Name ____________________ Date ____________

# BENCHMARK 1
*(Chapters 1 and 2)*



## A. Real Numbers and Expressions

The set of real numbers consists of all rational and irrational numbers. Rational numbers can be expressed as fractions, and irrational numbers can not be expressed as fractions. All real numbers can be graphed on a number line, as seen below. The operations that combine real numbers have specific properties, which can be extended directly to algebraic expressions.

### 1. Graph Real Numbers on a Number Line

**Vocabulary**

**Number Line** Real Numbers can be graphed as points on a line called a *real number line*, on which numbers increase from left to right.

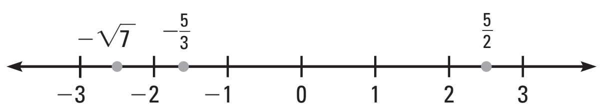
**EXAMPLE** **Graph the following real numbers on a number line.**

**a.** $\frac{5}{2}$ **b.** $-\frac{5}{3}$ **c.** $-\sqrt{7}$

**Solution:**

When graphing rational numbers, as in a. and b., consider the mixed number expression. That is, $\frac{5}{2} = 2\frac{1}{2}$ and $-\frac{5}{3} = -1\frac{2}{3}$. To graph the number on the number line, begin at zero and move to the integer part of the number. Then move the fractional part of the next unit (in the already determined direction) to graph the number.

Note that $\sqrt{7}$ is between $\sqrt{4} = 2$ and $\sqrt{9} = 3$. The calculator approximation of $\sqrt{7} \cong 2.6$ supports this insight.




**PRACTICE** **Graph the following real numbers on a number line.**

**1.** $\frac{3}{5}$ **2.** $-\frac{11}{3}$ **3.** $\sqrt{15}$

### 2. Use Properties and Definitions of Operations

**Vocabulary**

**Opposite** The opposite, or additive inverse, of any number $b$ is $-b$.

**Reciprocal** The reciprocal, or multiplicative inverse, of any nonzero number $b$ is $\frac{1}{b}$.

**Subtraction:** $a - b = a + (-b)$

**Division:** $a \div b = a \cdot \left(\frac{1}{b}\right), b \neq 0$

**Properties of addition and multiplication** Let $a$, $b$, and $c$ be real numbers:

1. **Closure:** $a + b$ and $a \cdot b$ are real numbers
2. **Commutative:** $a + b = b + a$ and $a \cdot b = b \cdot a$
3. **Associative:** $(a + b) + c = a + (b + c)$ and $(a \cdot b) \cdot c = a \cdot (b \cdot c)$
4. **Identity:** $a + 0 = 0 + a = a$ and $a \cdot 1 = 1 \cdot a = a$
5. **Inverse:** $a + (-a) = 0$ and $a \cdot \left(\frac{1}{a}\right) = 1, a \neq 0$
6. **Distributive:** $a \cdot (b + c) = a \cdot b + a \cdot c$

Name ______________________ Date ______________



# BENCHMARK 1

*(Chapters 1 and 2)*

**EXAMPLE**

Be aware that subtraction and division *do not* possess the commutative and associative properties. But by rewriting them as addition and multiplication, we can apply these useful properties!

**Use properties and definitions of operations to show that $10 \cdot (x \div 2) = 5x$. Justify each step.**

**Solution:**

$10 \cdot (x \div 2) = 10 \cdot \left(x \cdot \frac{1}{2}\right)$ **Definition of division.**

$= 10 \cdot \left(\frac{1}{2} \cdot x\right)$ **Commutative property of multiplication.**

$= \left(10 \cdot \frac{1}{2}\right) \cdot x$ **Associative property of multiplication.**

$= (5)\,x = 5x$ **By multiplication.**

**PRACTICE**

**Using real numbers, give and verify an example of the following properties.**

**4.** Closure property of multiplication

**5.** Commutative property of addition

**6.** Associative property of addition

**7.** Associative property of multiplication

**Use properties and definitions of operations to show the following. Justify each step.**

**8.** $(2 \div x) \cdot 7 = \frac{14}{x}, x \neq 0$

**9.** $(5 + c) - 5 = c$

## 3. Use Unit Analysis

**Vocabulary**

**Unit Analysis** Observing the units of measurement serves two main functions. First, it provides a quick check of the validity of a solution. Secondly, it can be used to help converting quantities from one unit of measurement to another.

**EXAMPLE**

Switching the numerator and denominator would yield the unit "minutes per word". In this rate, a smaller number would indicate a faster typist. Check this with a person who can type 500 words in 5 minutes.

**You can type 300 words in 5 minutes. What is your typing rate?**

**Solution:**

$\frac{300 \text{ words}}{5 \text{ minutes}} = 60$ words per minute

Using unit analysis, our answer is in words per minute, which is an acceptable unit for measuring a typing rate.

**EXAMPLE**

**The speed of light is approximately 300,000 kilometers per second. What is the approximate speed of light in miles per hour?**

**Solution:**

$\left(\frac{300{,}000 \text{ kilometers}}{1 \text{ second}}\right) \cdot \left(\frac{60 \text{ seconds}}{1 \text{ minute}}\right) \cdot \left(\frac{60 \text{ minutes}}{1\text{hour}}\right) \cdot \left(\frac{0.625 \text{ mile}}{1 \text{ kilometer}}\right)$

$= 675{,}000{,}000$ miles per hour

# BENCHMARK 1

*(Chapters 1 and 2)*



Note that unit analysis helped us determine how to write each unit multiplier we used. For instance, we multiplied our original rate by $\frac{60 \text{ seconds}}{1 \text{ minute}}$, which equals 1. Note that $\frac{1 \text{ minute}}{60 \text{ seconds}}$ also equal 1, but this would be an unfortunate choice. Our units "seconds" would fail to cancel.

**PRACTICE**

**10.** You travel 210 miles while driving at a rate of 60 mile per hour. How long have you been driving?

**11.** Convert 1 yard to centimeters, given 1 inch = 2.54 centimeters.

## 4. Evaluate an Algebraic Expression

**Vocabulary**

**Variable** A *variable* is a letter that is used to represent one or more numbers.

**Algebraic Expression** An expression involving variables is called an *algebraic expression.*

**Evaluate** When you substitute a number for each variable in an algebraic expression and simplify, you are *evaluating* the algebraic expression.

**EXAMPLE**

Conscientiously apply the order of operations. Don't take too many steps at once, as this can result in an incorrect answer.

**Evaluate $2k^2 - 3k + 10$ when $k = -2$.**

**Solution:**

| | |
|---|---|
| $2k^2 - 3k + 10 = 2(-2)^2 - 3(-2) + 10$ | **Substitute −2 for *k*.** |
| $= 2(4) - 3(-2) + 10$ | **Evaluate the powers first.** |
| $= 8 - (-6) + 10$ | **Perform the multiplications.** |
| $= 8 + 6 + 10$ | **Rewrite the subtraction as an addition.** |
| $= 24$ | **Add.** |

**PRACTICE**

**Evaluate the expression for the given value of the variable.**

**12.** $4x^2 - 3x + 11$ when $x = 0$

**13.** $2w - 10 \div (w + 6)$ when $w = -4$

## 5. Simplify an Algebraic Expression

**Vocabulary**

**Terms** In an expression that can be written as a sum, the parts added together.

**Variable Term** A term that has a variable part.

**Constant Term** A term that has no variable part.

**Coefficient** The number when a term is a product of a number and a power of a variable.

**Like Terms** Terms that have the same variable parts.

Name ______________________ Date ______________



# BENCHMARK 1

*(Chapters 1 and 2)*

**EXAMPLE**

Combine like terms by simply combining their coefficients. Be sure to include the sign of the coefficient when combining them.

