If the lengths of two sides of a triangle are known, is the measure of one of the angles of the triangle enough to determine a unique triangle? Explain.
    b. If the measures of two angles of a triangle are known, is the length of one side of the triangle sufficient to determine if the triangle formed is a unique triangle? Explain.

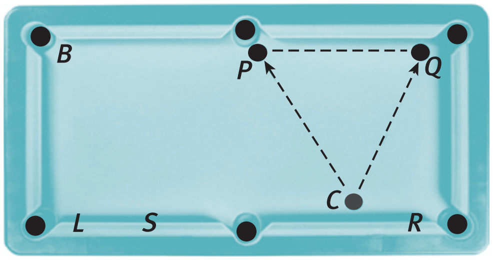
Angles and Triangles
POOL ANGLES

Pool is a game that requires talent and a knowledge of angles to play well. Bank and kick shots involve hitting a ball (B) into a rail of a rectangular pool table, and then into a pocket, somewhere on the other side of the table. As shown below, the angle at which the ball hits the side is equal to the angle at which it leaves the side.



1. Name a pair of adjacent, supplementary angles in the diagram.
2. Angle QPS is supplementary to $\angle PSR$. Also, $m\angle QPS = (2x + 12)°$ and $m\angle PSR$ is $x°$. Answer each question below and show your work.
   a. Find the value of x.
   b. Find $m\angle PSL$.
3. The measure of $\angle BLS$ is $90°$. Explain why $\angle LSB$ and $\angle LBS$ must be complementary.
4. The measures of segment BS and segment PS are both 4.5 feet and $m\angle PBS = 56°$.
   a. Find the measure of $\angle BSP$. Item 2 says that "Angle QPS is supplementary to $\angle PSR$."
   b. Is $\triangle BPS$ a unique triangle, or can more than one triangle be formed using the given segment lengths and angle measures? Explain.

Another type of pool shot involves aiming the ball at point C directly at a pocket, as shown below.



5. The measure of $\angle RQC = (x + 12)^\circ$ and $m\angle PQC = (2x)^\circ$. Set up and solve an equation to find the value of x.

6. In $\triangle QPC$, $m\angle C = (x + 11)^\circ$, $m\angle Q = (2x + 6)^\circ$, and $m\angle P = 70^\circ$.
- **a.** Set up and solve an equation to find the value of x.
- **b.** Use the value you found for x to find $m\angle C$ and $m\angle Q$. Show your work.
- **c.** Is $\triangle QPC$ a unique triangle, or can more than one triangle be formed using the three angle measures? Justify your answer.

7. Is it possible for $\triangle QPC$ to have side lengths of 4 feet, 1.5 feet, and 2 feet? Justify your answer.

8. The measures of two side lengths of a triangle are 6 centimeters and 8 centimeters, and the measure of one angle is 35°.
- **a.** Use a ruler and a protractor to draw a triangle or triangles that meet these conditions
- **b.** **Attend to precision.** Is there only one triangle or more than one triangle that meets these conditions? Explain.

Scoring Guide	Exemplary	Proficient	Emerging	Incomplete
	The solution demonstrates these characteristics:			
Mathematics Knowledge and Thinking (Items 1, 2a-b, 3, 4a-b, 5, 6a-c, 7, 8b)	• Clear and accurate understanding of adjacent angle relationships and angle relationships in a triangle.	• An understanding of adjacent angle relationships and angle relationships in a triangle.	• Partial understanding of adjacent angle relationships and angle relationships in a triangle.	• Incorrect or incomplete understanding of adjacent angle relationships and angle relationships in a triangle.
Problem Solving (Items 2a-b, 4a, 5, 6a-b)	• An accurate interpretation of a problem in order to find missing angle measurements.	• A somewhat accurate interpretation of a problem to find missing angle measurements.	• Difficulty interpreting a problem to find missing angle measurements	• Incorrect or incomplete interpretation of a problem.
Mathematical Modeling / Representations (Items 4b, 6c, 8a-b)	• An accurate drawing of a triangle given information on the side lengths and angles.	• A drawing of a triangle given information on the side lengths and angles.	• Difficulty in drawing a triangle given information on the side lengths and angles.	• An incorrect drawing of a triangle given information on the side lengths and angles.
Reasoning and Communication (Items 1, 3, 4b, 6c, 7, 8b)	• Precise use of appropriate terms to describe angle relationships and triangles.	• Use of appropriate terms to describe angle relationships and triangles.	• A partially correct use of terms to describe angle relationships and triangles.	• An incomplete or inaccurate use of terms to describe angle relationships and triangles.

Similar Figures

ACTIVITY 15

The Same but Different

Lesson 15-1 Identify Similar Figures and Find Missing Lengths

My Notes

Learning Targets:

- Identify whether or not polygons are similar.
- Find a common ratio for corresponding side lengths of similar polygons.

SUGGESTED LEARNING STRATEGIES: Marking the Text, Summarize/Paraphrase/Retell, Visualization, Create Representations, Identify a Subtask

The Pentagon, the headquarters of the United States Department of Defense, is located in Arlington County, Virginia. This building is named for its shape.

1. Study these two aerial photos of the Pentagon.

Photo 1

Photo 2

a. How are the photos alike?

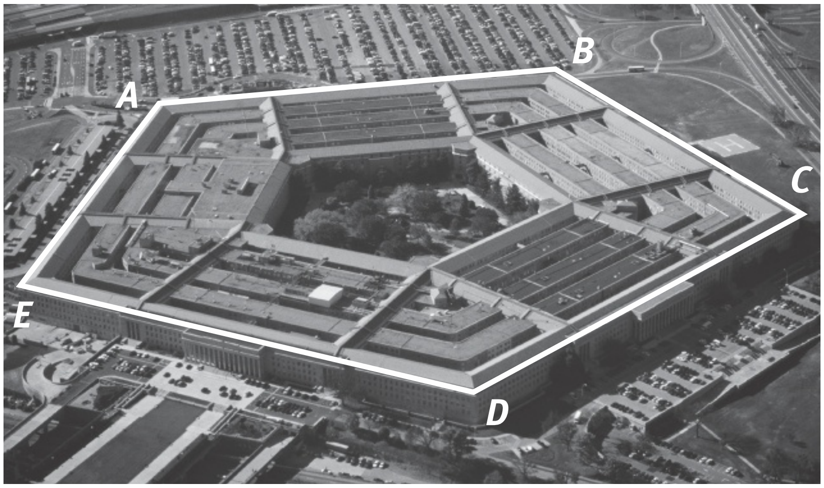
b. How are the photos different?

My Notes

MATH TIP

The shape of the Pentagon building is actually a regular pentagon, but the perspective in this picture makes the lengths appear to be different.

2. Use a protractor and ruler to measure the line segments and angles in both photos below. Measure segments to the nearest millimeter and angles to the nearest degree.



a. $AB =$ ______ **b.** $m\angle A =$ ______
c. $BC =$ ______ **d.** $m\angle B =$ ______
e. $CD =$ ______ **f.** $m\angle C =$ ______
g. $DE =$ ______ **h.** $m\angle D =$ ______
i. $EA =$ ______ **j.** $m\angle E =$ ______

k. $FG =$ ______ **l.** $m\angle F =$ ______
m. $GH =$ ______ **n.** $m\angle G =$ ______
o. $HI =$ ______ **p.** $m\angle H =$ ______
q. $IJ =$ ______ **r.** $m\angle I =$ ______
s. $JF =$ ______ **t.** $m\angle J =$ ______

My Notes

3. Use the measurements from Item 2 to find the following ratios to the nearest tenth.

a. $\frac{AB}{FG} =$ ______ **b.** $\frac{BC}{GH} =$ ______ **c.** $\frac{CD}{HI} =$ ______

d. $\frac{DE}{IJ} =$ ______ **e.** $\frac{EA}{JF} =$ ______

4. What can you conclude about the ratio of the lengths of the segments and the measures of the angles in the photos?

Similar figures are figures in which the lengths of the corresponding sides are in proportion and the corresponding angles are ***congruent***. ***Corresponding parts*** of similar figures are the sides and angles that are in the same relative positions in the figures.

5. Construct viable arguments. Are the two photographs of the Pentagon *similar*? Justify your reasoning.

MATH TERMS

Figures that are **congruent** have exactly the same size and the same shape

In a *similarity statement*, such as $\triangle ABC \sim \triangle DEF$, the order of the vertices shows which angles correspond. So, $\triangle ABC \sim \triangle DEF$ means that $\angle A$ corresponds to $\angle D$, $\angle B$ corresponds to $\angle E$, and $\angle C$ corresponds to $\angle F$. The corresponding sides follow from the corresponding angles. They are AB and DE, BC and EF, and CA and FD.

6. The lengths of the sides of quadrilateral $ABCD$ are 4, 6, 6, and 8 inches. The lengths of the sides of a similar quadrilateral $JKLM$ are 6, 9, 9, and 12 inches.

a. Write the ratios for the corresponding sides of the quadrilaterals.

b. What do you notice about the ratios of the sides of the similar quadrilaterals?

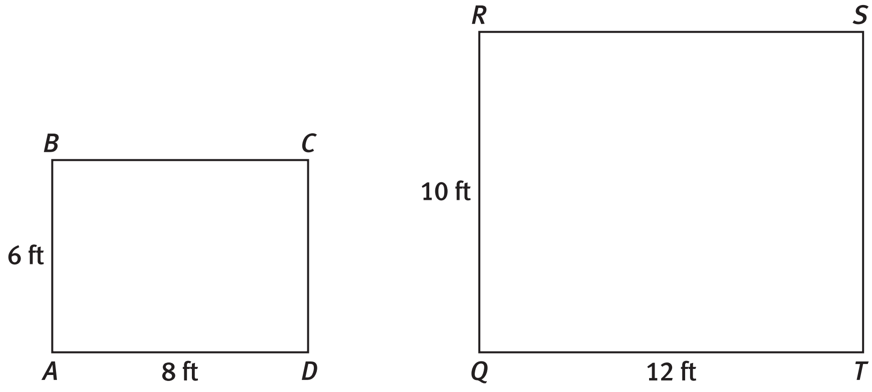
READING MATH

The symbol $\sim$ means "is similar to." Read the similarity statement $\triangle ABC \sim \triangle DEF$ as "Triangle ABC is similar to triangle DEF."

My Notes

If two figures have the same shape with corresponding angles congruent and proportional corresponding sides, then the two figures are similar. When the corresponding sides of similar figures are in proportion, they are equivalent to a common ratio.

7. Rectangle *ABCD* is 6 feet by 8 feet. Rectangle *QRST* is 10 feet by 12 feet.



a. Name the corresponding angles.

b. Are the corresponding angles congruent? Explain.

c. Name the corresponding sides.

d. Write the ratios of the corresponding widths and lengths of the rectangles.

e. Are the corresponding sides in proportion? Explain.

f. Is rectangle *ABCD* similar to rectangle *QRST*? Explain.

My Notes

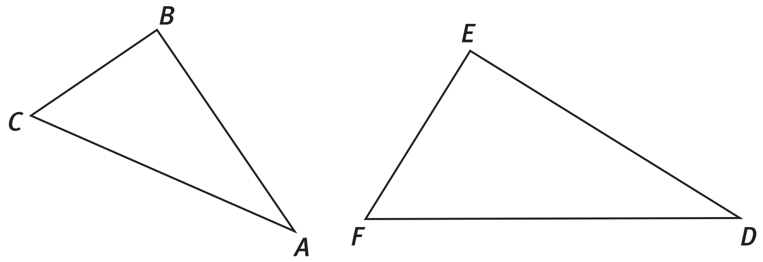
Check Your Understanding

8. Are two congruent figures similar? Explain.

9. When are two polygons with the same number of sides not similar?

LESSON 15-1 PRACTICE

Use a protractor and a ruler to measure the angles and sides of triangles *ABC* and *DEF*.



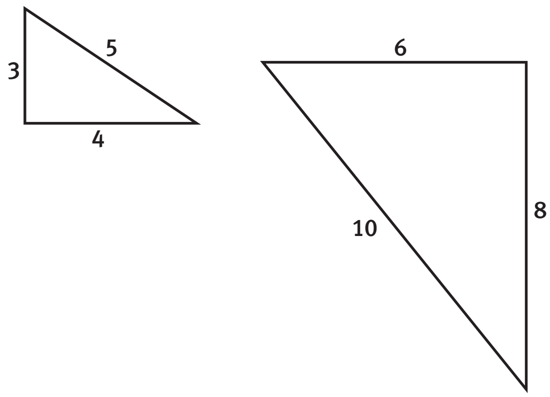
10. Are the corresponding angles congruent? Explain.

11. Are the corresponding sides in proportion? Explain.

12. Are the triangles similar? If so, explain why and write a similarity statement. If not, explain why not.

13. Model with mathematics. Sketch two similar rectangles. Explain why they are similar.

14. Are the ratios of the corresponding sides of the right triangles shown equal? Explain.



15. Make sense of problems. Rectangle *A* is 3 meters wide and 5 meters long. Rectangle *B* is 2.5 meters wide and 4.5 meters long. Rectangle *C* is 10 meters wide and 18 meters long. Are any of the rectangles similar? Explain.

16. Reason abstractly. Are all squares similar? Explain.

MATH TIP

When two figures have a different ***orientation***, it can be tricky to identify corresponding sides or angles. Try turning and/or flipping one of the figures until its shape looks like the other figure.
In Item 14, turn the larger triangle to the right until the 8-unit side is the base. Then flip the triangle over an imaginary vertical line. This can help you identify the corresponding sides.

ACADEMIC VOCABULARY

The ***orientation*** of a figure is the way in which the figure is positioned.

My Notes

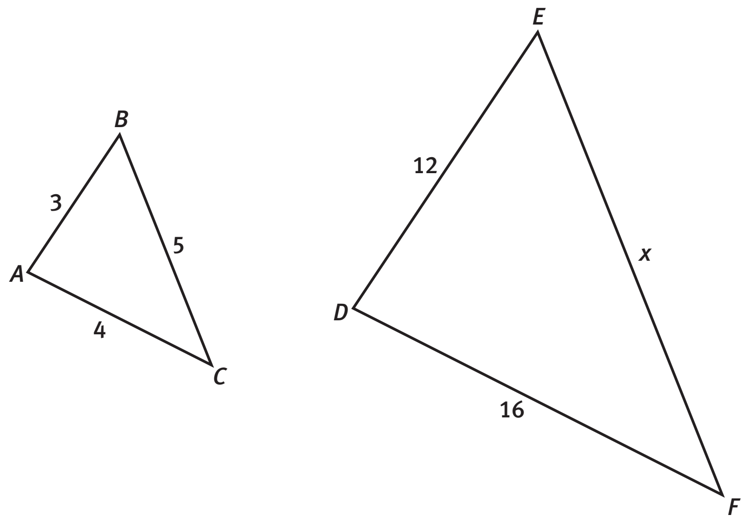
Learning Targets:

- Apply properties of similar figures to determine missing lengths.
- Solve problems using similar figures.

SUGGESTED LEARNING STRATEGIES: Graphic Organizer, KWL Chart, Think Aloud, Visualization, Discussion Groups, Create Representations

The corresponding sides of similar figures are in proportion and form a common ratio. This relationship can be used to find missing lengths.

1. $\triangle ABC \sim \triangle DEF$



a. Write the ratios of the corresponding sides.

b. What is the common ratio of the corresponding sides with known lengths?

c. Use the common ratio to write a proportion to find the value of x, the missing side length. Solve the proportion.

2. Model with mathematics. A rectangular postcard of a painting is similar to the original painting. The postcard is 4 inches wide and 6 inches long. The original painting is 34 inches wide.

a. Draw similar rectangles to model the problem.

b. Write and solve a proportion for the situation. Let x represent the length of the original painting.

