

AQR Notes on Graphs, Paths, and Circuits

Graph:

A collection of vertices (nodes) connected by edges. The degree of each vertex is the number of edges connected to that vertex.

Path:

Trace the graph without lifting the pencil from the paper.

Euler (or Eulerian) path:

Trace the graph without lifting the pencil from the paper and without tracing the same edge more than once. Vertices may be passed through more than once. The starting and ending points don't have to be the same. A graph has an Euler path if and only if it has no more than two vertices of odd degree.

Euler (or Eulerian) circuit:

An Euler circuit is similar to an Euler path, except that the starting and ending points must be the same. A graph has an Euler circuit if and only if every vertex has an even degree.

Hamilton (or Hamiltonian) path:

Trace the graph without lifting the pencil from the paper and without visiting a vertex more than once (visit every vertex exactly once). The starting and ending points don't have to be the same. The only known way to determine whether a given graph has a Hamilton path is by trial-and-error.

Hamilton (or Hamiltonian) circuit:

A Hamilton circuit is similar to a Hamilton path, but the starting and ending points must be the same.

Cycle:

A cycle is a path that starts and stops at the same vertex without using an edge more than once. A graph that doesn't have any cycles is an "efficient network."

To find an efficient network, identify any cycle in the graph and remove one of its edges. Repeat until there are no cycles.