Once the coefficients are combined, *do not* combine the variable parts of the like terms.

**Simplify by combining like terms.**

**a.** $7x^2 - 3x + 1 - 2x^2 - 4$

$7x^2 - 3x + 1 - 2x^2 - 4 = (7x^2 - 2x^2) - 3x + (1 - 4)$ **Group like terms**

$= 5x^2 - 3x - 3$ **Combine like terms**

**b.** $3(2x - 5y + 3) - 4(x + 2y - 1)$

$3(2x - 5y + 3) - 4(x + 2y - 1)$

$= 6x - 15y + 9 - 4x - 8y + 4$ **Apply distributive property**

$= (6x - 4x) + (-15y - 8y) + (9 + 4)$ **Group like terms**

$= 2x - 23y + 13$ **Combine like terms**

**PRACTICE**

**Simplify the expression.**

**14.** $2r + 3s - 4t - 3r - 4s + 3t$

**15.** $x^2 - 3xy + 2y^2 + 4x^2 - xy + y$

**16.** $3(w - 5) - 2(w + 2)$

**17.** $y(y - 3) + 2(y^2 + 5)$

## Quiz

**Graph the following real numbers on a number line.**

**1.** $-\frac{23}{8}$

**2.** $\frac{13}{7}$

**3.** $-\sqrt{31}$

**Using real numbers, give and verify an example of the following properties.**

**4.** Inverse property of multiplication

**5.** Distributive property

**Use properties and definitions of operations to show the following. Justify each step.**

**6.** $a + (b - c) = (b + a) - c$

**7.** $(6 \cdot b) \div 6 = b$

**8.** Given 1 U.S. dollar = 116 Japanese yen, and 1 Japanese yen = 11,442 Turkish liras, find the value of 25 U.S. dollars in Turkish liras.

**Evaluate the expression for the given value of the variable.**

**9.** $4d - 11d^2$ when $d = -1$

**10.** $2w - 10 \div (w + 6)$ when $w = -4$

**Simplify the expression.**

**11.** $3(x - y) - y(x - 3)$

**12.** $a(a^2 - a) + 3(a^3 - 2a^2)$

Name ____________________ Date ____________

# BENCHMARK 1
*(Chapters 1 and 2)*



## B. Problem Solving

The ability to solve word problems should be one the highest priorities for every student in a mathematics class. The skills necessary to successfully solve a word problem are accessible to everyone, but often neglected. Often times, solving these problems is attempted half-heartedly, or not at all! The three skills presented here can help make word problems a pleasure, instead of something to dread. The only way to achieve this is to invest the necessary practice time, with a positive attitude along the way.

### 1. Write a Verbal Model

**Vocabulary**

**Verbal Model** A verbal model is a mathematical expression with words, instead of numbers and variables.

**EXAMPLE**

**You are planning on baking cookies for holiday gifts. You already have 36 cookies baked, and you can bake 24 cookies every hour.**

- Write an expression that shows how many cookies you will have ready after $h$ hours.
- How many cookies will you have after 5 hours?

**Solution:**

**STEP 1:** Write a verbal model. Then write an algebraic expression.

Cookies baked per hour • Hours + Number of cookies already baked

⇩ ⇩ ⇩

$24 \cdot h + 36$

An expression that shows how many cookies you will have after $h$ hours is $24h + 36$.

A word problem should have a word answer. Always write your final answer as a sentence.

**STEP 2:** Evaluate the expression from step 1 when $h = 5$.

$24h + 36 = 24(5) + 36$ **Substitute 5 for *h*.**

$= 120 + 36$ **Multiply.**

$= 156$ **Add.**

You will have 156 cookies after 5 hours.

**PRACTICE**

**Write an expression and answer the question.**

**1.** A boat traveling 22 miles per hour can cover a distance of 121 miles before needing to refuel. At this speed, how many hours can the boat travel before needing to refuel?

**2.** A supermarket has 8 pounds of a bulk trail mix that is worth \$2.25 per pound. How much money does the supermarket have invested in this trail mix?

Name ______________________ Date ______________

BENCHMARK 1
B. Problem Solving

# BENCHMARK 1

*(Chapters 1 and 2)*

## 2. Look for a Pattern

**EXAMPLE** **A coffee shop has fixed daily expenses which include rent, insurance, utilities and wages. The coffee shop also has variable expenses per cup of coffee served. The table below shows the total daily expense *e* when *c* cups of coffee are served. Find the total daily expenses for the coffee shop if 150 cups of coffee were served.**

| **Cups Served, *c*** | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| **Daily Expenses ($), *e*** | 85.00 | 85.30 | 85.60 | 85.90 | 86.20 |

**Solution:**

Always investigate and interpret what happens when the independent variable is zero. In this case, you find the fixed expenses. Most often, you will gain some insight to the problem, if not key information.

The expenses increase by $0.30 for each cup of coffee served.

85.00 → 85.30 → 85.60 → 85.90 → 86.20

0.30 0.30 0.30 0.30

You can use this pattern to write a verbal model for the expenses.

Total Expenses = Fixed Expenses + Expense per cup • cups served

($) ($) ($/cup) (cups)

$e = 85.00 + 0.30 \cdot c$

An equation for the total daily expenses is $e = 85.00 + 0.30c$.

So the total daily expenses for the coffee shop when 150 cups of coffee are served is $e = 85.00 + 0.30(150) = 85.00 + 45.00 = \$130.00$.

**PRACTICE** **Look for a pattern in the following tables. Then write an equation that represents the table.**

**3.**

| ***x*** | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| ***y*** | −35 | −27 | −19 | −11 |

**4.**

| ***x*** | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| ***y*** | $\frac{3}{5}$ | $\frac{7}{5}$ | $\frac{11}{5}$ | $\frac{15}{5}$ |

## 3. Draw a Diagram

**EXAMPLE** **You decide to build a picture frame for an 8″ × 10″ portrait, using 2″ wide molding. How many linear inches of molding will you need? That is, what is the outside perimeter of the frame?**

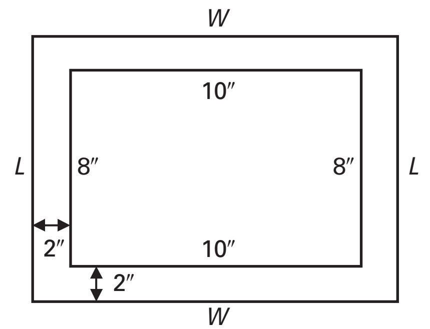
# BENCHMARK 1
*(Chapters 1 and 2)*



**Solution:**

Begin by drawing and labeling a diagram. Try to include all the information given in the problem.

When possible, include your variable(s) in the drawing. It may help you see a physical relationship between what you know, and what you are looking for.




From the diagram, we can write and solve an equation to find the outside perimeter of the frame.

Perimeter $= L + W + L + W = 2L + 2W$ — **Definition of perimeter.**

$= 2(8 + 2 + 2) + 2(10 + 2 + 2)$ — **Because $L = 8 + 2 + 2$ and $W = 10 + 2 + 2$**

$= 2(12) + 2(14)$ — **Add.**

$= 24 + 28 = 52$ — **Multiply, then add.**

You will need 52 linear inches of molding to make this frame.