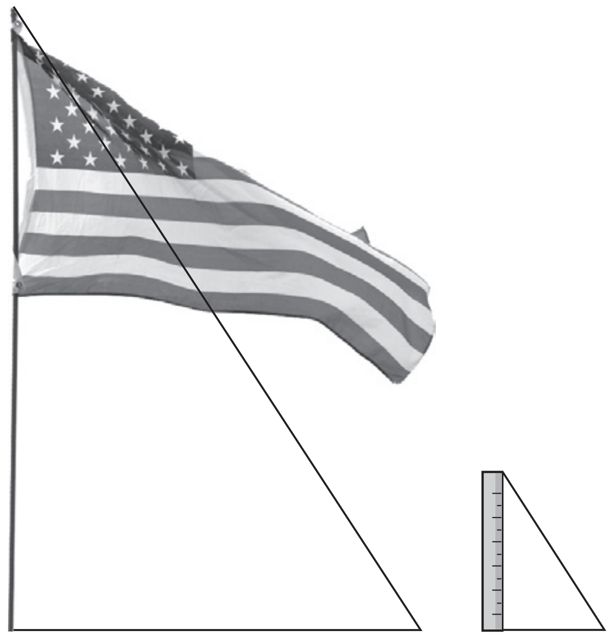
My Notes

c. How long is the original painting?

d. Is your answer reasonable? Explain.

Some objects may be too tall to measure with rulers. Similar triangles can be used to indirectly measure the heights of these objects in real life.

3. A flagpole casts a shadow 8 feet long. At the same time, a yardstick 3 feet tall casts a shadow 2 feet long. The drawing shows how similar triangles can be used to model the situation.



a. Label the picture with the appropriate lengths. Let x represent the height of the flagpole.

b. Use the corresponding sides of similar triangles to write and solve a proportion for the situation.

c. How tall is the flagpole?

d. Is your answer reasonable? Explain.

e. Lars claims that he can solve the flagpole problem using measures within each figure and writes $\frac{x}{8} = \frac{3}{2}$. Is Lars correct? Explain.

My Notes

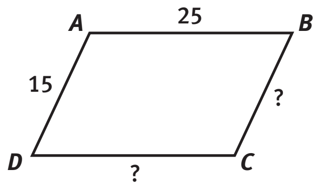
Check Your Understanding

4. If you know the length and width of one rectangle and the length of a second rectangle, can you always use corresponding sides to find the width of the second rectangle? Support your answer.

5. Do you need to know the lengths of all the sides of one triangle to find a missing length of a similar triangle? Explain.

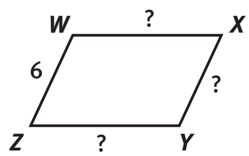
LESSON 15-2 PRACTICE

parallelogram *ABCD* ~ parallelogram *WXYZ*

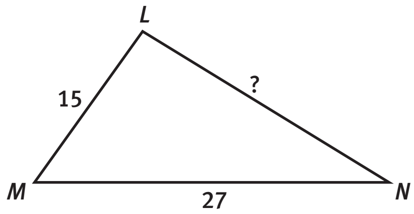


6. What is the common ratio of side *AD* to side *WZ*?

7. Find the length of segment *WX*.

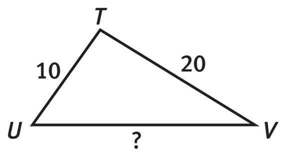


$\triangle LMN \sim \triangle TUV$. Use what you know about common ratios to answer Items 8–9.



8. How long is segment *LN*?

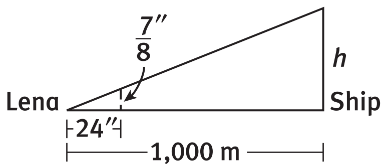
9. How long is segment *UV*?



10. A 4-meter-tall flagpole casts a 6-meter shadow at the same time that a nearby building casts a 24-meter shadow. What is the height of the building? Solve this problem two different ways. First, set up and solve a proportion in which each ratio compares corresponding side lengths in the two figures. Then set up and solve a proportion in which each ratio compares side lengths within each figure.

11. Make sense of problems. Lena is standing on the beach when she sees a tall sailing ship pass by 1,000 meters offshore. She holds a ruler vertically 24 inches in front of her eyes, and the ship appears to be $\frac{7}{8}$ inch high. The figure in the My Notes column represents the situation as two similar right triangles. Find the approximate height (h) of the sailing ship above the water. Explain your answer.

(Figure not to scale)

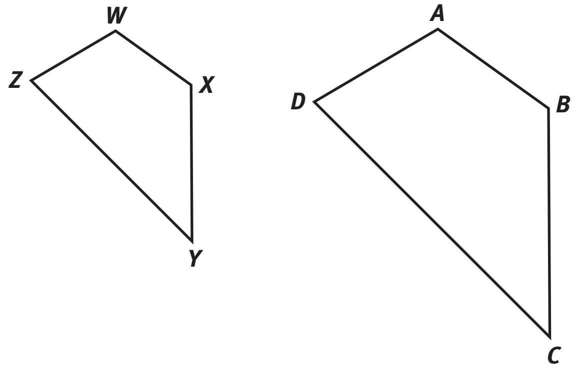


ACTIVITY 15 PRACTICE

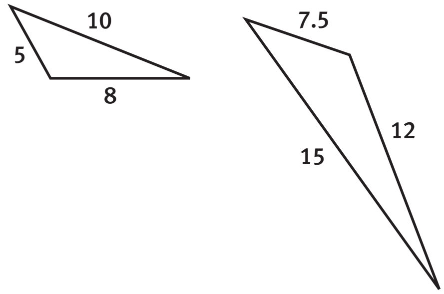
Write your answers on a separate piece of paper. Show your work.

Lesson 15-1

1. Use a ruler and protractor to decide if the figures are similar. Justify your decision.



2. a. Identify the corresponding sides on the figures below.
   b. Are the ratios of the corresponding sides of the triangles equal? Explain.



3. $\triangle ABC \sim \triangle EFG$, $m\angle A = 25°$, and $m\angle F = 100°$. What is $m\angle C$?
   A. 125°
   B. 100°
   C. 55°
   D. 25°

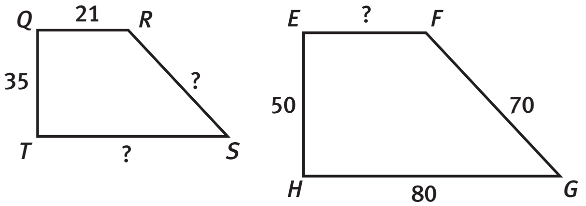
4. Rectangle J is 6 feet wide and 9 feet long. Rectangle K is 9 feet wide and 12 feet long. Rectangle L is 15 feet wide and 22.5 feet long. Are any of the rectangles similar? Explain.

5. a. Are all equilateral triangles similar? Explain.
   b. Are all right triangles similar? Explain.

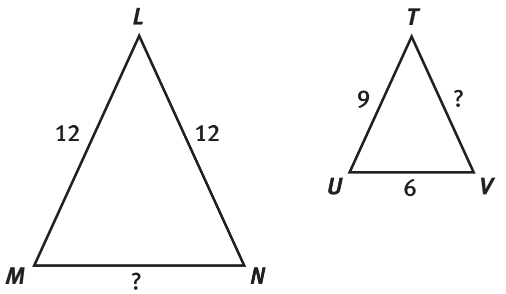
6. How is the mathematical meaning of the word "similar" the same as or different from the word "similar" in everyday conversation?

Lesson 15-2

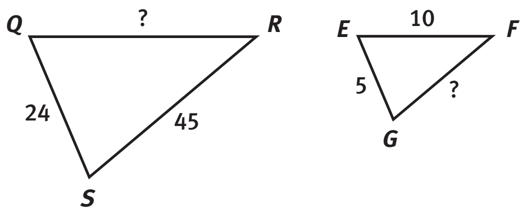
7. Trapezoid $QRST \sim$ trapezoid $EFGH$. Find the measures of the missing sides.



8. $\triangle LMN \sim \triangle TUV$. Find the measures of the missing sides.

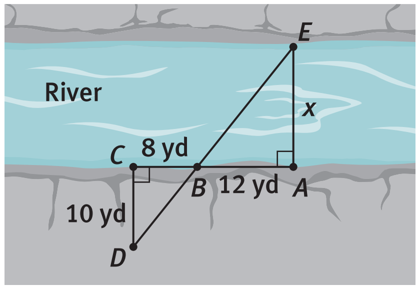


9. $\triangle QRS \sim \triangle EFG$. Find the measures of the missing sides.



10. A rectangular room is 42 feet wide and 69 feet long. On a blueprint, the room is 14 inches wide. How long is the room on the blueprint?

11. John wants to find the width of a river. He marks distances as shown in the diagram. Which of the following ratios can be used to find the width of the river?



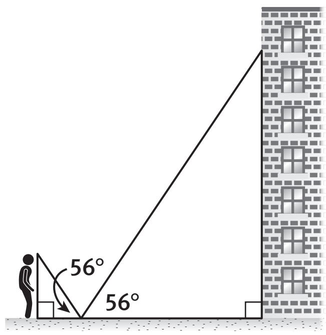
A. $\frac{10}{8} = \frac{12}{x}$ **B.** $\frac{10}{12} = \frac{8}{x}$

C. $\frac{8}{10} = \frac{12}{x}$ **D.** $\frac{x}{12} = \frac{8}{10}$

12. Miguel is 5 feet 10 inches tall. On a sunny day he casts a shadow 4 feet 2 inches long. At the same time, a nearby electric tower casts a shadow 8 feet 9 inches long. How tall is the tower?

MATHEMATICAL PRACTICES
Make Sense of Problems

13. Sam wants to find the height of a window in a nearby building but it is a cloudy day with no shadows. Sam puts a mirror on the ground between himself and the building. He tilts it toward him so that when he is standing up, he sees the reflection of the window. The base of the mirror is 1.22 meters from his feet and 7.32 meters from the base of the building. Sam's eye is 1.82 meters above the ground. How high up on the building is the window?



Circles: Circumference and Area

Gardens Galore

Lesson 16-1 Circumference of a Circle

Learning Targets:

- Investigate the ratio of the circumference of a circle to its diameter.
- Apply the formula to find the circumference of a circle.

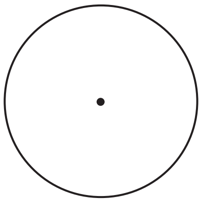
SUGGESTED LEARNING STRATEGIES: Think Aloud, Create Representations, Discussion Groups, Summarizing, Paraphrasing

Rose wants to create several circular gardens in her yard. She needs to find the distance around and the area of each garden.

A ***circle*** is the set of points in the same ***plane*** that are an equal distance from a given point, called the ***center***. The distance around a circle is called the ***circumference***.

A line segment through the center of a circle with both endpoints on the circle is called the ***diameter***. A line segment with one endpoint on the center and the other on the circle is called the ***radius***.

1. A circle is shown.



a. Use the information given above to label the center and the circumference of the circle

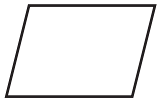
b. Draw and label a diameter and a radius in the circle.

2. What is the relationship between the length of the diameter and the length of the radius of a circle?

My Notes

MATH TERMS

A **plane** is a flat surface that extends in all directions. A parallelogram is usually used to model a plane.



My Notes

There is also a relationship between the circumference and the diameter of a circle. Work with your group to complete the activity below to find the relationship.

3. Measure the circumference and diameter of the circular objects provided by your teacher. Use the table to record the data. Then calculate the ratios.

Description of object	Object 1	Object 2	Object 3	Object 4	Object 5
Circumference					
Diameter					
Ratio of circumference to diameter (as a fraction)					
Ratio of circumference to diameter (as a decimal)					

The ratio of the circumference to the diameter of a circle is called **pi**, denoted by the Greek letter π. A commonly used approximation for *pi* is $\pi \approx 3.14$.

4. Which measurement tools used by your class gave the most accurate approximation of *pi*? Why do you think this is true?

5. Express regularity in repeated reasoning. Using the data on your table, write the equation that relates the circumference of a circle, C, to *pi* (π) and the diameter, d, of a circle.

6. Rewrite the equation from Item 5 to show the relationship of the circumference of a circle, C, to its radius (r) and π.

The equations you wrote for Items 5 and 6 above are the formulas for finding the circumference of a circle given its diameter or radius.

WRITING MATH

The digits following the decimal point in the decimal representation for *pi* never end or repeat, so all values found using *pi* are approximations. The symbol $\approx$ means "approximately equal to."

CONNECT TO LANGUAGE

The Greek letter π is the first letter in the Greek words for perimeter and periphery.

My Notes

7. Sometimes $\frac{22}{7}$ is used as an approximation of π. Why is this fraction a good approximation?

8. **Attend to precision.** Should the circumference of a circle be labeled in units or in square units? Explain.

Now you have an equation you can use to find the circumference of a circle. Use what you know to help Rose find the distance around one of her gardens.

9. One of the circular gardens Rose wants to make has a diameter of 6 feet.
   a. Use a circumference formula to find the amount of decorative fencing that Rose needs to enclose this garden. Use 3.14 or $\frac{22}{7}$ for π. Show your work. Tell which value you used for *pi*.

   b. Decorative fencing is sold in packages of 12-foot sections. How many packages must Rose buy to enclose this garden? Explain your reasoning and show your work.

My Notes

Check Your Understanding

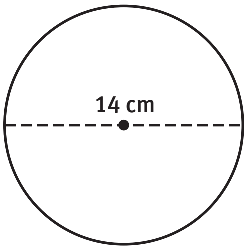
10. Explain how you could decide which approximation of π—3.14 or $\frac{22}{7}$—to use to compute the circumference of a circle.

11. Explain how the circumference of a circle and the definition of *pi* are related.

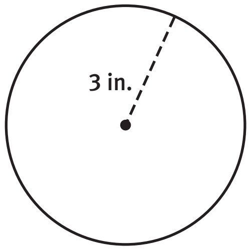
LESSON 16-1 PRACTICE

12. For Items a–d, find the circumference of each circle expressed as a decimal.

a.



b.



c. a circle with a radius of 8 ft

d. a circle with a diameter of 25 m

13. Find the circumference of a circular dog pen that has a radius of 35 meters. Use $\frac{22}{7}$ for π.

14. A window shaped like a circle has a diameter of 15 centimeters. What is the circumference of the window? Use 3.14 for π.

15. A circular tablecloth has a radius of 18 inches. What is the circumference of the tablecloth? Use 3.14 for π.

16. **Make use of structure.** A circle has a circumference of 125.6 centimeters. What is the diameter of the circle to the nearest centimeter? What is the radius to the nearest centimeter?

17. **Make sense of problems.** The diameter of a bicycle wheel is 26 inches. About how many revolutions does the wheel make during a ride of 500 feet? Use 3.14 for π. Explain your answer.

My Notes

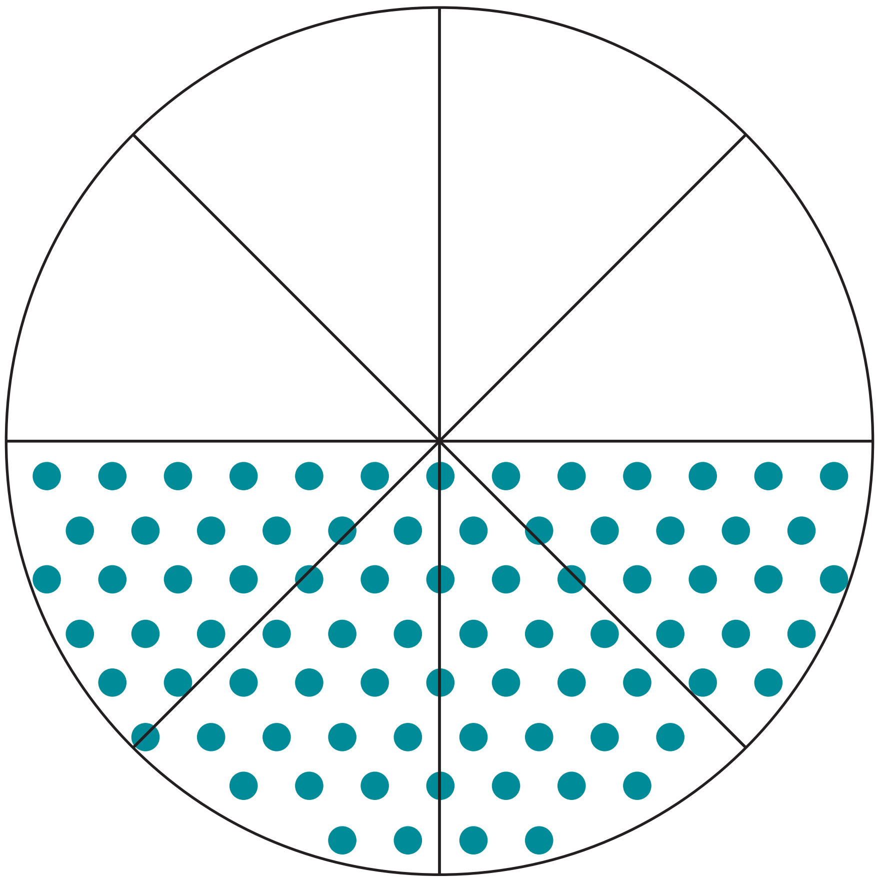
Learning Targets:

- Approximate the area of a circle.
- Apply the formula to find the area of a circle.

SUGGESTED LEARNING STRATEGIES: Create Representations, Think Aloud, Think-Pair-Share, Use Manipulatives

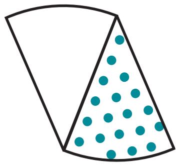
Rose also needs to know the area of each of the circular gardens.

1. On a separate sheet of paper, trace the circle shown. Shade half of the circle. Then cut your circle into eight congruent pie-shaped pieces.



My Notes

2. Arrange the eight pieces using the alternating pattern shown.



3. Sketch the shape you made with the circle pieces.

a. What geometric shape does the shape resemble?

b. What do you know about the area of the circle and the area of the figure you make?

4. On your sketch, draw and label the height of the figure. What part of the circle does the height represent?

5. What other measure of the circle do you need to know to determine the area of the shape you sketched? Label it on your sketch and explain your reasoning.

6. **Model with mathematics.** Use words, symbols, or both to describe how you can now calculate the area of the circle. Start with the formula $A = b \times h$ and substitute into the formula. Refer to your labeled sketch as needed.

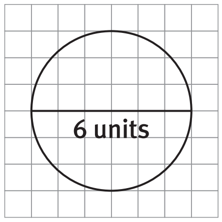
7. Use your answer from Item 6 to write the formula for the area of a circle A in terms of its radius r and π. Explain how you found the formula.

8. Should the area of a circle be labeled in units or in square units? Explain.

My Notes

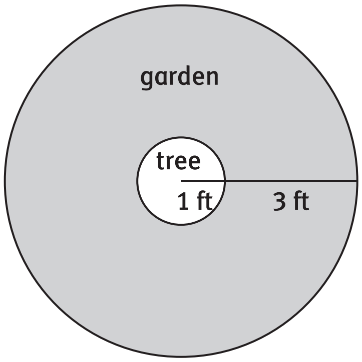
9. A circle with a radius of 3 units is graphed.

a. Estimate the area by counting the number of enclosed squares. Show the calculations that lead to your estimation.



b. Use the formula you wrote in Item 7 to find the area of the circle. Use 3.14 for π.

10. Make sense of problems. One of Rose's gardens will have a tree in the center as shown. The radius of the large outer circle will be 4 feet while the radius of the inner circle for the tree will be 1 foot. Find the area of the garden without the tree. Show your work.



Check Your Understanding

11. Is the circumference of a circle enough information to determine the area of the circle? Explain.

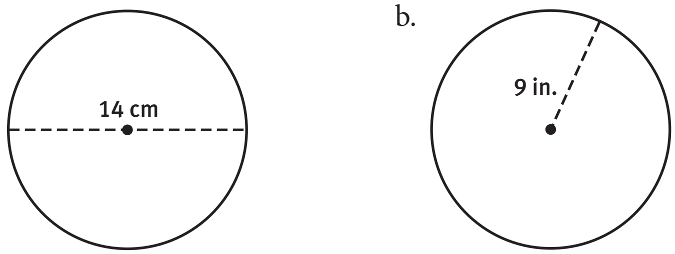
12. What is the formula for the area of a circle in terms of its diameter, *d*?

My Notes

LESSON 16-2 PRACTICE

13. Find the approximate area of each circle. Use the value for π that makes the math simplest for you.

a. b.



c. a circle with a radius of 8 km

d. a circle with a diameter of 25 ft

14. What is the approximate area of a circular pond that has a radius of 35 meters? Tell what value you used for π.

15. A penny has a diameter of about 26.5 millimeters. What is the area of a penny to the nearest hundredth? Use 3.14 for π.

16. One trampoline has a diameter of 12 feet. A larger trampoline has a diameter of 14 feet. How much greater is the area of the larger trampoline? Use 3.14 for π.

17. **Make sense of problems.** A painting shaped like a circle has a diameter of 20 inches. A circular frame extends 2 inches around the edge of the painting. How much wall space does the framed painting need? Use 3.14 for π.

18. The circumference of a circle is 31.4 centimeters. What is the area of the circle?

19. **Reason abstractly.** In ancient Egypt, a scribe equated the area of a circle with a diameter of 9 units to the area of a square with a side length of 8 units. What value of π does this method produce? Explain.

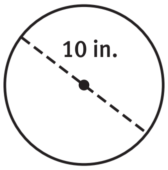
ACTIVITY 16 PRACTICE

Write your answers on a separate piece of paper. Show your work.

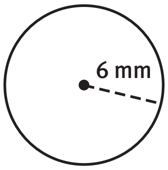
Lesson 16-1

1. Find the circumference of each circle below. Use 3.14 for π.

   a.

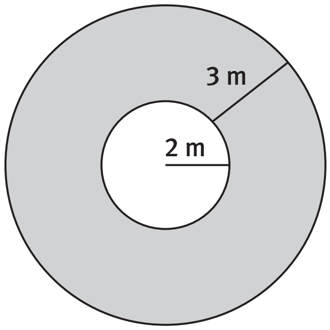
   b.

2. The diameter of a pizza is 14 inches. What is the circumference of the pizza? Tell what value you used for π.
3. The radius of a circular mirror is 4 centimeters. What is the circumference of the mirror? Tell what value you used for π.
4. The radius of a circular garden is 28 feet. What is the circumference of the garden? Tell what value you used for π.
5. Find the diameter of a circle if $C = 78.5$ feet. Use 3.14 for π.
6. Find the radius of a circle if $C = 88$ yards. Use $\frac{22}{7}$ for π.
7. Multiple Choice. A standard circus ring has a radius of 6.5 meters. Which of the following is the approximate circumference of the circus ring?
   A. 13 meters
   B. 20.4 meters
   C. 40.8 meters
   D. 132.7 meters

Lesson 16-2

8. What is the area of a pizza with a diameter of 12 inches?
9. A circle has circumference 28.26 cm. What is the area of the circle? Use 3.14 for π.
10. Find the area of the shaded region. Use 3.14 for π.



11. Multiple Choice. The circular base of a traditional tepee has a diameter of about 15 feet. Which of the following is the approximate area of the base of the tepee?
    A. 23.6 square feet
    B. 47.1 square feet
    C. 176.6 square feet
    D. 706.5 square feet

12. A window is shaped like a semicircle. The base of the window has a diameter of $3\frac{1}{2}$ feet. Find the area of the window to the nearest tenth of a foot. Explain how you found the answer.

13. A circle has a diameter of 5 meters and a square has a side length of 5 meters.
   a. Which has the greater perimeter? How much greater?
   b. Which has the greater area? How much greater?

14. A circle with center at $(1, -1)$ passes through the point $(1, 2)$. Find the radius and then the area of the circle. Use 3.14 for π. Make a sketch on graph paper if it is helpful.

15. A quiche with a diameter of 12 inches can feed 6 people. Can a quiche with a diameter of 10 inches feed 4 people, assuming the same serving size? Explain your thinking.

16. A pizza with a diameter of 8 inches costs $10. A pizza with a diameter of 14 inches costs $16. Which is the better buy? Explain your thinking.

17. The radius of a circle is doubled.
   a. How does the circumference change?
   b. How does the area change?

MATHEMATICAL PRACTICES
Reason Abstractly and Quantitatively

18. Is it possible for a circle to have the same numerical value for its circumference and area? Explain your reasoning.

Composite Area

Tile Designs

Lesson 17-1 Area of Composite Figures

My Notes

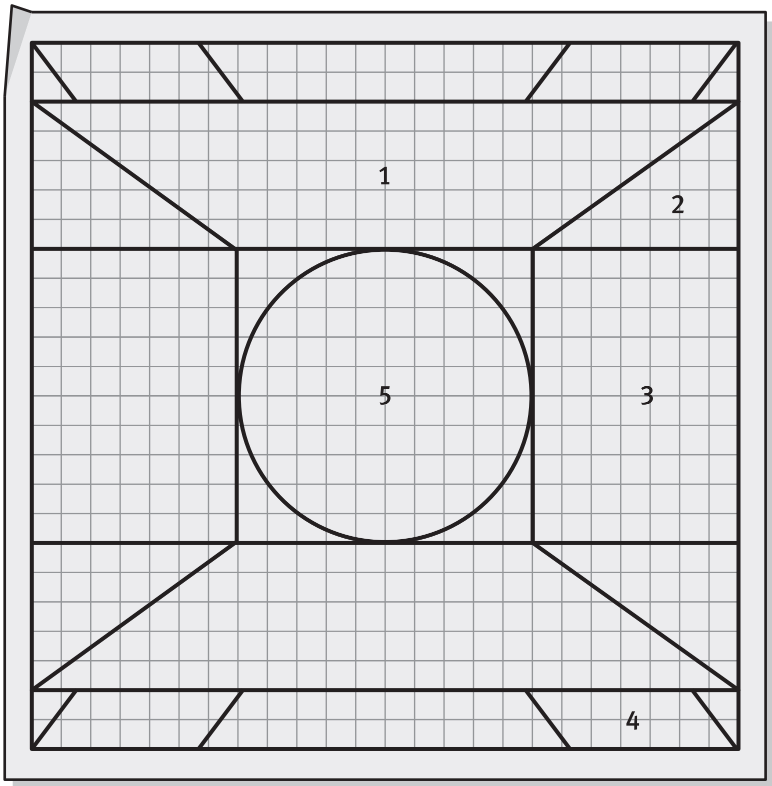
Learning Targets:

- Determine the area of geometric figures.
- Determine the area of composite figures.

SUGGESTED LEARNING STRATEGIES: Create Representations, Discussion Groups, Identify a Subtask, Think-Pair-Share, Visualization

Each year the students in Ms. Tessera's classes create a design for a stained-glass window. They draw two-dimensional figures on grid paper to create the design for their stained-glass windows.

1. This drawing shows the design for one of the projects done last year.



a. What is the area of the entire stained-glass window? Explain.

b. What is the most precise geometric name for each of the numbered shapes in the design?

My Notes

MATH TIP

Recall the following area formulas.

Triangle: $A = \frac{1}{2}bh$

Parallelogram: $A = bh$

Trapezoid: $A = \frac{1}{2}h(b_1 + b_2)$

Circle: $A = \pi r^2$

ACADEMIC VOCABULARY

In geometry, when you divide a composite figure into smaller figures, you ***decompose*** the figure.

c. Find the area of each numbered shape.

Figure 1 Figure 2 Figure 3

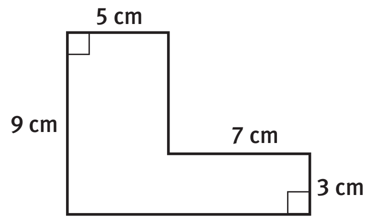
Figure 4 Figure 5

d. Explain why the students might be interested in finding the areas of the different shapes used in the design of the stained-glass window.

Some geometric figures have formulas that can be used to find the area of the figure. Other shapes do not have their own formulas.

A ***composite figure*** is made up of two or more geometric figures. You can find the area of a composite figure by dividing, or ***decomposing***, it into simpler geometric shapes with known formulas.

2. The composite figure below can be divided into two rectangles.



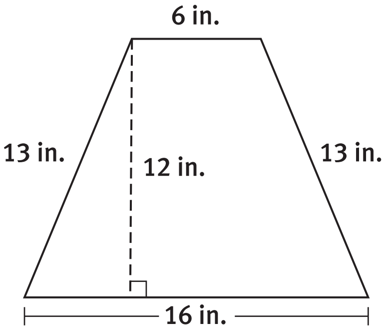
a. Draw a line segment on the diagram to divide the figure into two rectangles. Label the length and width of each rectangle.

b. Find the total area of the composite figure. Show your work. Label your answer with the appropriate unit of measure.

c. Find the perimeter of the composite figure. Show your work. Label your answer with the appropriate unit of measure.

Check Your Understanding

3. Use the trapezoid shown.



a. Explain how to find the area of the trapezoid by dividing it into simpler geometric shapes.

b. Find the area of the trapezoid using the simpler geometric shapes you found in part a.

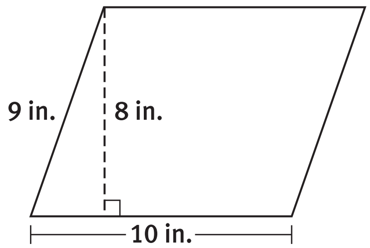
c. Use the formula for the area of a trapezoid to find the area. Compare this area to the area you found in Part b.

4. **Construct viable arguments.** When dividing a composite figure into simpler geometric shapes to find the area, explain why the simpler figures cannot overlap or have gaps.

LESSON 17-1 PRACTICE

For Items 5–10, find the perimeter and area of each figure.

5.



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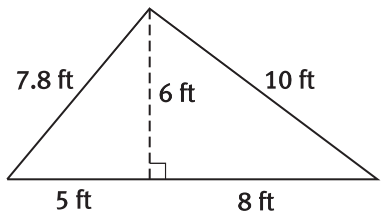

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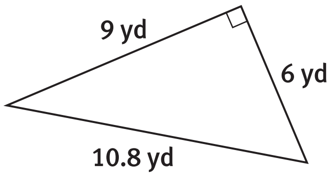

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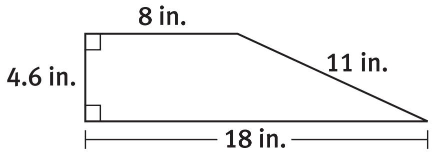

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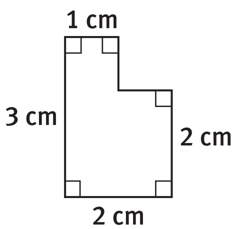

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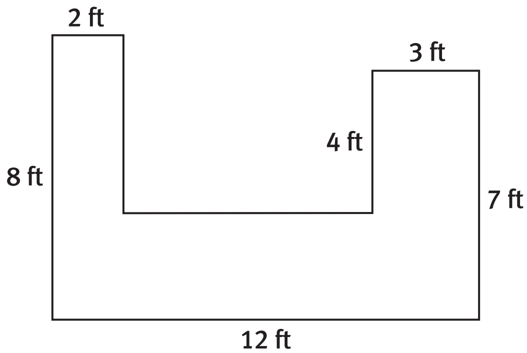
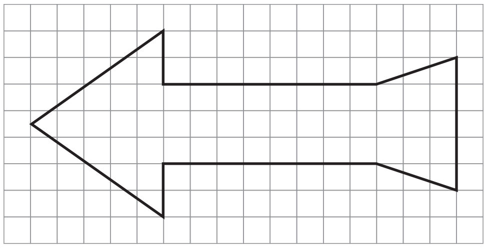


My Notes

11. What is the area of the arrow in square units? Justify your answer.



12. Make sense of problems. A 4-inch-wide by 6-inch-long picture is placed on a solid mat that forms a frame around it. The mat is 8 inches long. The mat and the picture are similar rectangles. What is the area of the mat?