**PRACTICE**

**5.** A company prints targets for archery ranges, and is designing a new target. The bull's eye will have a diameter of 3 inches. Then there will be an additional 5 concentric circles, each with 4 inch width. What are the dimensions of the smallest size paper the company can use to print this target?

**6.** A 12″ × 12″ ceramic wall tile has a 3″ × 2″ rectangle removed for an electrical outlet. What is the area of the tile that remains?

## Quiz

**1.** Kendra pays \$35 plus \$5 per hour to rent a scooter. Write an expression for how much she pays for renting a scooter *h* hours. How much does she pay for renting the scooter for 6 hours?

**2.** Look for a pattern in the table. Then write an equation that represents the table.

| ***x*** | 0 | 5 | 10 | 15 |
|---|---|---|---|---|
| ***y*** | 6 | 21 | 36 | 51 |

**3.** An equilateral triangle with side length 2″ will have a height of $\sqrt{3}$″. A yield sign is an equilateral triangle with a perimeter of 7.5 feet. To the nearest inch, what is the height of a yield sign?

Name ______________________ Date ____________

# BENCHMARK 1
*(Chapters 1 and 2)*



## C. Linear Equations

The title "Linear Equations" actually refers to two separate ideas. The first type of linear equations we work with are in *one variable*. We will be asked to solve these linear equations. That is, we will need to use fundamental transformations to determine what value(s) of the variable make the equation true. The second type of linear equations we will work with are in *two variables*. These types of equations have so many solutions that the only way to accurately present them is graphically. And naturally, the graph of these solutions form a line.

### 1. Solve a Linear Equation

**Vocabulary**

**Equation** A statement that two expressions are equal.

**Linear equation** An equation that can be written in the form $ax + b = 0$ where $a$ and $b$ are constants and $a \neq 0$.

**Solution** A number is a solution of an equation in one variable if substituting the number for the variable results in a true statement.

**Equivalent equations** Two equations are equivalent equations if they have the same solution(s).

**EXAMPLE** **Solve $3x - 2(5x + 4) = -4(5 - x)$**

**Solution:**

Be sure to distribute the negative two to every term in the parenthesis.

| | |
|---|---|
| $3x - 2(5x + 4) = -4(5 - x)$ | **Write original equation.** |
| $3x + (-2)(5x + 4) = -4(5 - x)$ | **Definition of subtraction.** |
| $3x + (-10x - 8) = -20 + 4x$ | **Distributive property** |
| $-7x - 8 = -20 + 4x$ | **Combine like terms.** |
| $-8 = -20 + 11x$ | **Add 7*x* to both sides.** |
| $12 = 11x$ | **Add 20 to both sides.** |
| $\frac{12}{11} = x$ or $x = \frac{12}{11}$ | **Divide each side by 11.** |
| *CHECK* $3 \cdot \frac{12}{11} - 2\left(5 \cdot \frac{12}{11} + 4\right) \stackrel{?}{=} -4\left(5 - \frac{12}{11}\right)$ | **Substitute $\frac{12}{11}$ for *x*.** |
| $\frac{36}{11} - \frac{120}{11} - 8 \stackrel{?}{=} -20 + \frac{48}{11}$ | **Simplify.** |
| $-\frac{84}{11} - \frac{88}{11} \stackrel{?}{=} -\frac{220}{11} + \frac{48}{11}$ | **Simplify.** |
| $-\frac{172}{11} = -\frac{172}{11}$ | **Solution checks.** |

When checking, always use the original printed problem. A mistake could have occurred when you wrote down the original problem.

**PRACTICE** **Solve the equation. Check your solution.**

**1.** $4x = 15$

**2.** $2 - 3y = 14$

**3.** $-2b - 6 = -3b - 10$

**4.** $n - (n - 1) = 1 - (1 - n)$

Name ______________________ Date ______________



# BENCHMARK 1

*(Chapters 1 and 2)*

## 2. Rewrite a Formula

**Vocabulary**

**Formula** An equation that relates two or more quantities, usually represented by variables.

**Solve for a variable** Rewrite an equation as an equivalent equation in which the variable is on one side and does not appear on the other side.

**EXAMPLE** **Solve the formula $F = \frac{9}{5}C + 32$ for $C$. Then find the Celsius temperature of 77° Fahrenheit.**

Before you begin, find all the places where the desired variable occurs. Using fundamental properties, try to combine multiple occurrences of the variable into one.

**STEP 1:** Solve the formula for $C$.

$F = \frac{9}{5}C + 32$ **Write temperature conversion formula.**

$F - 32 = \frac{9}{5}C$ **Subtract 32 from both sides.**

$\frac{5}{9}(F - 32) = C$ **Multiply both sides by $\frac{5}{9}$.**

**STEP 2:** Substitute the given value into the rewritten formula.

$C = \frac{5}{9}(F - 32) = \frac{5}{9}(77 - 32) = 25$ **Substitute 77 for $F$ and simplify.**

77° Fahrenheit is equivalent to 25° Celsius.

**PRACTICE**

**5.** Solve the formula $x - 4y = -80$ for $y$. Find $y$ when $x = 100$.

**6.** Solve the girth formula $G = l + 2w + 2h$ for $h$. Find $h$ when $G = 23''$, $l = 10''$, and $w = 3''$.

## 3. Write an Equation of a Line

**Vocabulary**

**Slope-Intercept Form of a Line** $y = mx + b$ given slope $m$ and $y$-intercept $b$.

**Point-Slope Form of a Line** $y - y_1 = m(x - x_1)$ given slope $m$ and point $(x_1, y_1)$.

**EXAMPLE** **Write an equation of the following lines.**

**a.** That passes through (0, −3) and has a slope of 5.

**b.** That passes through (−2, −1) and (3, 5).

**Solution:**

When asked to write the equation of a graphed line, try to use points that have integer coordinates.

**a.** Since (0, −3) is actually the $y$-intercept, we can use the slope-intercept form of a line.

$y = mx + b$ **Use slope-intercept form of a line.**

$y = 5x + (-3)$ **Substitute 5 for $m$, and −3 for $b$.**

$y = 5x - 3$ **Simplify.**

**b.** The first thing we must do is find the slope of this line.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - (-1)}{3 - (-2)} = \frac{6}{5}$$

Now use either of the two given points and the point-slope form of a line.

Name ______________________ Date ______________

# BENCHMARK 1

*(Chapters 1 and 2)*



We could have used any point on the line for the point-slope form. Expressing our final equation in slope-intercept form yields the same answer, regardless of which point was chosen.

| | |
|---|---|
| $y - y_1 = m(x - x_1)$ | **Use point slope form of a line.** |
| $y - 5 = \frac{6}{5}(x - 3)$ | **Substitute $\frac{6}{5}$ for $m$, and (3, 5) for $(x_1, y_1)$.** |
| $y - 5 = \frac{6}{5}x - \frac{18}{5}$ | **Distributive property** |
| $y = \frac{6}{5}x + \frac{7}{5}$ | **Write in slope-intercept form.** |

**PRACTICE** **Write an equation of the line that has the given slope and *y*-intercept.**

**7.** $m = 4, b = -3$ **8.** $m = 0, b = 11$

**Write an equation of the line that passes through the given point and has the given slope.**

**9.** $(2, 5), m = 3$ **10.** $(-1, -3), m = -2$

**Write an equation of the line that passes through the given points.**

**11.** $(3, -2), (-2, 3)$ **12.** $(-0.5, 1.2), (2.1, -1.1)$

## 4. Use Ratios to Identify Direct Variation

**Vocabulary**

**Direct Variation** The equation $y = ax$ represents *direct variation* between $x$ and $y$, and $y$ is said to *vary directly* with $x$.