13. Make use of structure. Parallelogram 1 has a base of 26 centimeters and a height of 15 centimeters. Parallelogram 2 is identical to it. A triangle with a base of 8 centimeters is cut from parallelogram 2 and placed so its base rests on the top of the original figure. How does the area of the resulting composite figure compare to the area of parallelogram 1? Explain.

14. Reason quantitatively. A kite is formed by connecting the bases of two triangular frames. The height of the top frame of the kite is 8 inches. The height of the lower section is 14 inches. The bases of the frames are 10 inches long. What is the least amount of material needed to make two kites?

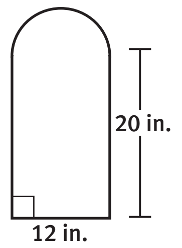
Learning Targets:

- Determine the area of composite figures.
- Solve problems involving area.

SUGGESTED LEARNING STRATEGIES: Chunking the Activity, Group Presentation, Summarizing, Paraphrasing, Identify a Subtask, Visualization

Composite figures may contain parts of circles. To find the area of these figures, it is necessary to identify the radius or the diameter of the circle.

1. The composite figure shown can be divided into a rectangle and a semicircle.



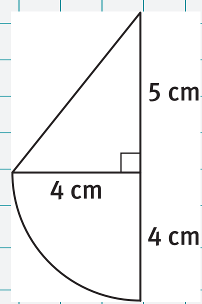
a. What is the diameter of the ***semicircle***?
b. Find the total area of the figure. Use $\pi = 3.14$. Show your work.

c. Find the distance around the figure. Show your work.

My Notes

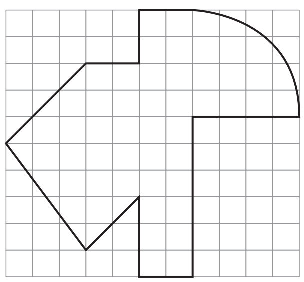
MATH TERMS

A **semicircle** is an arc whose measure is half of a circle. The area of a semicircle is half of the area of a circle with the same radius.



2. The figure in the My Notes column is divided into a right triangle and a quarter-circle. Find the area of the composite figure. Use $\pi = 3.14$.

3. A student dropped a piece of stained glass. A fragment has the shape shown below.
a. Divide the fragment into smaller shapes you can use to find its total area.

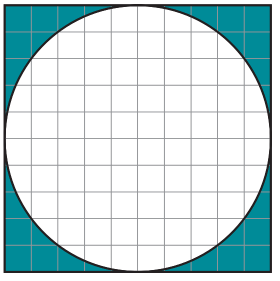


My Notes

b. Find the area of the fragment if each square on the grid represents 1 cm^2. Use $\pi = 3.14$. Show the calculations that led to your answer.
Student calculations will differ depending on how they divide the figure. Sample answer: Area $= 10.5 + 12 + 20 + 12.56 \approx 55.06$ cm^2

You can break a composite figure into geometric shapes and add the areas of the shapes to find the area. You can also subtract to find the area of part of a composite figure.

4. The stained-glass design on page 179 contains a circle that fits exactly in a square, as shown.



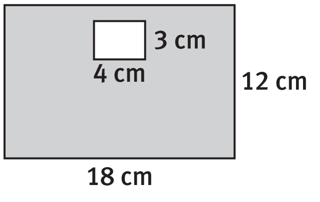
a. Shade the region that is inside the square but outside the circle.

b. Describe a method for finding the area of the shaded region.
Sample answer: Find the area of the square and subtract the area of the circle.

c. Find the area of the shaded region if each square on the grid represents 1 cm^2. Use $\pi = 3.14$. Show your work.
$100 - 78.5 \approx 21.5$ cm^2

Check Your Understanding

5. What is the area of the shaded region? Explain your thinking.



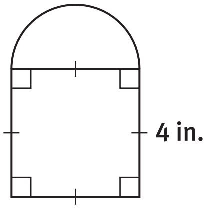
6. Construct viable arguments. Explain how to find the area of a composite figure composed of simpler geometric shapes.

My Notes

LESSON 17-2 PRACTICE

Find the area of each figure. Use $\pi = 3.14$.

7.



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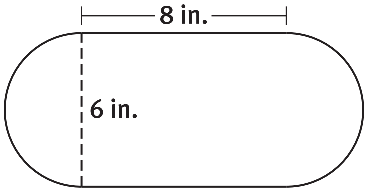
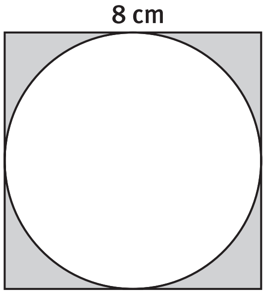
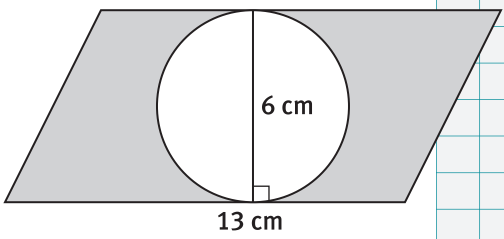


Find the area of the shaded region. Use $\pi = 3.14$.

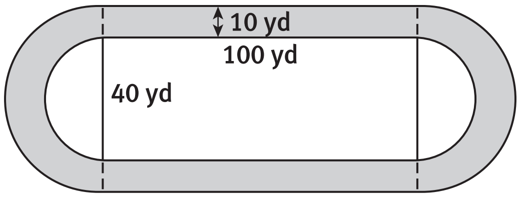
9.



10.



11. An athletic field has the shape of a 40-yard-by-100-yard rectangle with a semicircle at each end. A running track that is 10 yards wide surrounds the field. Use this information to answer the questions below. Use $\pi = 3.14$.



a. Find the area of the athletic field, without the track.
b. Find the area of the athletic field with the track included.
c. Find the area of the track, the shaded portion of the diagram.
d. Suppose a rectangular fence of 180 yards by 80 yards encloses the athletic field and running track. How much of the fenced area is not a part of the field and track?

12. Make sense of problems. A circular plate has a diameter of 6 inches. A pancake in the center of the plate has a radius of 2 inches. How much of the plate is not covered by the pancake? Use $\pi = 3.14$.

13. Reason quantitatively. A section of stained glass is made by placing a circle at the top of a triangle with a base of 10 centimeters and a height of 8 centimeters. The diameter of the circle is equal to the height of the triangle. What is the area of the section of stained glass? Show your work. Use $\pi = 3.14$.

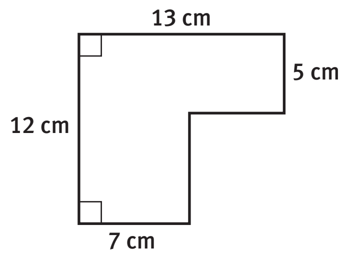
ACTIVITY 17 PRACTICE

Write your answers on a separate piece of paper. Show your work.

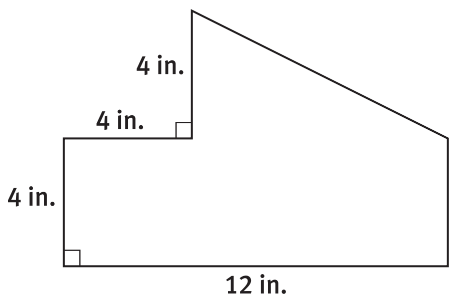
Lesson 17-1

For Items 1–4, find the area of the figure. Use $\pi = 3.14$.

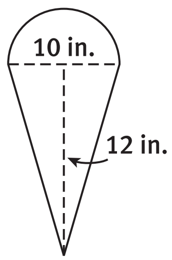
1.



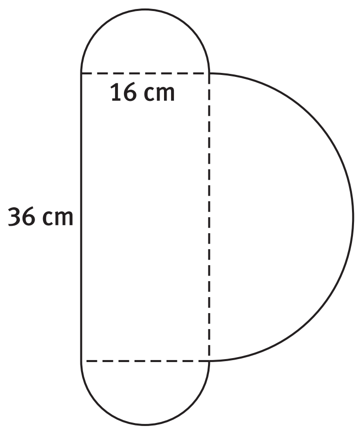
2.



3.

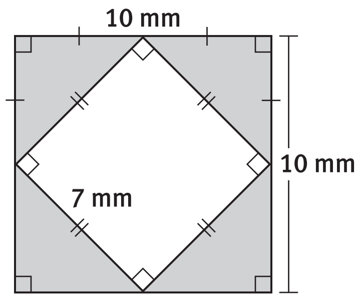


4.



For Items 5–8, find the area of the shaded region. Use $\pi = 3.14$.

5.



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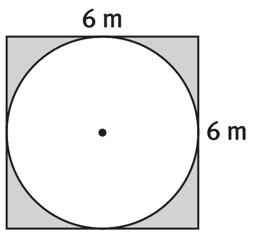

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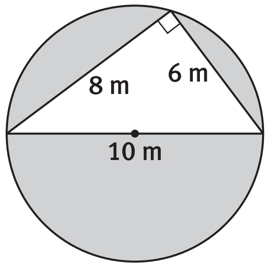

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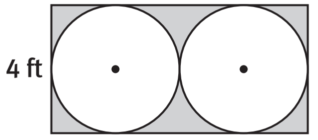
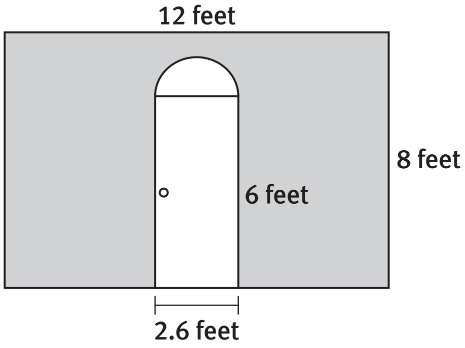
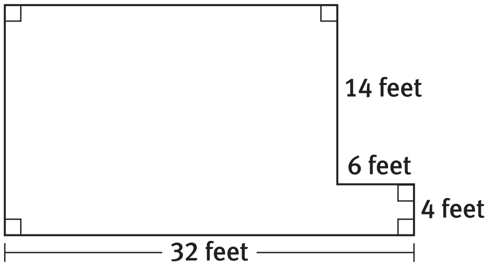


Lesson 17-2

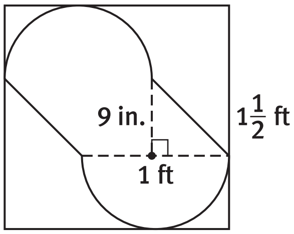
9. Sue wants to paint the wall shown. What is the area of the wall to the nearest tenth? Use $\pi = 3.14$.
A. 72.2 ft^2
B. 75.1 ft^2
C. 77.7 ft^2
D. 80.4 ft^2



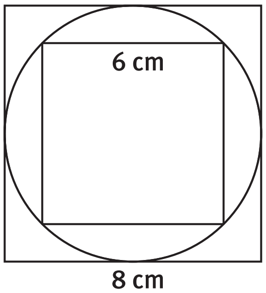
10. A room with the dimensions shown needs carpet. How much carpet is needed to cover the entire floor of the room?
A. 576 sq ft
B. 492 sq ft
C. 472 sq ft
D. 388 sq ft



11. A square blanket has a design on it as shown. Find each of the following in square inches and in square feet. Use $\pi = 3.14$.
a. area of the design
b. area of the blanket without the design



12. Each square section of a quilt has the design shown. Use $\pi = 3.14$.
a. What is the area of the circular section between the two squares?
b. What is the area of the four corner sections?



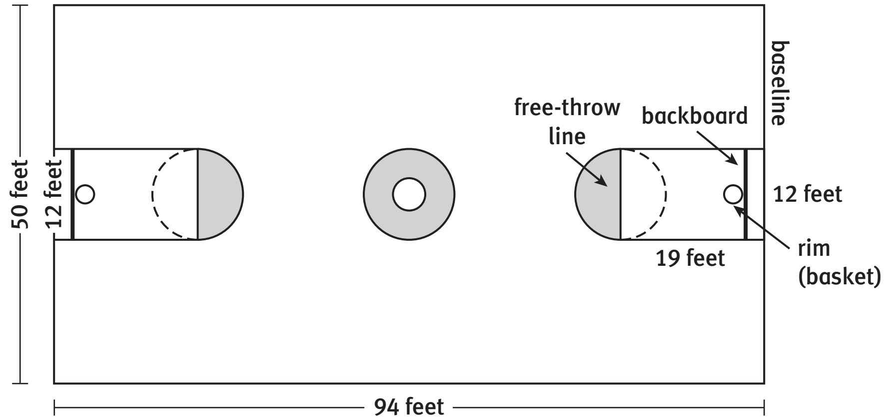
MATHEMATICAL PRACTICES
Look for and Make Use of Structure

13. How does knowing the area formulas of simple geometric shapes help you find the area of composite figures?

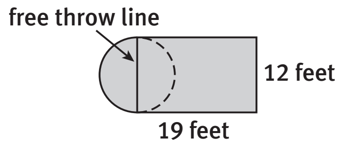
Circumference and Area

IN THE PAINT

An NBA basketball court is 94 feet long and 50 feet wide. It contains three circles, each with a diameter of 12 feet. Two of these circles are located at the free-throw lines, and the third circle is at the center of the court. Within the third circle is another circle with a radius of 2 feet.



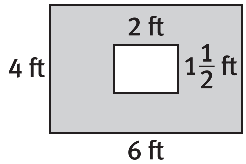
1. One gallon of paint will cover 110 square feet. How many gallons of paint will be needed to paint the shaded regions on the court? Use $\pi \approx 3.14$. Explain your thinking.
2. The region including the circle at the free-throw line to the baseline is shown. Find the area of this region. Use $\pi \approx 3.14$.



3. The key is the rectangular region on the basketball court from the free-throw line to the backboard. The backboard is 4 feet from the baseline.
   a. Is the key similar to the basketball court? Explain.
   b. Is the inner circle similar to the entire circle in the center of the court? Explain.

Circumference and Area
IN THE PAINT

A vertical *backboard* located 4 feet from the baseline supports the rim of the basketball net. The backboard measures 6 feet wide and 4 feet high. The shooter's square is a white box above the rim of the basket. It must measure $1\frac{1}{2}$ feet high and 2 feet wide, as shown at right.

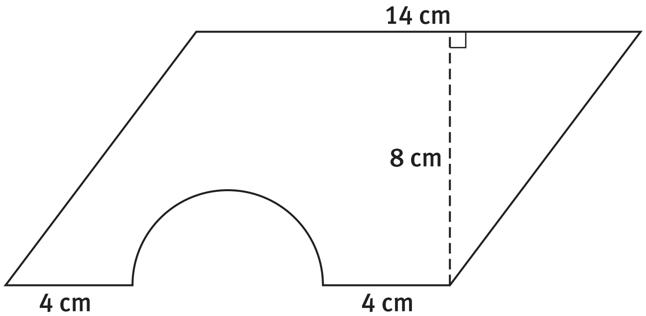


4. What is the area of the portion of the backboard that is NOT white?

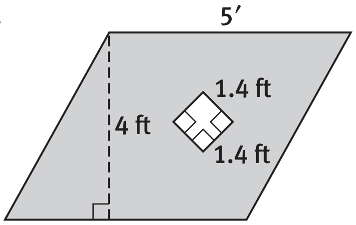
5. The rim of the basket has a radius of 9 inches.
   a. What is the approximate circumference of the basket? Use $\pi \approx 3.14$.
   b. Explain why 3.14 is used when finding the circumference of circles.

6. The design of a basketball team's logo sometimes includes geometric designs. The shapes below are from the logos of two teams. Find the area of each shape.

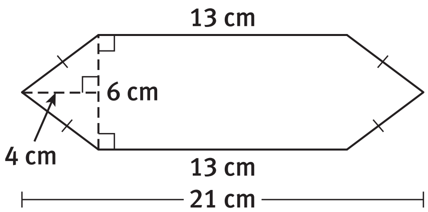
a.



b.



7. Michael claims he can find the area of the composite shape shown by inscribing it in a rectangle and subtracting. Devora claims that to find the area you need to use addition. Which student is correct? Justify your answer.



Scoring Guide	Exemplary	Proficient	Emerging	Incomplete
	The solution demonstrates these characteristics:			
Mathematics Knowledge and Thinking (Items 1, 2, 3a-b, 4, 5a-b, 6a-b, 7)	• Accurately and efficiently finding the circumference and area of circles and the area of composite figures.	• Finding the circumference and area of circles and the area of composite figures.	• Difficulty finding the circumference and area of circles and the area of composite figures.	• No understanding of finding the circumference and area of circles and the area of composite figures.
Problem Solving (Items 1, 2, 4, 5a, 6a-b)	• An appropriate and efficient strategy that results in a correct answer.	• A strategy that may include unnecessary steps but results in a correct answer.	• A strategy that results in some incorrect answers.	• No clear strategy when solving problems.
Mathematical Modeling / Representations (Items 3a-b, 7)	• Clear and accurate understanding of similar figures. • Solving composite figures by adding or subtracting.	• An understanding of similar figures. • Recognizing that composite figures are made up of simpler figures.	• Difficulty recognizing similar figures. • Difficulty in working with composite figures.	• No understanding of similar figures. • No understanding of composite figures.
Reasoning and Communication (Items 1, 3a-b, 5b, 7)	• Precise use of appropriate terms to explain similar figures, finding area, and π.	• An adequate explanation of similar figures, finding area, and π.	• A partially correct explanation of similar figures, finding area, and π.	• An incomplete or inaccurate explanation of similar figures, finding area, and π.

Sketching Solids

Putt-Putt Perspective

ACTIVITY 18

Lesson 18-1 Shapes That Result from Slicing Solids

Learning Targets:

- Draw different views of three-dimensional solids.
- Identify cross sections and other views of pyramids and prisms.

SUGGESTED LEARNING STRATEGIES: Visualization, Look for a Pattern, Use Manipulatives, Create Representations

The Service Club at Park Middle School is creating a miniature golf course to raise funds for a food bank. The theme is interesting structures around the world. Buildings that will be included are the Washington Monument, the Flatiron Building, the Louvre Pyramid, and the Pentagon Building.

1. Compare and contrast the two- and three-dimensional shapes in these four buildings.

My Notes

CONNECT TO GEOGRAPHY

The Washington Monument is located in Washington, DC. The Pentagon is nearby in Arlington County, VA. The Flatiron Building is in New York, NY. The Louvre Pyramid is the entrance to the Louvre Museum in Paris, France.

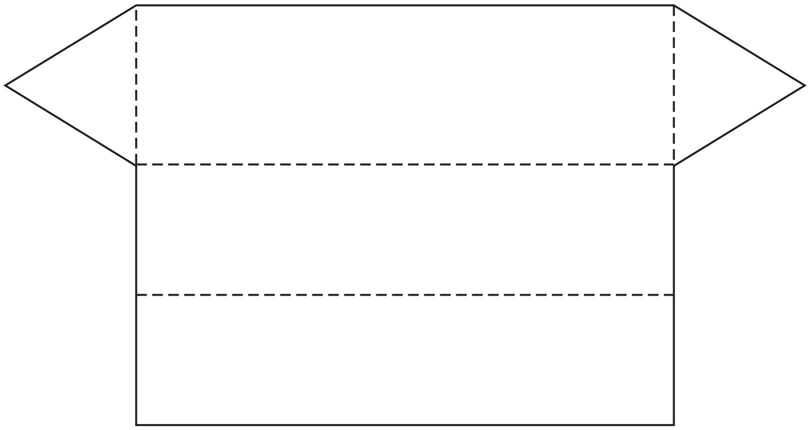
My Notes

To prepare for drawing and building the structures for the mini-golf course, the students explore relationships between shapes, views of shapes, and drawings of shapes.

A ***prism*** is a solid with two parallel congruent bases that are both polygons.

2. The ***net*** shows a two-dimensional pattern for a prism.

Figure 1



a. Cut out Figure 1 on the next page. Fold it along the dashed lines to form a prism.

b. A prism is named using the shape of its bases. Name the solid formed by the net in Figure 1.

c. The **faces** of a prism are the sides that are not the bases. What is the shape of the three faces?

d. **Reason abstractly.** Imagine making a slice through the prism parallel to the bases. What is the shape of the two-dimensional slice?

e. **Reason abstractly.** Imagine making a slice through the prism perpendicular to the bases. What is the shape of the two-dimensional slice?

MATH TERMS

A **net** is a a two-dimensional pattern that can be folded to form a solid.

My Notes

Figure 1

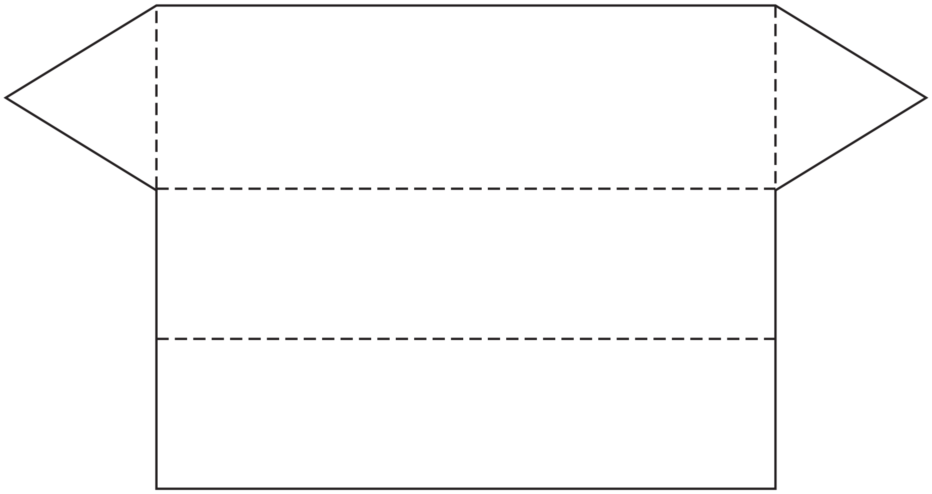
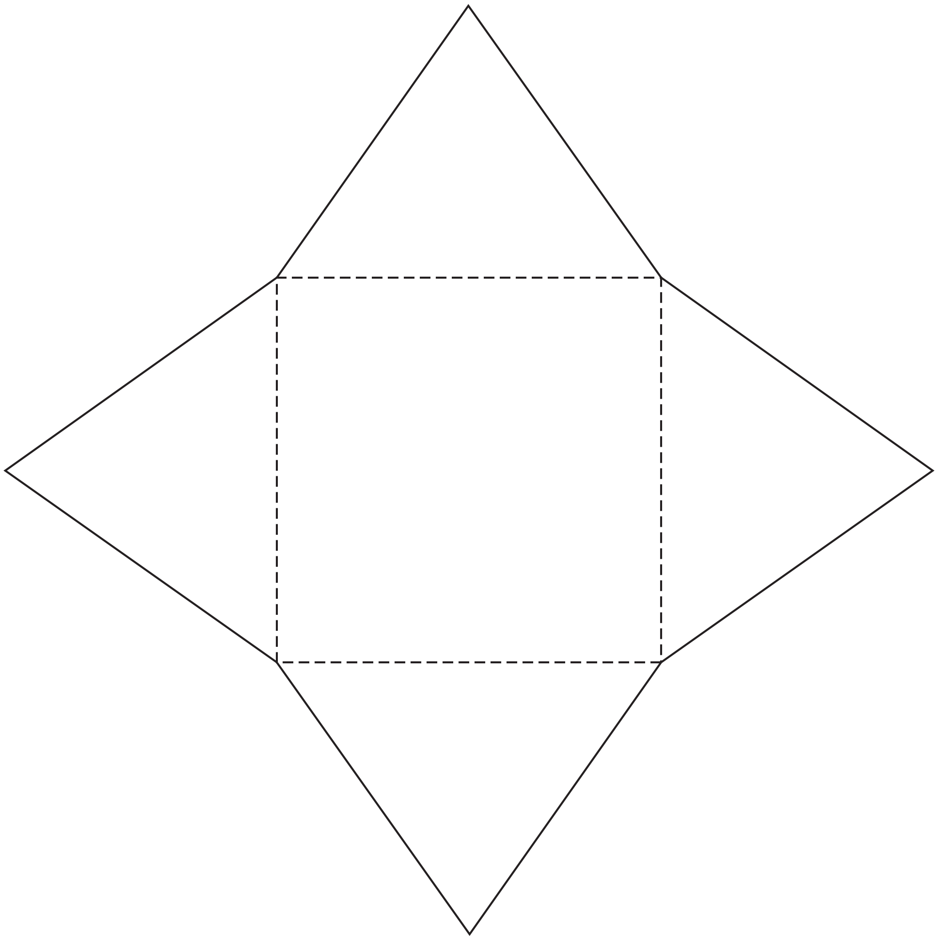


Figure 2



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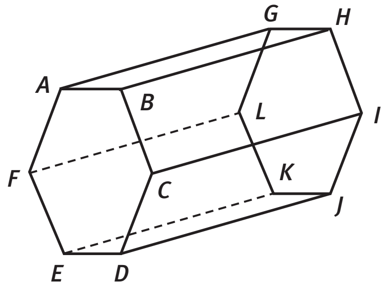
My Notes

3. Consider the hexagonal prism shown on the right.



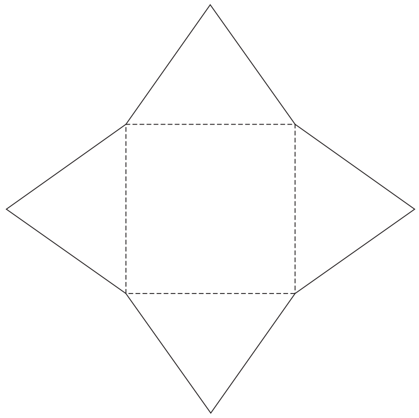
a. Why do you think some of the lines are dotted?

b. Imagine making a slice that goes through points *B*, *D*, *J*, and *H* of the prism. What is the shape of the two-dimensional slice?

A ***pyramid*** is a solid that has only one base. That base is a polygon. The faces of a pyramid are all triangles.

4. The net shows a pattern for a pyramid.

Figure 2



a. Cut out Figure 2 on page 193. Fold it to form a pyramid.

b. A pyramid is named using the shape of its base. What is the name of the solid formed by Figure 2?

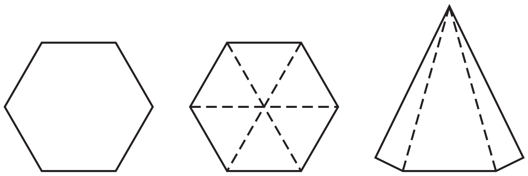
c. Reason abstractly. Imagine making a slice through the pyramid parallel to the base. What is the shape of the two-dimensional slice?

d. Model with mathematics. Sketch and label the view of the base, side, and top of the pyramid.

My Notes

5. Three views of a hexagonal pyramid are shown.

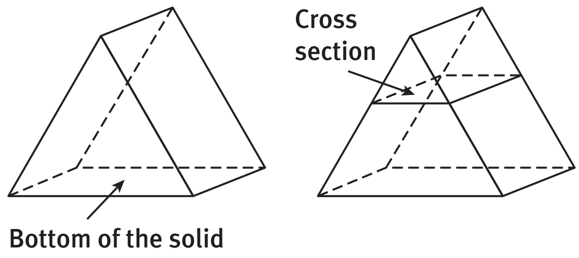
a. Label each view as *side*, *top*, or *base*.



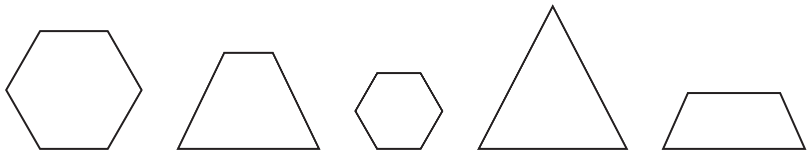
b. Explain the significance of the dashed segments in the second view.

6. Model with mathematics. Sketch and label the bottom, top, and side views of the Pentagon Building.

A ***cross section*** of a solid figure is the intersection of that figure and a plane.



7. Reason abstractly. Several cross sections of a hexagonal pyramid are shown. Label each cross section as *parallel* or *perpendicular* to the base of the pyramid.



Check Your Understanding

For Items 8–10, consider a rectangular pyramid.

8. Describe the shape of the base and the faces of the pyramid.

9. a. What shapes are formed by cross sections parallel to the base? Explain your thinking.

b. Are all of the cross sections parallel to the base the same size?

10. Construct viable arguments. Are all of the cross sections perpendicular to the base the same shape and size? Justify your answer.

My Notes

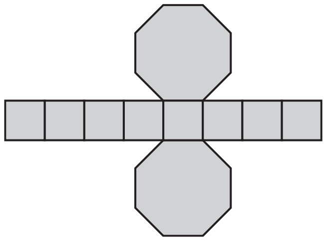
MATH TIP

You can think of a cross section as a slice of a solid that is parallel or perpendicular to the base of the solid.

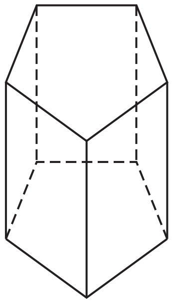
My Notes

LESSON 18-1 PRACTICE

11. What is the name of the solid formed by the net?



Use the solid to the right for Items 12–14.

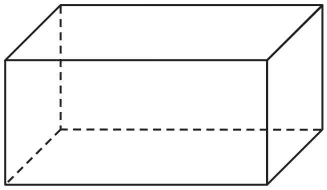


12. What is the name of the solid?

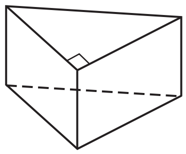
13. Reason abstractly. Imagine making slices through the solid parallel to the bases. What two-dimensional shapes are formed?

14. Reason abstractly. Imagine making slices through the solid perpendicular to the bases. What two-dimensional shapes are formed?

15. Model with mathematics. Sketch and label the bottom, top, and side views of the rectangular prism.



Use the solid to the right for Items 16 and 17.



16. Sketch the cross section that is parallel to the bases.

17. Sketch three different cross sections that are perpendicular to the bases.

For items 18 and 19, consider an octagonal pyramid.

18. Sketch two different cross sections that are parallel to the base of the pyramid.

19. Sketch three different cross sections that are perpendicular to the base of the pyramid.

20. Construct viable arguments. Can the cross section of a solid ever be a point? Explain your thinking.

21. Reason abstractly. How can the name of a prism or a pyramid help you visualize the cross sections of the solid?

My Notes

Learning Targets:

- Calculate the lateral and total surface area of prisms.