**Constant of variation** The nonzero constant $a$ is called the *constant of variation*.

**EXAMPLE** **A batch of cookies is made, and certain data about each cookie is recorded. The table below gives the area of the bottom of the circular cookies, and their respective weight. Tell whether cookie area and weight show direct variation. If so, write an equation that relates the quantities.**

| | | | | | | |
|---|---|---|---|---|---|---|
| **Area of cookie, *a* (cm²)** | 46 | 56 | 56 | 51 | 54 | 49 |
| **Weight of cookie, *w* (grams)** | 23 | 28 | 29 | 25 | 27 | 24 |

We could have looked at the ratio of area to weight and still discovered direct variation between *a* and *w*. In that case our equation would have been $a = 2w$.

**Solution:**

Find the ratio of weight $w$ to area $a$ for each cookie.

$\frac{23}{46} = 0.5$ $\frac{28}{56} = 0.5$ $\frac{29}{56} \approx 0.52$

$\frac{25}{51} \approx 0.49$ $\frac{27}{54} = 0.5$ $\frac{24}{49} \approx 0.49$

Since the ratios are approximately equal, the data show direct variation. An equation relating the area of a cookie to its weight is $\frac{w}{a} = 0.5$, or $w = 0.5a$.

Name ______________________ Date ____________



# BENCHMARK 1

*(Chapters 1 and 2)*

**PRACTICE** **Tell whether the data in the table show direct variation. If so, write an equation relating *x* and *y*.**

**13.**

| *x* | −6 | −3 | 0 | 9 | 15 |
|---|---|---|---|---|---|
| *y* | −8 | −4 | 0 | 12 | 20 |

**14.**

| *x* | −2 | −1 | 1 | 3 | 4 |
|---|---|---|---|---|---|
| *y* | 5 | 3 | −1 | −5 | −7 |

## Quiz

**Solve the equation. Check your solution.**

**1.** $\frac{2}{3}z + 5 = -3$

**2.** $5a - 4 = -3a + 12$

**3.** $3(c - 1) = 2(2c + 5)$

**4.** $4(m - 1) + 2m = m - 3(m - 7)$

**5.** Solve the formula $3s + 12t = -100$ for $t$. Find $t$ when $s = 50$.

**6.** Write an equation of the line with slope $-\frac{1}{3}$ and $y$-intercept $\frac{5}{2}$.

**7.** Write an equation of the line that passes through $\left(-1, \frac{1}{5}\right)$ and has slope $\frac{3}{2}$.

**8.** Write an equation of the line that passes through $\left(\frac{2}{3}, -4\right)$ and $\left(\frac{7}{3}, 2\right)$.

**9.** Tell whether the data in the table show direct variation. If so, write an equation relating $x$ and $y$.

| *x* | −3 | −1 | 2 | 2.5 | 4 |
|---|---|---|---|---|---|
| *y* | 0.9 | 0.3 | −0.6 | −0.75 | −1.2 |

Name ________________________________ Date ______________



# BENCHMARK 1

*(Chapters 1 and 2)*

## D. Functions and Graphs

The idea of the function is one of the most fundamental ideas in the field of mathematics. It actually transcends mathematics and is extremely important in many other fields, including the sciences, engineering, and business. By studying the graphs of functions, we can often see characteristics that are obscure algebraically.

### 1. Identify Functions

**Vocabulary**

**Relation** A mapping, or pairing, of input values with output values.

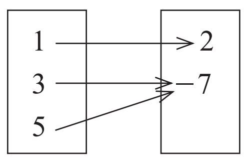
**Domain** The set of input values for a relation.

**Range** The set of output values for a relation.

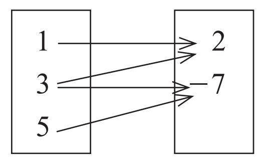
**Function** A relation for which each input has exactly one output. If any input of a relation has more than one output, the relation is *not* a function.

**EXAMPLE** **Tell whether the relation is a function.**

**a.** Input Output




**b.** Input Output




Functions are so useful because for a given input, there is exactly one output. Functions are definitely not ambiguous!

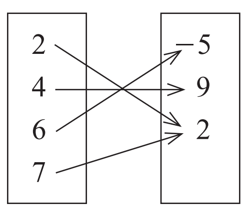
**Solution:**

**a.** This relation is a function because each input is mapped onto exactly one output.

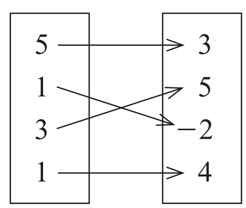
**b.** This relation is not a function because the input 3 is mapped onto both −7 and 2.

**PRACTICE** **Tell whether the relation is a function. Explain.**

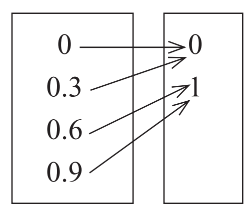
**1.** Input Output




**2.** Input Output




**3.** Input Output




### 2. Use the Vertical Line Test

**Vocabulary**

**Vertical line test** A relation is a function if and only if no vertical line intersects the graph of the relation at more than one point.

Name ______________________ Date ______________

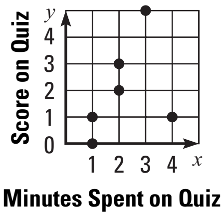


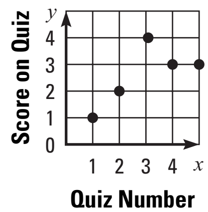
# BENCHMARK 1

*(Chapters 1 and 2)*

**EXAMPLE** **The first graph below presents 7 students' scores on a 10 point quiz versus time spent on the quiz. The second graph presents an individual student's scores on 5 such quizzes. Are the relations represented by the graphs functions? Explain.**

A vertical line intersecting the graph more than once indicates one input of the relation has more than one output.

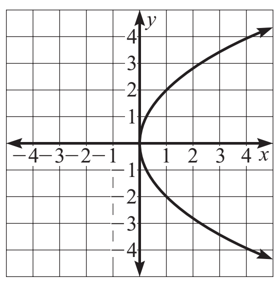


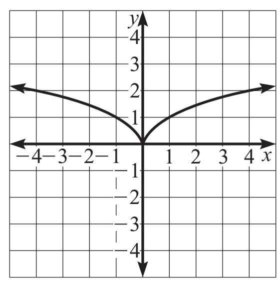

The graph for the group of seven students does not represent a function. The vertical lines at $x = 1$, 2, and 3 each intersect the graph at more than one point. The graph for the individual student's scores does represent a function, because there is no vertical line that intersects the graph at more than one point.

**PRACTICE** **Use the vertical line test to tell whether the relation is a function or not.**

**4.**



**5.**



## 3. Find a Slope

**Vocabulary**

**Slope** The *slope m* of a non-vertical line is the ratio of vertical change (the rise) to horizontal change (the run).

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$ where $(x_1, y_1)$ and $(x_2, y_2)$ are any two distinct points on the line.

**Undefined Slope** A vertical line has an *undefined slope*. In the above formula, any two points on a vertical line will lead to division by zero, which is undefined.

**EXAMPLE** **What is the slope of the line passing through (5, −2) and (−2, 5)?**

When calculating the slope of a line, it is not important which point is first and second. However, the order of subtraction in the numerator and denominator must be preserved.