SUGGESTED LEARNING STRATEGIES: Use Manipulatives, Create Representations, Summarizing, Paraphrasing, Think-Pair-Share, Visualization, Discussion Groups

The students in the service club will paint the structures in the golf course. They first investigate how to find the surface area of prisms.

1. Work with your group. Look at Net 1, Net 2, and Net 3 on pages 203 and 204. What solids do the nets form?

A ***lateral face*** of a solid is a face that is not a base. A ***right prism*** is a prism on which the bases are directly above each other, making the lateral faces perpendicular to the bases. As a result, all the lateral faces are rectangles. The ***lateral area*** of a solid is the sum of the areas of the lateral faces.

2. For Nets 1–3, what are the shapes of the lateral faces of the figures? Explain.

3. **Attend to precision.** Find the area of each lateral face and the lateral area of each prism.

	Face 1	Face 2	Face 3	Face 4	Face 5	Lateral Area
Net 1						
Net 2						
Net 3						

GROUP DISCUSSION TIP

With your group, read the text carefully. Reread definitions of terms as needed to help you comprehend the meanings or words, or ask your teacher to clarify vocabulary terms.

MATH TIP

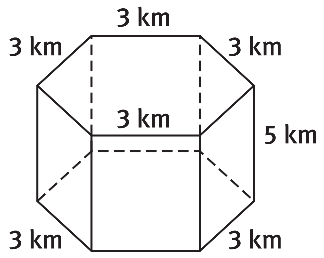
Net 1: Use the square units on the figure to find the lateral areas.

Net 2: Use the given measurements to find the lateral areas.

Net 3: Measure the side lengths to the nearest whole centimeter. Then find the lateral areas.

My Notes

4. Examine the right hexagonal prism.



a. Find the area of each lateral face. Show your work.

b. Find the lateral area of the solid. Show all your work.

c. Determine the perimeter of the base, *P*.

d. Multiply the perimeter of the base, *P*, times the height of the prism, *h*.

e. Compare your responses to Parts b and d. What do you notice?

Lesson 18-2
Lateral and Total Surface Area of Prisms

My Notes

The following formula can be used to find the lateral area of a prism: $L = P \times h$, where L represents the lateral area, P represents the perimeter of the base, and h represents the height of the prism.

5. Use the formula to find the lateral area of the prisms in Item 24 for Nets 1 and 2. Are the lateral areas the same as the ones you recorded in the table?
Net 1:

Net 2:

The ***surface area*** of a prism is the sum of the areas of the lateral faces and the areas of the bases.

6. Attend to precision. Describe the relationship between the lateral area and the surface area of a prism.

7. Reason quantitatively. Find the surface area of each of the prisms in Item 24. Explain your thinking.

Net 1:

Net 2:

Net 3:

Check Your Understanding

8. Make use of structure. Why do you think that the lateral area of a prism is equal to the product of the perimeter and the height of the prism?

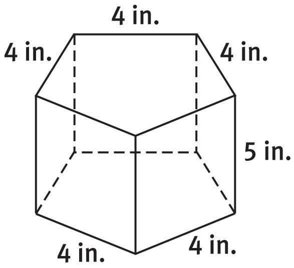
9. Construct viable arguments. Explain how to use a net to find the lateral and total surface area of a prism.

My Notes

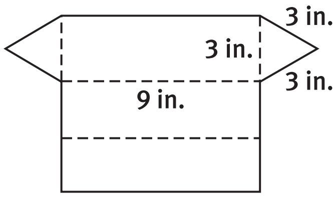
LESSON 18-2 PRACTICE

Find the lateral area of the prisms in Items 10 and 11.

10.



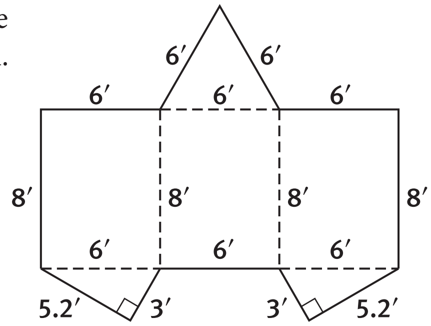
11.



Use the net to the right for Items 12 and 13.

12. Reason quantitatively. Find the lateral area of the triangular prism.

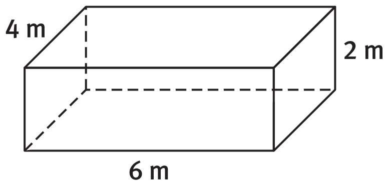
13. Find the surface area of the triangular prism.



Use the prism to the right for Items 14 and 15.

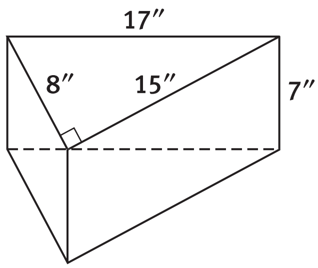
14. Model with mathematics. Draw a net to find the lateral area of the prism.

15. Find the surface area of the prism.



Use the prism for Items 16–18.

16. A display case is shaped like the prism shown. The case needs to be covered with a plastic film. How much film is needed to cover the lateral area?

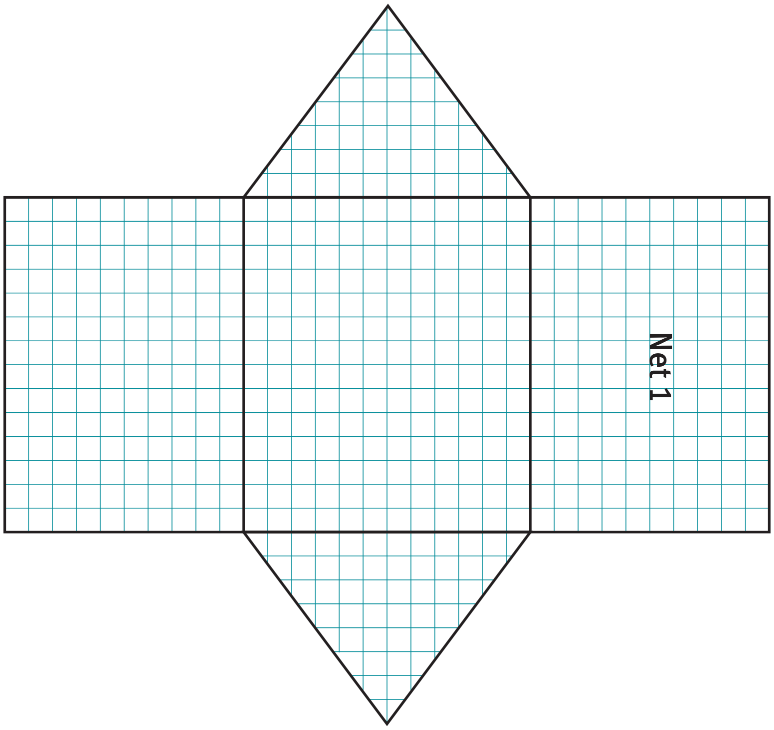


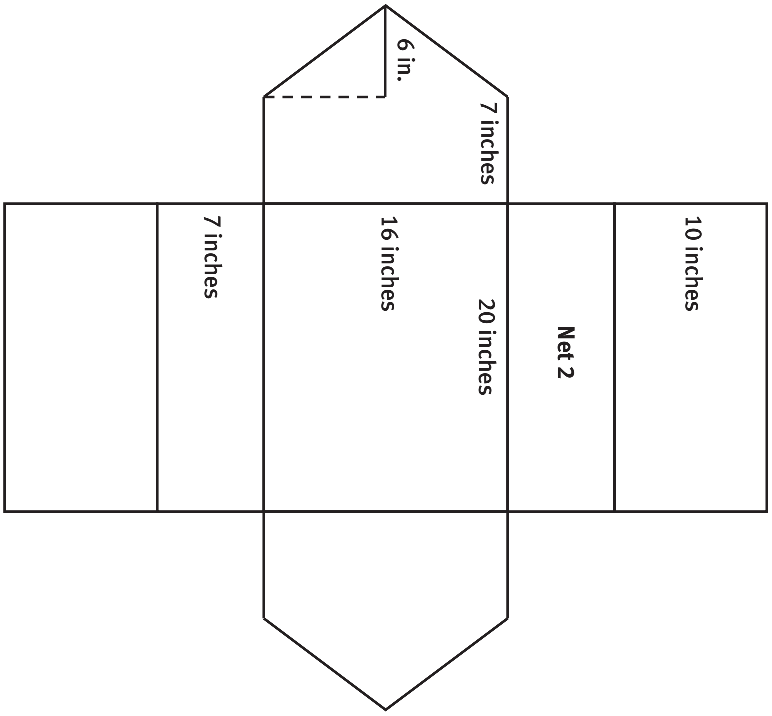
17. How much film is needed to cover the surface area of the display case?

18. Make sense of problems. How much film is needed to cover all but the face the case rests on?

19. Attend to precision. A cube-shaped block has edges that are 3 inches long. A larger block has edges that are twice as long. Compare the surface area of the smaller block to the surface area of the larger block. Support your answer.

My Notes





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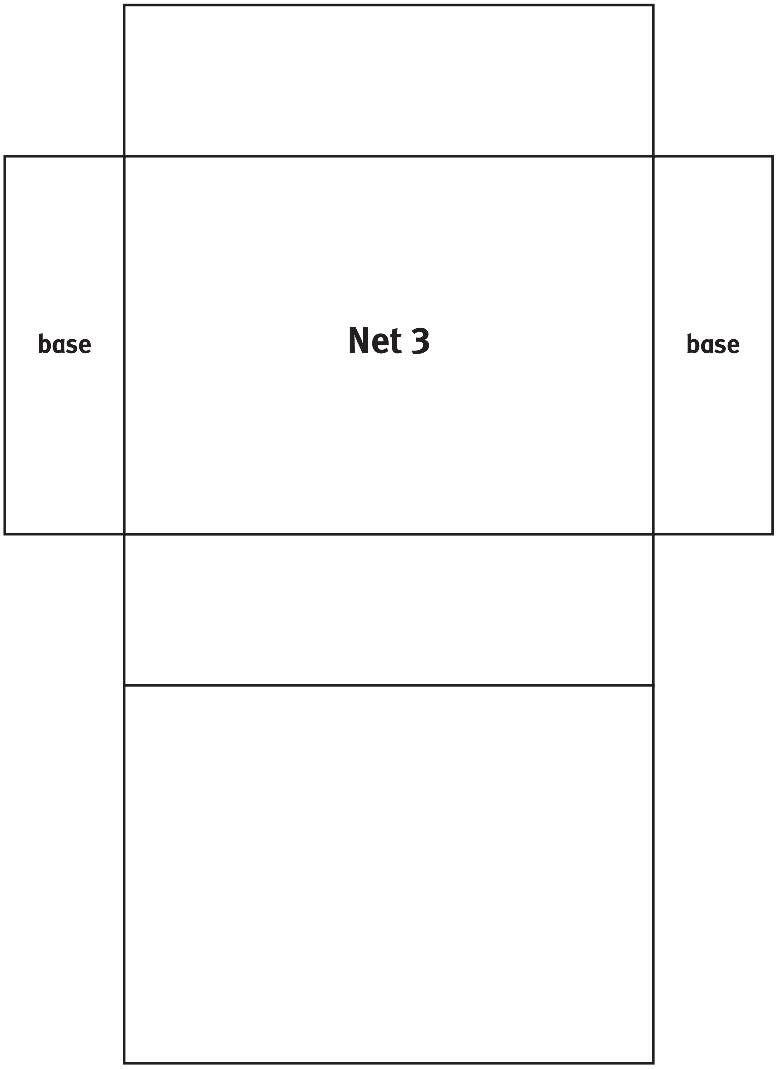


Learning Targets:

- Calculate the lateral and total surface area of pyramids.

SUGGESTED LEARNING STRATEGIES: Create Representations, Group Presentation, Marking the Text, Use Manipulatives, Visualization, Vocabulary Organizer

The students in the service club also investigate how to find the surface area of pyramids.

Two nets of pyramids the students use are on page 210.

The lateral area of a pyramid is the combined area of the faces. The height of a triangular face is the ***slant height*** of the pyramid.

1. Use Net 4, the net of the square pyramid.
   a. Draw the slant height on the net.

   b. Why do you think it is called the slant height?

2. Use Net 4 to find the lateral area of the square pyramid. Explain your thinking.

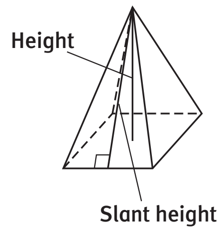
The surface area of a pyramid is the sum of the areas of the triangular faces and the area of the base.

3. Use Net 4 to find the surface area of the square pyramid. Explain your thinking.

My Notes

MATH TIP

Be sure you do not confuse the *slant height* of a pyramid with its *height*. Slant height is a measure along a triangular face. Height is an internal measure from the vertex to the base.



My Notes

CONNECT TO History

The triangular pyramid that can be created using Net 5 has four identical faces. It is an example of a Platonic solid. The Platonic solids are sometimes also called "cosmic figures." There are only five Platonic solids: cube, tetrahedron, dodecahedron, octahedron, and icosahedron.

4. **Attend to precision.** Use a centimeter ruler to measure Net 5. Then find the surface area of the triangular pyramid.

5. **Model with mathematics.** Consider a square pyramid with base edges 12 cm and slant height 10 cm.
   a. Draw a net to represent the pyramid and label the dimensions of the base edges and the slant height.

   b. Students found the lateral area of the square pyramid using the method shown below. Explain the steps of the method.

Step 1: $\frac{1}{2}\times12\times10+\frac{1}{2}\times12\times10+\frac{1}{2}\times12\times10+\frac{1}{2}\times12\times10$

Step 2: $=\frac{1}{2}\times(12+12+12+12)\times10$

Step 3: $=\frac{1}{2}\times(4\times12)\times10$

Step 4: $=\frac{1}{2}\times48\times10$

Step 5: $=240$ cm2

My Notes

The following formula can be used to find the lateral area of a regular pyramid:

$L = \frac{1}{2}P \times \ell$, where L represents the lateral area, P represents the perimeter of the base, and ℓ represents the slant height of the pyramid.

6. **Construct viable arguments.** A student says that the above formula can be used to find the lateral area of a rectangular pyramid. Is the student correct? Explain your reasoning.

7. **Make sense of problems.** The base of a regular triangular pyramid has sides that are 8 meters long and a height of 6.9 meters. The slant height of the pyramid is 6.9 meters.
   a. Find the lateral area of the pyramid. Explain your thinking.

   b. Find the surface area of the pyramid. Explain your thinking.

MATH TERMS

A **regular polygon** is a polygon with congruent sides and congruent angles.

A **regular pyramid**, also called a right regular pyramid, is a pyramid with a base that is a regular polygon, and all the lateral faces are congruent.

Check Your Understanding

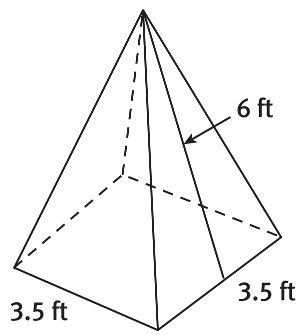
Write your answers on a separate piece of paper.

8. Why do you need to know the slant height, rather than the height, of a regular pyramid to find the surface area of the pyramid?
9. **Attend to precision.** Explain how to use a net to find the lateral and the total surface area of a pyramid.

My Notes

LESSON 18-3 PRACTICE

Use the square pyramid for Items 10 and 11.



10. **Model with mathematics.** Use a net to find the lateral area of the pyramid.

11. Find the surface area of the pyramid.

Use the following information for Items 12 and 13.

A regular triangular pyramid has a base length of 4.6 centimeters and a slant height of 4 centimeters.

12. Use a net to find the lateral area of the pyramid.

13. If the height of the triangular base is approximately 4 cm, what is the surface area of the pyramid?

Use the following information for Items 14 and 15.