Let $(x_1, y_1) = (5, -2)$ and $(x_2, y_2) = (-2, 5)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - (-2)}{-2 - 5} = \frac{7}{-7} = -1$$

The slope of this line is $m = -1$.

**PRACTICE** **Find the slope of the line passing through the given points.**

**6.** (3, −4), (5, −3) **7.** (−2, 3), (5, 3) **8.** (−2, 4), (1, −1) **9.** (0, 3), (0, 7)

Name ______________________ Date ______________

# BENCHMARK 1

*(Chapters 1 and 2)*



## 4. Graph an Equation

**Vocabulary**

**Slope-Intercept Form** A line with equation $y = mx + b$ has slope $m$ and $y$-intercept $b$. The equation $y = mx + b$ is said to be in *slope-intercept form*.

**Standard Form** The standard form of a linear equation is $Ax + By = C$ where $A$ and $B$ are not both zero.

**Intercepts** A $x$- and *$y$-intercept* is the $x$- and $y$-coordinate of the points where a graph intersects the $x$- and $y$-axis, respectively.

**EXAMPLE**

**a.** Graph $y = \frac{3}{2}x - 2$

**b.** Graph $4x - 3y = -12$

**Solution:**

Slope-intercept form is characterized by having $y$ isolated on one side of the equation.

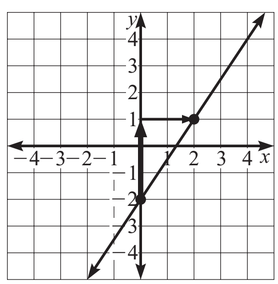
**a.**

**STEP 1:** The equation is in slope-intercept form.

**STEP 2:** Identify the $y$-intercept. The $y$-intercept is $-2$, so plot the point $(0, -2)$ where the line crosses the $y$-axis.

**STEP 3:** Identify the slope. The slope is $\frac{3}{2}$, so plot a second point on the line by starting at $(0, -2)$ and then moving up 3 units and right 2 units. The second point is $(2, 1)$.

**STEP 4:** Draw a line through the two points.




**b.**

**STEP 1:** The equation is in standard form.

**STEP 2:** Identify the $x$-intercept.

$4x - 3(0) = -12$ **Let $y = 0$.**

$x = -3$ **Solve for $x$.**

The $x$-intercept is $-3$. So, plot the point $(-3, 0)$.

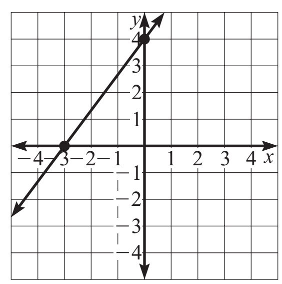
**STEP 3:** Identify the $y$-intercept.

$4(0) - 3y = -12$ **Let $x = 0$.**

$y = 4$ **Solve for $y$.**

The $y$-intercept is 4. So, plot the point $(0, 4)$.

**STEP 4:** Draw a line through the two points.




Both of these equations are linear. Because of their different forms, we use different methods to graph them. Be sure to recognize the different forms, and know how to convert between them.

**PRACTICE**

### Graph the linear equation.

**10.** $y = -3x + 4$ **11.** $y = -3$ **12.** $x + 5y = -10$

**13.** $3x = 10$ **14.** $x = 2y - 8$ **15.** $3x - 2y = 6$

## 5. Estimate Correlation Coefficients

**Vocabulary**

**Scatter Plot** A *scatter plot* is a graph of a set of data pairs $(x, y)$.

**Positive Correlation** If $y$ tends to increase as $x$ increases, then the data have a *positive correlation*.

Name ______________________________ Date ______________



# BENCHMARK 1

*(Chapters 1 and 2)*

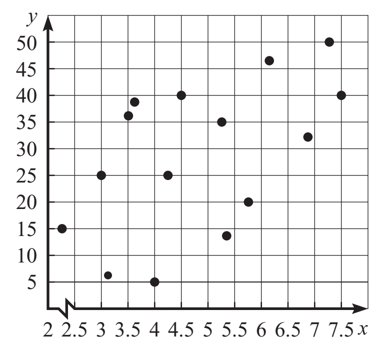
**Negative Correlation** If $y$ tends to decrease as $x$ increases, then the data have a *negative correlation*.

**Correlation Coefficient** A *correlation coefficient*, denoted by $r$, is a number from $-1$ to 1 that measures how well a line fits a set of data pairs $(x, y)$. If $r$ is near 1, the points lie close to a line with positive slope. If $r$ is near $-1$, the points lie close to a line with negative slope. If $r$ is near 0, the points do not lie close to any line.

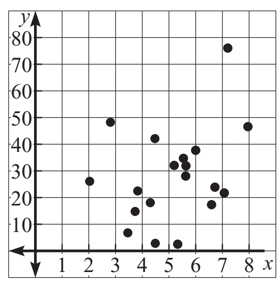
**EXAMPLE** **Tell whether the correlation coefficient for the data is closest to −1, −0.5, 0, 0.5, or 1.**

If the data lie exactly on a horizontal line, $r$ will equal 0. This is because there is no correlation between the $x$ and $y$ values. Any $x$ produces the same $y$!

**a.**



**b.**

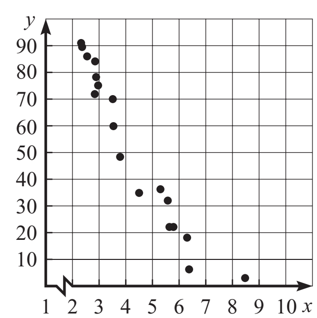


**a.** The scatterplot shows a clear but fairly weak positive correlation. So, $r$ is between 0 and 1, but not too close to either one. The best estimate given is $r = 0.5$. (The actual value is $r \approx 0.53$.)

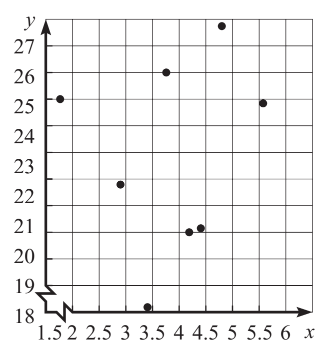
**b.** The scatter plot shows approximately no correlation. So, the best estimate given is $r = 0$. (The actual value is $r \approx 0.10$.)

**PRACTICE** **Tell whether the correlation coefficient for the data is closest to −1, −0.5, 0, 0.5, or 1.**

**16.**



**17.**



Name ______________________ Date ____________

# BENCHMARK 1
*(Chapters 1 and 2)*



## 6. Approximate a Best-Fitting Line

**Vocabulary**

**Best-Fitting Line** The *best-fitting line* is the line that lies as close as possible to all the data points.

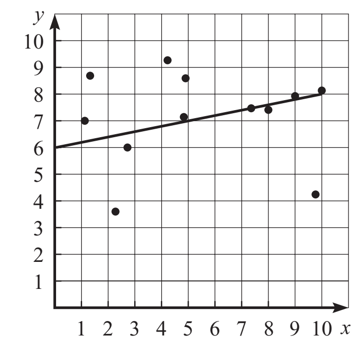
**EXAMPLE** **Twelve cooking students are required to prepare a main course entrée and a dessert plate. Their dishes are then tasted and scored by a panel of judges. Their average scores on a scale from 1 to 10 are presented in the table below. Approximate the best-fitting line for this data.**

| **Entrée** | 9.0 | 7.4 | 9.7 | 2.2 | 1.0 | 8.0 | 9.9 | 2.8 | 1.4 | 4.8 | 4.9 | 4.3 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Dessert** | 7.9 | 7.5 | 4.4 | 3.5 | 7.0 | 7.4 | 8.2 | 6.0 | 8.7 | 7.2 | 8.6 | 9.3 |

From statistics, there is an exact formula for the **one** best-fitting line. It is best calculated with the use of a calculator or computer.

When you are approximating the best-fitting line, try to use points that are convenient. Also, be aware that the points may or may not be original data points.

**STEP 1:** Draw a scatter plot of the data.




**STEP 2:** Sketch the line that appears to best fit the data. One possibility is shown.

**STEP 3:** Choose two points on the line. For the line shown, you might choose (0, 6) and (10, 8). Note that neither is an original data point.

**STEP 4:** Write an equation of the line. First find the slope using (0, 6) and (10, 8).

$m = \frac{8 - 6}{10 - 0} = \frac{2}{10} = 0.2$

Since one of the points is the $y$-intercept, use the slope-intercept form to write the equation.

$y = mx + b$ **Slope-intercept form.**

$y = 0.2x + 6$ **Substitute for *m* and *b*.**

An approximation of the best-fitting line is $y = 0.2x + 6$.

**PRACTICE** **In the following, (a) draw a scatter plot of the data, and (b) approximate the best-fitting line.**

**18.**

| ***x*** | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| ***y*** | 4 | 2 | 7 | 8 | 8 |

**19.**

| ***x*** | 0 | 1 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| ***y*** | 50 | 35 | 42 | 22 | 17 |

**20.**

| ***x*** | 2 | 4 | 5 | 7 | 8 |
|---|---|---|---|---|---|
| ***y*** | 3 | 7 | 9 | 13 | 11 |

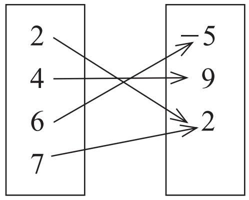
Name ______________________________ Date ______________

# BENCHMARK 1

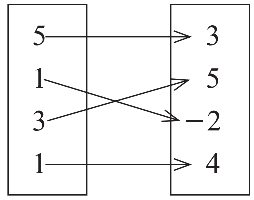
*(Chapters 1 and 2)*

## Quiz

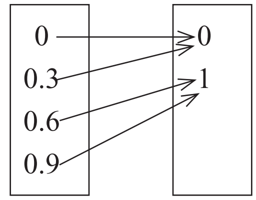
**1.** Input Output

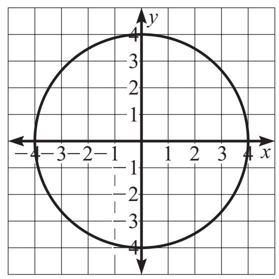



**2.** Input Output




**3.** Input Output




**4.** $y = 2.5x - 4$

**5.** $2x - 3y = 6$

**6.** $2x - 4y = 11$

Name ______________________________ Date ______________



# BENCHMARK 1
*(Chapters 1 and 2)*

## E. Inequalities

Here two basic types of inequalities are presented. First we look at inequalities in one variable, and then inequalities in two variables. They are closely related, but the methods we use to solve them are dramatically different. Also, both types of inequalities generally have an infinite number of solutions. Therefore, the only way we can accurately represent the solution set is graphically.

### 1. Graph Inequalities

**Vocabulary**

**Linear inequality in one variable** An inequality that can be written in one of the following forms, where $a$ and $b$ are real numbers and $a \neq 0$:

$ax + b < 0$ $\quad$ $ax + b > 0$ $\quad$ $ax + b \leq 0$ $\quad$ $ax + b \geq 0$

**Solution of an inequality in one variable** A value that when substituted for the variable, results in a true statement.

**Graph of an inequality in one variable** Consists of all points on a number line that represent solutions.

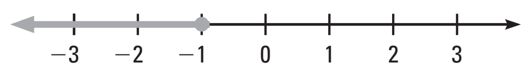
**EXAMPLE** **Graph the following:**

**a.** $x \leq -1$ $\quad$ **b.** $x > 2$ $\quad$ **c.** $-2 \leq x < 3$ $\quad$ **d.** $x < 1$ or $x \geq 3$

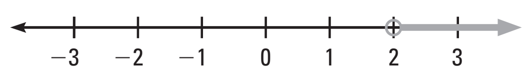
**Solution:**

In a compound inequality like problem c., make sure the inequality makes sense. For instance, there are no solutions to $3 < x < -1$. Without the $x$, this is an always false statement, $3 < -1$.

**a.** The solutions are all real numbers less than or equal to $-1$. A closed dot is used in the graph to indicate $-1$ *is* a solution.




**b.** The solutions are all real numbers greater than 2. An open dot is used in the graph to indicate 2 is *not* a solution.




**c.** The solutions are all real numbers that are greater than or equal to $-2$ *and* less than 3.


**d.** The solutions are all real numbers that are less than 1 or greater than or equal to 3.


**PRACTICE** **Graph the inequality.**

**1.** $x < 3$ $\quad$ **2.** $x \leq -1$ $\quad$ **3.** $-3 < x \leq -1$ $\quad$ **4.** $x < -2$ or $x > 1$

# BENCHMARK 1

*(Chapters 1 and 2)*

## 2. Solve an Inequality

**Vocabulary** **Equivalent inequalities** Inequalities that have the same solutions.

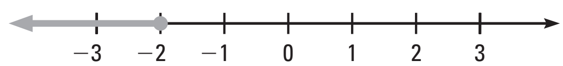
**EXAMPLE** **Solve $3 - 2x \geq 3x + 13$. Then graph the solution.**

We could have added $2x$ to both sides and arrived at the same solution. It would be written as $-2 \geq x$.

**Solution:**

| | |
|---|---|
| $3 - 2x \geq 3x + 13$ | **Write original inequality.** |
| $3 - 5x \geq 13$ | **Subtract $3x$ from each side.** |
| $-5x \geq 10$ | **Subtract 3 from each side.** |
| $x \leq -2$ | **Divide each side by $-5$ and reverse the inequality.** |

The solutions are all real numbers less than or equal to $-2$. The graph is below.



**PRACTICE** **Solve the inequality and then graph the solution.**

**5.** $-x + 10 < 9$ **6.** $3x + 5 > 4x + 2$ **7.** $\frac{2}{5}x \leq 2$

## 3. Solve Compound Inequalities

**EXAMPLE** **Solve the inequality and then graph the solution.**

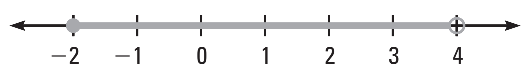
**a.** $-5 < 3 - 2x \leq 7$ **b.** $3x + 1 > 10$ or $3 - 4x \geq 11$

**Solution:**

This type of compound inequality can be written as two inequalities. In problem a. they would be $-5 < 3 - 2x$ and $3 - 2x \leq 7$.

| | |
|---|---|
| **a.** $-5 < 3 - 2x \leq 7$ | **Write original inequality.** |
| $-5 - 3 < -2x \leq 7 - 3$ | **Subtract 3 from each expression.** |
| $-8 < -2x \leq 4$ | **Simplify.** |
| $4 > x \geq -2$ | **Divide all expressions by $-2$ and reverse inequalities.** |
| $-2 \leq x < 4$ | **Rewrite the inequality, preserving the relationships.** |

The solutions are all real numbers greater than or equal to $-2$ *and* less than 4. The graph is shown below.