The Louvre Pyramid has a square base with sides that are 35 meters long. The slant height of each triangular face of the pyramid is 27.84 meters.

14. What is the lateral area of the Louvre Pyramid?

15. What is the surface area of the Louvre Pyramid?

My Notes

16. A square pyramid has a slant height of 5 meters. The perimeter of the base is 32 meters. Find the surface area of the pyramid.

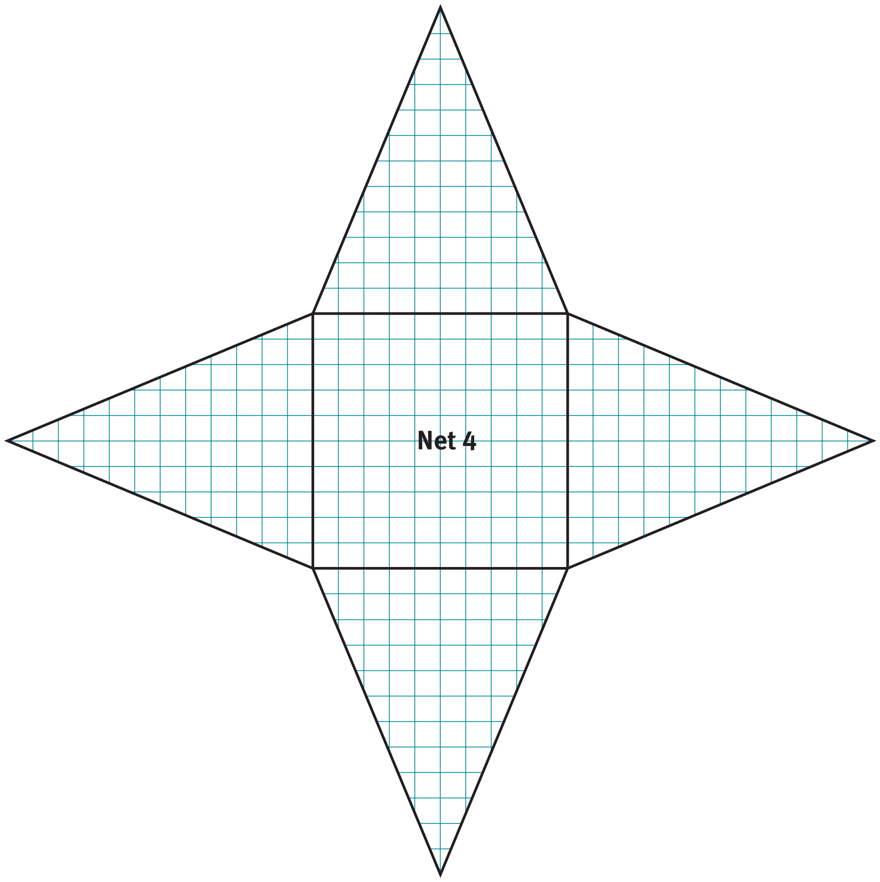
17. Construct viable arguments. A regular triangular pyramid has a base length of 3.5 meters, a height of 1.3 meters, and a surface area of 21 square meters. What is the approximate slant height of the pyramid? Justify your answer by showing how you found it.

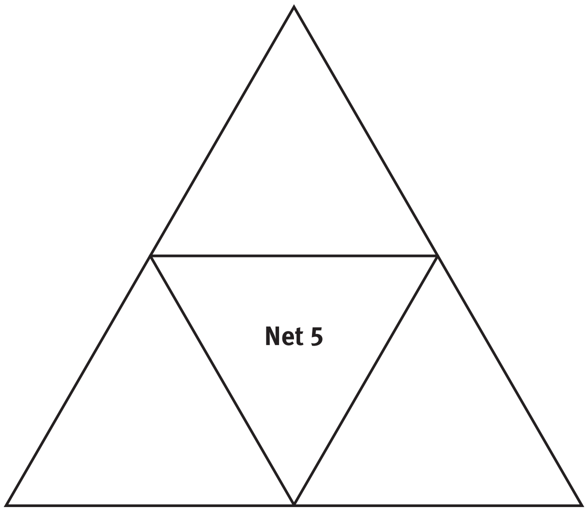
18. The pyramid of Khufu in Giza, Egypt, is a square pyramid with a base length of 756 feet. The slant height of this great pyramid is 612 feet. What is the lateral area of the pyramid of Khufu?

19. Make use of structure. A party favor is made from two square pyramids joined at their bases. Each edge of the square base is 3 centimeters. The slant height of the triangular faces is 4 centimeters. What is the surface area of the party favor?

20. Make sense of problems. A model of a Mayan pyramid has a square base with sides that are 1.3 meters long. The slant height of the pyramid is 0.8 meter. It costs $4.59 per square meter to paint the pyramid. How much will it cost to paint the lateral area of the model?

My Notes



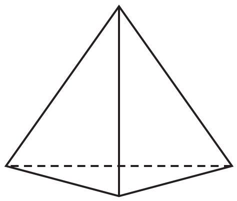


ACTIVITY 18 PRACTICE

Write your answers on a separate piece of paper. Show your work.

Lesson 18-1

Use the solid for Items 1 and 2.

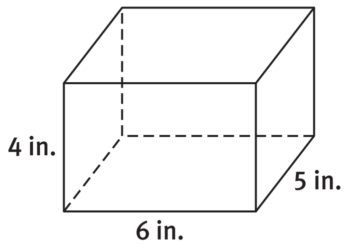


1. Imagine making slices through the solid parallel to the base. What two-dimensional shapes are formed?
2. Imagine making slices through the solid perpendicular to the base. What two-dimensional shapes are formed?

Lesson 18-2

Find the lateral and surface area of the figures in Items 3–5.

3.



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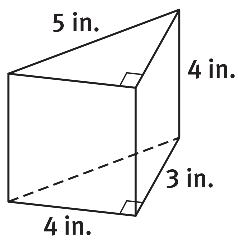
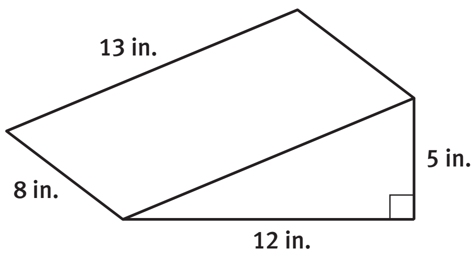
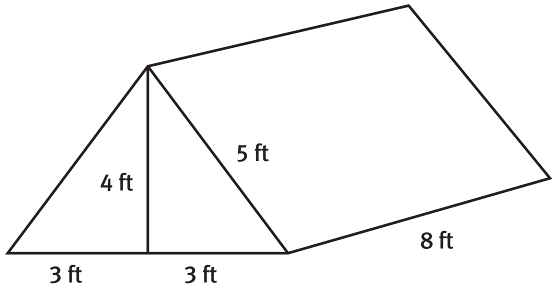


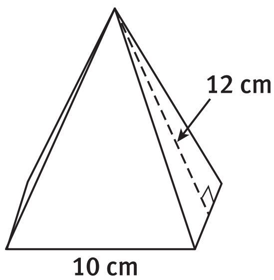
5.



6. A tent with canvas sides and a floor is shown. How much canvas is used to make the sides and floor of the tent?

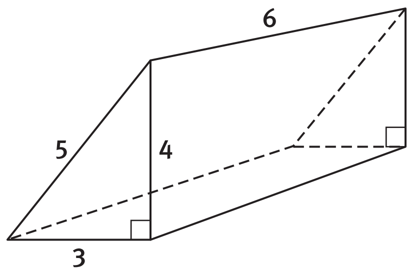


7. A rectangular prism is 10 meters tall. It has a square base with sides that are 4 meters long. What is the surface area of the prism?
8. Find the lateral and the surface area of the square pyramid.



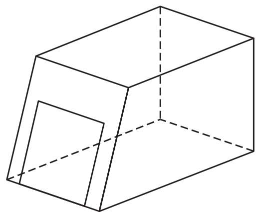
Lesson 18-3

9. The diagram shows the dimensions of a wooden block. The block will be covered with a reflective film. How much of the film is needed to cover the entire block?



A. 72 cm^2
B. 84 cm^2
C. 96 cm^2
D. 108 cm^2

Use the information and the drawing for Items 10 and 11. The Pup Company has a new model of dog kennel. The left and right sides are trapezoids and all other faces are shown in the diagram.



10. Sketch the top, front, and side views of the kennel.

11. Which cross sections of the kennel described below will be congruent?
A. all cross sections that are perpendicular to the bottom and parallel to the front and back faces
B. all cross sections that are perpendicular to the bottom and parallel to the left and right faces
C. all cross sections that are parallel to the top and bottom
D. none of the above

12. A cardboard box is 32 centimeters long, 15 centimeters wide, and 6 centimeters tall. The box does not have a top. How much cardboard was used to make the box?

13. The length of a side of the base of a square pyramid is 15 inches. The pyramid has a lateral area of 270 square inches. What is the slant height of the pyramid?

14. Three identical boxes are stacked by placing the bases on top of each other. Each box has a base that is 18 inches by 9 inches and is 4 inches tall. The stack of boxes will be shrink-wrapped with plastic. How much shrink-wrap is needed to cover the boxes?

15. A shed has the shape of a cube with edges that are 6 feet long. The top of the shed is a square pyramid that fits on top of the cube. The slant height of the faces is 5 feet. The shed has a single rectangular door that is 5 feet tall by 4 feet wide. All but the door and the bottom of the shed need to be painted. What is the area of the surface that needs to be painted?

MATHEMATICAL PRACTICES
Attend to Precision

16. Describe the similarities and differences in finding the lateral areas of a prism and a pyramid that have congruent bases.

Volume—Prisms and Pyramids

Berneen Wick's Candles

Lesson 19-1 Find the Volume of Prisms

Learning Targets:

- Calculate the volume of prisms.

SUGGESTED LEARNING STRATEGIES: Graphic Organizer, Look for a Pattern, Predict and Confirm, Use Manipulatives

Berneen makes all the candles that she sells in her shop, Wick's Candles. The supplies for each candle cost $0.10 per cubic inch. Berneen wants to find the volume of every type of candle she makes to determine the cost for making the candles.

Volume measures the space occupied by a solid. It is measured in cubic units.

1. Berneen uses unit cubes as models of 1-inch cubes.

a. Use unit cubes to build models of 2-inch cubes and 3-inch cubes. Then complete the table.

Length of Edge (in.)	Area of Face ($in.^2$)	Volume of Cube ($in.^3$)
1		
2		
3		

b. Make use of structure. Describe any relationships you see in the data in the table.

My Notes

MATH TIP

Cubes are named by the lengths of their edges. A 1-inch cube is a cube with edges that are 1 inch in length. A 2-inch cube is a cube with edges that are 2 inches in length.

Cubes of any size can be used to build larger cubes.

My Notes

The formula for the volume, V, of a cube, with edge length e, is $V = e^3$.

2. **Reason quantitatively.** One of Berneen's candles is a cube with sides that are 1.5 inches long.

   a. Use the formula to find the volume of this candle. Show your work.
   $V = e^3 = 1.5 \times 1.5 \times 1.5 =$

   b. Recall that supplies for each candle cost $0.10 per cubic inch. How much does it cost to make this candle, to the nearest cent?

Most of the candles Berneen makes are in the shape of rectangular prisms.

3. The formula for the volume of a cube is also equal to the area of the base times the height of the cube.

   a. Consider this relationship to help complete the table. Use cubes to build the prisms as needed.

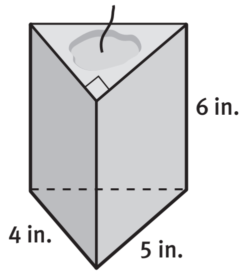
Dimensions of Candle (in.)	Area of Base (in.^{2})	Candle Height (in.)	Candle Volume (in.^{3})
$l = 4$ $w = 2$ $h = 1$			
$l = 4$ $w = 2$ $h = 2$			
$l = 4$ $w = 2$ $h = 3$			
$l = 5$ $w = 3$ $h = 1$			
$l = 5$ $w = 3$ $h = 2$			

   b. Describe any pattern you see for finding volume.

My Notes

The volume, V, of a prism is the area of the base, B, times the height, h: $V = B \times h$.

4. Berneen makes a candle in the shape of a triangular prism as shown. The candle is very popular with many customers because of its interesting shape.



a. What is the volume of the candle? Explain your thinking.

b. Make sense of problems. Remember that the cost of the supplies for each candle is $0.10 per cubic inch. How much profit will Berneen make if she sells this candle for $8.99? Show your work.

Check Your Understanding

5. Construct viable arguments. Can a rectangular prism and a cube have the same volume? Support your opinion with an example or counterexample.

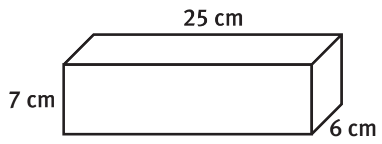
6. Make use of structure. How does the volume of a prism change when one dimension is doubled? When two dimensions are doubled? When three dimensions are doubled? Explain your thinking.

My Notes

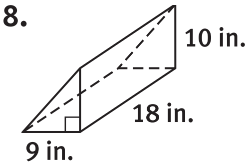
LESSON 19-1 PRACTICE

Find the volume of each figure.

7.



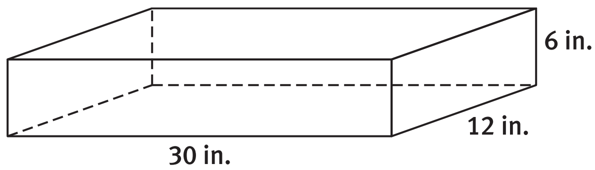
8.



9. A cube with edge length 7 centimeters.

10. A cube that has a face area of 25 square inches.

Use the prism for Items 11 and 12.



11. How many cubes with a side length of 3 inches will fit into the rectangular prism? Explain.

12. Find the volume of the prism.

13. A small refrigerator has a square base with sides that are 3 feet long. The refrigerator has a capacity of 40.5 cubic feet. How tall is the refrigerator?

14. **Reason quantitatively.** How many cubic feet are equivalent to 1 cubic yard? Explain.

15. **Make sense of problems.** The Gray family is putting in a pool in the shape of a rectangular prism. The first plan shows a pool that is 15 feet long, 12 feet wide, and 5 feet deep. The second plan shows a pool with the same length and width, but a depth of 7 feet. How much more water is needed to fill the second pool if both pools are filled to the top?

16. **Attend to precision.** A student says that the volume of a triangular prism with a base area of 12 meters and a height of 5 meters is 60 square meters. Is the student correct? If not, what is wrong with the student's statement?

My Notes

Learning Targets:

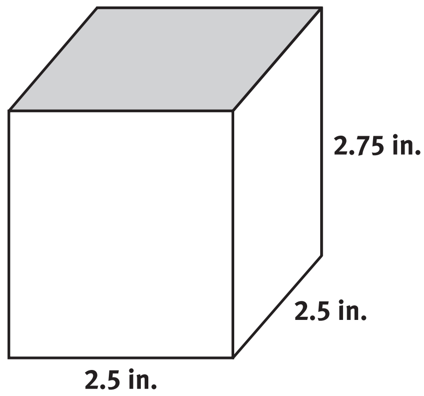
- Calculate the volume of pyramids.
- Calculate the volume of complex solids.
- Understand the relationship between the volume of a prism and the volume of a pyramid.

SUGGESTED LEARNING STRATEGIES: Create Representations, Look for a Pattern, Predict and Confirm, Think-Pair-Share, Use Manipulatives

1. Other candles in Wick's Candles are in the shape of pyramids. To find the volumes, Berneen makes models to look for a relationship between the volume of a prism and the volume of a pyramid.