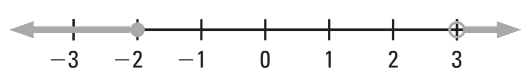


This type of compound inequality may have the entire number line as its solution set. For instance, $x < 2$ or $x > -1$.

**b.** A solution of this compound inequality is a solution of *either* of its parts.

| **First Inequality** | | **Second Inequality** | |
|---|---|---|---|
| $3x + 1 > 10$ | **Write original inequality.** | $3 - 4x \geq 11$ | **Write original inequality.** |
| $3x > 9$ | **Subtract 1 from each side.** | $-4x \geq 8$ | **Subtract 3 from each side.** |
| $x > 3$ | **Divide each side by 3.** | $x \leq -2$ | **Divide each side by $-4$ and reverse inequality.** |

The solutions are all real numbers greater than 3 *or* less than or equal to $-2$. The graph is shown below.



Name ________________________ Date ____________

# BENCHMARK 1
*(Chapters 1 and 2)*

**PRACTICE** **Solve the inequality and then graph the solution.**

**8.** $-10 < 2x - 6 < 0$ **9.** $4x - 3 < -15$ or $2x + 1 \geq 5$

## 4. Solve a Linear Inequality in Two Variables

**Vocabulary** **Linear inequality in two variables** An inequality that can be written in one of these forms:

$Ax + By < C$ $Ax + By \leq C$ $Ax + By > C$ $Ax + By \geq C$

**Solution of an inequality in two variables** An ordered pair $(x, y)$ is a solution if the inequality is true when the values of $x$ and $y$ are substituted into the inequality.

**EXAMPLE** **Tell whether the ordered pair is a solution of $7x - 4y \leq 14$.**

**a.** (1, −3) **b.** (2, 1) **c.** (2, 0) **d.** (2, −1)

Pay attention to the inequality sign. If it is strict, equality is not permitted. If your substitution yields a statement like 5 < 5, your ordered pair is not a solution.

**Solution:**

| | Ordered pair | Substitute | Conclusion |
|---|---|---|---|
| **a.** | (1, −3) | $7(1) - 4(-3) = 7 + 12 = 19 \not\leq 14$ | (1, −3) is not a solution |
| **b.** | (2, 1) | $7(2) - 4(1) = 14 - 4 = 10 \leq 14$ | (2, 1) is a solution |
| **c.** | (2, 0) | $7(2) - 4(0) = 14 - 0 = 14 \leq 14$ | (2, 0) is a solution |
| **d.** | (2, −1) | $7(2) - 4(-1) = 14 + 4 = 18 \not\leq 14$ | (2, −1) is not a solution |

**PRACTICE** **Tell whether the given ordered pair is a solution of $-2x + 3y > 10$.**

**10.** (2, 5) **11.** (5, 2) **12.** $\left(-\frac{1}{2}, 3\right)$

**13.** (−5, −4) **14.** (5, 10)

## 5. Graph a Linear Inequality in Two Variables

**Vocabulary** **Graph a linear inequality in two variables** The set of all points in a coordinate plane that represent solutions of the inequality.

**Boundary line** Line that divides the plane into two halves—one half is made up entirely of solutions of the inequality, and the other has no solutions.The boundary line is the graph of the equation formed by replacing the inequality symbol with the equal sign.

**EXAMPLE** **Graph a. $3x - y > 6$ and b. $x \geq -1$ in a coordinate plane.**

**Solution:**

**a.** Graph the boundary line $3x - y = 6$. Use a dashed line because the inequality symbol is >. Ordered pairs that yield equality are *not* solutions.

BENCHMARK 1
E. Inequalities

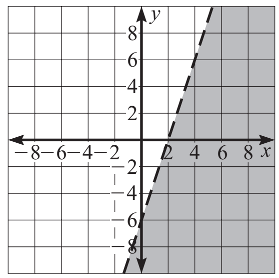
Name ______________________________ Date ______________

# BENCHMARK 1

*(Chapters 1 and 2)*

In both of these examples, we use the point (0, 0) to determine which side to shade. If the boundary line passes through (0, 0), select a different test point.

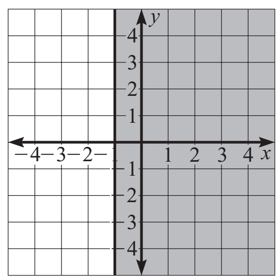
Test the point (0, 0). Because (0, 0) is not a solution of the inequality, shade the half-plane that does not contain (0, 0).




Notice that the inequality $x \geq 1$ could be considered a linear inequality in one variable. The directions of graphing in a coordinate plane clarify the ambiguity.

**b.** Graph the boundary line $x = -1$. Use a solid line because the inequality symbol is $\geq$. Ordered pairs that yield equality *are* solutions.

Test the point (0, 0). Because (0, 0) is a solution of the inequality, shade the half-plane that contains (0, 0).




**PRACTICE** **Graph the linear inequality in a coordinate plane.**

**15.** $y > 1$ **16.** $x - 2y < 6$ **17.** $2x \leq -7$

## Quiz

**Graph the inequality.**

**1.** $x \geq 0$ **2.** $x > -2$ **3.** $0 \leq x \leq 3$ **4.** $x \leq 2$ or $x > -1$

**Solve the inequality and then graph the solution.**

**5.** $-3x + 4 \leq 13$ **6.** $2(3x - 1) \geq 4x + 6$ **7.** $3 - \frac{2}{3}x < 7$

**8.** $-3 \leq -3x + 6 < 9$ **9.** $3x - 2 < -2$ or $-2(x + 3) < -6$

**Tell whether the given ordered pair is a solution of the inequality.**

**10.** $y \leq -4$; (0, 4) **11.** $y < -4x + 3$; (−1, 7)

**12.** $5x - 3y \geq 12$; (3, 1)

**Graph the linear inequality in a coordinate plane.**

**13.** $y \leq x$ **14.** $y \geq \frac{4}{3}x - 2$ **15.** $5x + 3y > -15$

Name ______________________ Date ______________

# BENCHMARK 1
*(Chapters 1 and 2)*



## F. Absolute Value and Transformations

The set of real numbers consists of all rational and irrational numbers. Rational numbers can be expressed as fractions, and irrational numbers can not be expressed as fractions. All real numbers can be graphed on a number line, as seen below. The operations that combine real numbers have specific properties, which can be extended directly to algebraic expressions.

### 1. Solve an Absolute Value Equation

**Vocabulary**

**Absolute value** For a number $x$, written $|x|$, the distance the number is from 0 on a number line. It is always positive.

**Extraneous solution** An apparent solution that must be rejected because it does not satisfy the original equation.

**EXAMPLE** **Solve the following: a. $|3x - 2| = 4$ and b. $|-2x - 10| = 3x$**

When solving an absolute value equation, always isolate the absolute value expression on one side. If the other side is negative, there can be no solution.

**Solution:**

**a.** $|3x - 2| = 4$ — **Write original equation.**

$3x - 2 = 4$ or $3x - 2 = -4$ — **Expression can equal 4 or −4.**

$3x = 6$ or $3x = -2$ — **Add 2 to each side.**

$x = 2$ or $x = -\frac{2}{3}$ — **Divide each side by 2.**

Check these solutions in the original equation.

| | |
|---|---|
| $\|3x - 2\| = 4$ | $\|3x - 2\| = 4$ |
| $\|3(2) - 2\| \stackrel{?}{=} 4$ | $\left\|3\left(-\frac{2}{3}\right) - 2\right\| \stackrel{?}{=} 4$ |
| $\|4\| \stackrel{?}{=} 4$ | $\|-4\| \stackrel{?}{=} 4$ |
| $4 = 4$ ✓ | $4 = 4$ ✓ |

The solutions are 2 and $-\frac{2}{3}$.