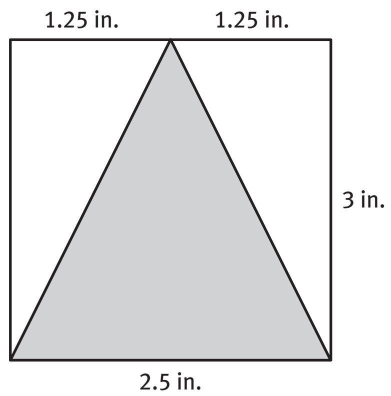
a. Make a model of the prism candle mold.

- Use index cards or card stock to cut out 1 square with side length 2.5 inches and 4 rectangles with length 2.5 inches and width 2.75 inches.
- Tape them together to form a net for a rectangular prism with no top. Then fold the net and tape it together to form a rectangular prism with no top as shown.

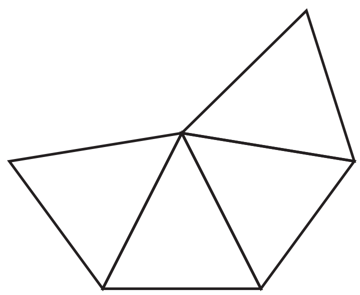


b. Make a model of the pyramid candle mold.

- Use index cards or card stock to cut out 4 isosceles triangles with the dimensions shown in the diagram.



- Tape the triangles together along their congruent sides to form a net for a square pyramid, as shown.
- Tape the net together to form a square pyramid.



My Notes

2. Compare the dimensions of the prism and the square pyramid you built in Item 1. What relationships do you notice?

3. Using the material your teacher distributes, fill your pyramid to the top. Predict the number of times you can fill and empty the pyramid into the rectangular prism to fill the prism to the top. Confirm your prediction by filling and emptying the pyramid into the prism.

The volume, V, of a pyramid is one-third the area of the base, B, times the height, h:

$$V = \frac{1}{3} \times B \times h$$

4. **Reason quantitatively.** Two of Berneen's candles have congruent rectangular bases. One candle is shaped like a rectangular prism, while the other is shaped like a rectangular pyramid. Both candles are 5 inches tall. What is the relationship between the volumes of the two candles? Explain.

5. **Make sense of problems.** A candle in the shape of a square pyramid has a base edge of 6 inches and a height of 6 inches.

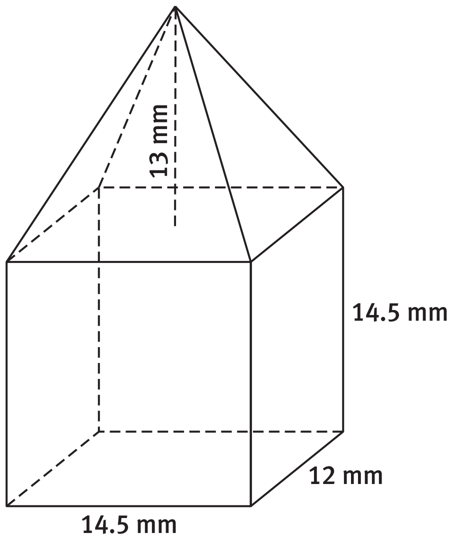
a. What is the volume of the candle?

b. How much does it cost Berneen to make the candle? Show your work.

My Notes

A **complex solid** is formed when two or more solids are put together. The volume of the complex solid is the sum of the volumes of the smaller solids.

6. Consider the complex solid shown.



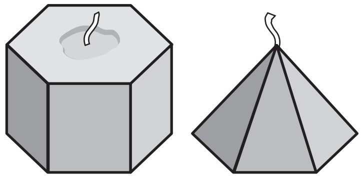
a. Find the volume of the rectangular prism.

b. Find the volume of the rectangular pyramid.

c. Find the volume of the complex solid.

Check Your Understanding

Use the candles for Items 7 and 8.



7. Express regularity in repeated reasoning. The candles shown have congruent bases and heights. What is true about the relationship between the volumes of the candles?

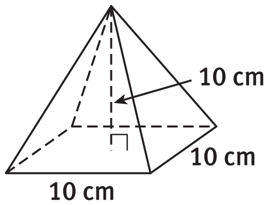
8. Model with mathematics. Suppose Berneen makes a candle by setting the pyramid on top of the prism. Write a formula for the volume of this candle.

My Notes

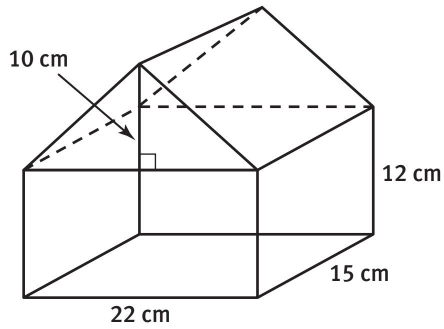
LESSON 19-2 PRACTICE

Find the volume of each figure in Items 9–11.

9.



10.



11. A triangular pyramid with a base area of 18 square inches and a height of 15 inches.

12. How does the volume of the triangular pyramid in item 11 compare with the volume of a triangular prism with a base area of 18 square inches and a height of 15 inches? Use words and symbols to explain.

13. The area of the base of a pyramid is 85 square centimeters. If its volume is 255 cubic centimeters, find the height of the pyramid.

14. A square pyramid 9 feet tall has a volume of 507 cubic feet. How long is each side of the base of the pyramid?

15. Two square pyramids are joined at their bases. Each base is 30 millimeters long. The distance between the vertices of the combined pyramids is 28 millimeters. What is the volume of the solid formed?

16. **Reason quantitatively.** A square pyramid with base lengths of 6 inches is 14 inches tall. The top part of the pyramid is cut off to form a smaller pyramid with base lengths that are 3 inches long and a height of 7 inches. How many square inches greater was the volume of the larger pyramid than that of the new smaller pyramid?

17. **Make sense of problems.** The Pyramid of Cestius in Rome today stands about 27 meters tall with a square base whose sides are about 22 meters long. The pyramid was based on ancient Nubian pyramids. These pyramids average a base area of 25.5 square meters and a height of 13.5 meters. How does the volume of the Pyramid of Cestius compare to the volume of an average Nubian pyramid?

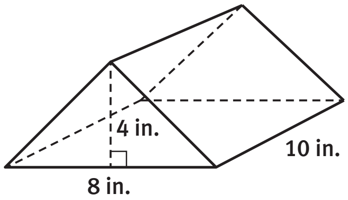
ACTIVITY 19 PRACTICE

Write your answers on a separate piece of paper. Show your work.

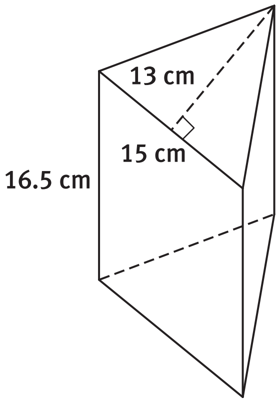
Lesson 19-1

For Items 1–4, find the volume of each figure.

1.



2.



3. A cube with edge length 8 inches.
4. A rectangular prism with sides that are 1.2, 1.8, and 2.5 meters long.
5. A rectangular prism with a square base is 6.4 meters tall. The prism has a volume of 409.6 cubic meters. What are the dimensions of the base of the prism?
6. Mariah is filling a terrarium in the shape of a rectangular prism with sand for her tarantula. The sand will be one-quarter of the way to the top. If the length of the terrarium is 17 inches, the width 12 inches, and the height 12 inches, what is the volume of the sand she uses?

Lesson 19-2

7. A container in the shape of a rectangular prism has a base that measures 20 centimeters by 30 centimeters and a height of 15 centimeters. The container is partially filled with water. A student adds more water to the container and notes that the water level rises 2.5 centimeters. What is the volume of the added water?
   A. 1,500 cm^3
   B. 3,600 cm^3
   C. 4,500 cm^3
   D. 9,000 cm^3

For Items 8–11, find the volume of the figure described.

8. A triangular pyramid with a base area of 43.3 meters and a height of 12 meters.
9. A square pyramid with base edge 10 centimeters and height 12 centimeters.
10. A triangular pyramid with a base length of 9 inches, a base height of 10 inches, and a height of 32 inches.
11. A square pyramid with a base length of 4 centimeters and a height of 6 centimeters resting on top of a 4-centimeter cube.
12. The area of the base of a triangular pyramid is 42 square feet. The volume is 1,197 cubic feet. Find the height of the pyramid.
13. The square pyramid at the entrance to the Louvre Museum in Paris, France, is 35.42 meters wide and 21.64 meters tall. Find the volume of the Louvre Pyramid.

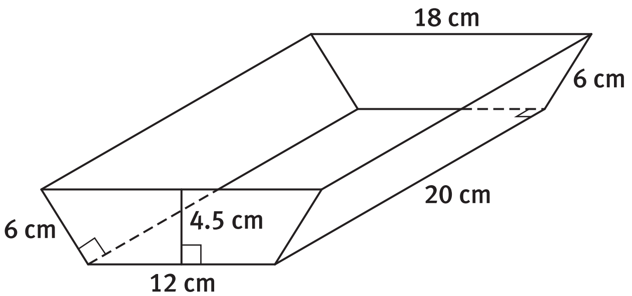
14. For prisms and pyramids, how are the area of the base of the solid *and* the shape of the solid related to the volume?

15. a. A triangular prism and a triangular pyramid have congruent bases and heights. What is the relationship between the volumes of the two figures? Explain in words using an example.

b. Explain the relationship between the volumes using their formulas.

16. A plastic tray is shown, with the dimensions labeled. The bottom and two of the sides are rectangles. The other two sides are congruent isosceles trapezoids. What is the volume of the tray?

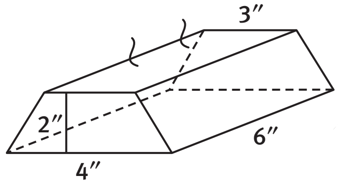
A. 1,350 cm^3
B. 1,080 cm^3
C. 1,440 cm^3
D. 1,620 cm^3



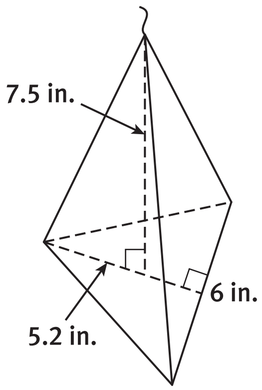
MATHEMATICAL PRACTICES
Attend to Precision

17. Berneen Wick wants to offer a gift set containing the three candles shown. Remember: The cost per cubic inch of a candle is $0.10. Prepare a report for Berneen in which you provide her with:

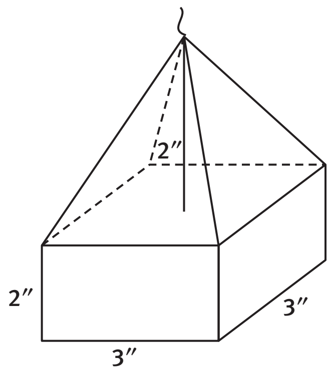
- a name and a cost for each candle and the method of calculating each cost
- your recommendation for the price of the gift set
- your reasons for the recommendation



Candle 1



Candle 2



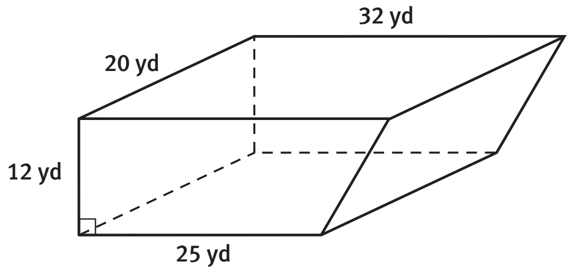
Candle 3

Surface Area and Volume
UNDER THE SEA

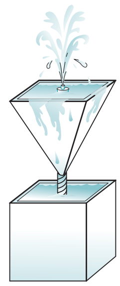
Mackeral "Mack" Finney is designing a new aquarium called Under the Sea. He plans to include several different types of saltwater tanks to house the aquatic life.

1. Mack begins by designing the smallest fish tank. This tank is a rectangular prism with dimensions 4 feet by 2 feet by 3 feet.
  a. Draw and label a net to represent the aquarium.
  b. The tank will have a glass covering on all six sides. Find the surface area of the tank. Explain your reasoning.
  c. Find the volume of the tank. Show your work.

2. Near the main entrance to the aquarium, Mack has decided to put a larger pool for four dolphins. Its shape is the trapezoidal prism shown.
  a. Sketch and label the dimensions of a cross section parallel to the bases of the prism.
  b. Find the amount of water needed to fill the pool. Explain your thinking.

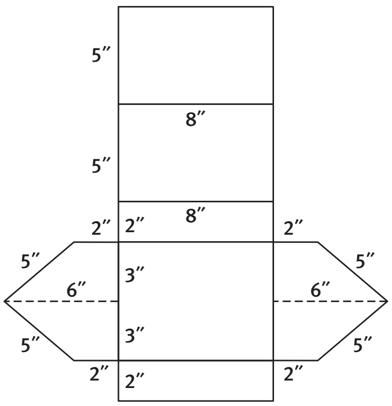


3. Mack designed a water fountain with a square pyramid flowing into a cube, as shown at right. The edges of the bases of the pyramid and the cube have the same length and the heights of the pyramid and the cube are the same. Describe the relationship between the volume of the cube and the volume of the pyramid.

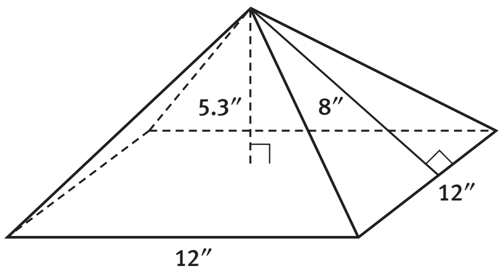


In addition to tanks for the aquatic life, Mack designs some hanging birdhouses for the trees around the aquarium.

4. The net for one birdhouse is shown below. What is the total surface area of the solid? Show your work.



5. Another birdhouse design is in the shape of a square pyramid, as shown below. Find the surface area and volume of the birdhouse.



Surface Area and Volume
UNDER THE SEA

Scoring Guide	Exemplary	Proficient	Emerging	Incomplete
	The solution demonstrates these characteristics:			
Mathematics Knowledge and Thinking (Items 1a-c, 2b, 3, 4, 5)	• Accurately and efficiently finding the surface area and volume of prisms and pyramids.	• Finding the surface area and volume of prisms and pyramids.	• Difficulty finding the surface area and volume of prisms and pyramids.	• No understanding of finding the surface area and volume of prisms and pyramids.
Problem Solving (Items 1b-c, 2b, 4, 5)	• An appropriate and efficient strategy that results in a correct answer.	• A strategy that may include unnecessary steps but results in a correct answer.	• A strategy that results in some incorrect answers.	• No clear strategy when solving problems.
Mathematical Modeling / Representations (Items 1a-b, 2a, 4, 5)	• Clear and accurate understanding of how a net represents a three-dimensional figure.	• Relating a net to the surfaces of a three-dimensional figure.	• Difficulty recognizing how a net represents a three-dimensional figure.	• No understanding of how a net represents a three-dimensional figure.
Reasoning and Communication (Items 1b, 2b, 3)	• Precise use of appropriate terms to explain finding surface area and volume of solids. • A precise and accurate description of the relationship between the volume of a pyramid and a cube.	• An adequate explanation of finding surface area and volume of solids. • A basically correct description of the relationship between the volume of a pyramid and a cube.	• A partially correct explanation of finding surface area and volume of solids. • A partial description of the relationship between the volume of a pyramid and a cube.	• An incomplete or inaccurate explanation of finding surface area and volume of solids. • A partial description of the relationship between the volume of a pyramid and a cube.