**b.** $|-2x - 10| = 3x$ — **Write original equation.**

$-2x - 10 = 3x$ or $-2x - 10 = -3x$ — **Expression can equal $3x$ or $-3x$.**

$-10 = 5x$ or $-10 = -x$ — **Add $2x$ to both sides.**

$x = -2$ or $x = 10$ — **Solve for $x$.**

Check these solutions in the original equation.

$|-2x - 10| = 3x$

| | |
|---|---|
| $\|-2(-2) - 10\| \stackrel{?}{=} 3(-2)$ | $\|-2(10) - 10\| \stackrel{?}{=} 3(10)$ |
| $\|-6\| \stackrel{?}{=} -6$ | $\|-30\| \stackrel{?}{=} 30$ |
| $6 \neq -6$ | $30 = 30$ ✓ |

The only solution is 10. −2 is an extraneous solution.

Name ______________________ Date ______________



# BENCHMARK 1

*(Chapters 1 and 2)*

**PRACTICE** **Solve.**

**1.** $|x| = 4$ **2.** $|x + 2| = 5$ **3.** $|4x + 3| = 3x$

## 2. Write an Absolute Value Inequality

**Vocabulary** **Tolerance** The maximum acceptable deviation of a product from some ideal or mean measurement.

**EXAMPLE** **A marina can hold boats between 8 feet and 50 feet, inclusive. Write an absolute value inequality describing the acceptable boat size range.**

**STEP 1:** Calculate the mean of the extreme boat sizes.

Mean of extremes $= \dfrac{8 + 50}{2} = 29$

Subtracting the lower extreme from the mean will also give you the tolerance.

**STEP 2:** Find the tolerance by subtracting the mean from the upper extreme.

Tolerance $= 50 - 29 = 21$

**STEP 3:** Write a verbal model. Then write an inequality.

|Actual length (feet) − Mean of extremes (feet)| ≤ Tolerance (feet)

$|\ell - 29| \le 21$

A boat can dock at the marina if its length $\ell$ satisfies $|\ell - 29| \le 21$.

**PRACTICE**

**4.** A German Shepherd is considered to be of "normal" size if it weighs between 65 and 80 pounds. Write an absolute value inequality describing the "normal" size range.

**5.** From Exercise 4, write an absolute value inequality describing the "abnormal" size range of German Shepherds.

## 3. Graph an Absolute Value Equation

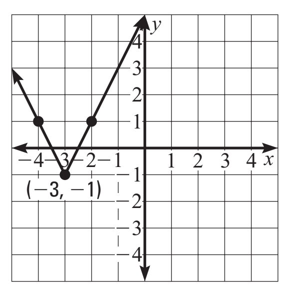
**Vocabulary** **Vertex of an absolute value graph** The highest or lowest point on the graph of an absolute value function. The graph of $y = a|x - h| - k$ has the vertex at $(h, k)$.

**EXAMPLE** **Graph $y = 2|x + 3| - 1$. Compare it with the graph of $y = |x|$.**

Consider writing $x + 3$ as $x - (-3)$. Compare this expression with the general $x - h$. This is why the $x$-coordinate of the vertex is $-3$.

**STEP 1:** Identify and plot the vertex, $(h, k) = (-3, -1)$.

**STEP 2:** Plot another point on the graph, such as $(-2, 1)$. Use symmetry to plot a third point, $(-4, 1)$.




**STEP 3:** Connect the points with a V-shaped graph.

**STEP 4:** Compare with $y = |x|$. The graph of $y = 2|x + 3| - 1$ is the graph of $y = |x|$ first stretched vertically by a factor of 2, then translated left 3 units and down 1 unit.

Name ______________________ Date ____________



# BENCHMARK 1
*(Chapters 1 and 2)*

**PRACTICE** **Graph the following and compare them with the graph of $y = |x|$.**

**6.** $y = |x + 4|$

**7.** $f(x) = 0.5|x - 3| - 2$

## 4. Apply Transformations to a Graph

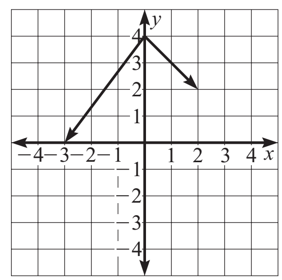
**Vocabulary** **Transformations of General Graphs** The graph of $y = a \cdot f(x - h) + k$ can be obtained from the graph of $y = f(x)$ by performing these steps:

**1.** Stretch or shrink the graph of $y = f(x)$ vertically by a factor of $|a|$ if $|a| \neq 1$. If $|a| > 1$, stretch the graph. If $|a| < 1$, shrink the graph.

**2.** Reflect the resulting graph from step 1 in the $x$-axis if $a < 0$.

**3.** Translate the resulting graph from step 2 horizontally $h$ units and vertically $k$ units.

**EXAMPLE** **The graph of the function $y = f(x)$ is shown. Sketch the graph of the given function.**



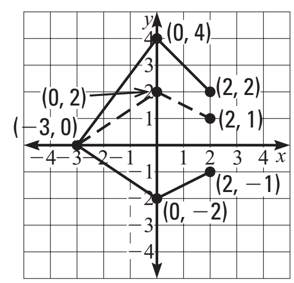
**a.** $y = -\frac{1}{2} f(x)$

**b.** $y = f(x - 3) - 2$

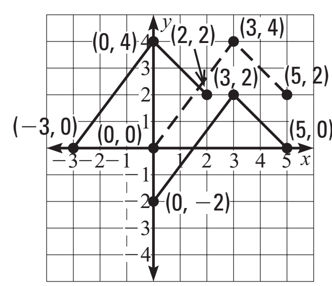
**Solution:**

The order of the transformations can sometimes be very important. For instance, shifting up, then reflecting across the $x$-axis will not yield the same graph as reflecting, then shifting up. Follow the steps outlined here.

**a.** The graph of $y = -\frac{1}{2} f(x)$ is the graph of $y = f(x)$ shrunk vertically by a factor of $\frac{1}{2}$, and then reflected across the $x$-axis. To draw the graph, multiply the $y$-coordinate of each labeled point by $\frac{1}{2}$ and connect their images. Then reflect this across the $x$-axis to form the final image.




**b.** The graph of $y = f(x - 3) - 2$ **is** the graph of $y = f(x)$ translated right 3 units and down 2 units. To draw the graph, first translate the graph 3 units to the right. Do so by adding 3 to the $x$-coordinates of each labeled point and connecting their images. Then translate this graph down 2 units by subtracting 2 from the $y$-coordinate of each point, and connect their images.



Name ______________________________ Date ______________



# BENCHMARK 1

*(Chapters 1 and 2)*

**PRACTICE** **Using the same graph of the function $y = f(x)$ from the above example, sketch the graph of the given functions.**

**8.** $y = f(x + 3)$ **9.** $y = -3f(x) + 2$ **10.** $y = -2f(x + 1) - 4$

## Quiz

**Solve.**

**1.** $|2x| = -5$ **2.** $|3x - 1| = 11$ **3.** $|4x - 3| = 3x$

**4.** The width of a wire on a certain transistor can vary. The least acceptable diameter is 0.21 mm while the maximum acceptable diameter is 0.47 mm. Write an absolute value inequality describing the acceptable size range.

**Graph the following and compare them with the graph of $y = |x|$.**

**5.** $y = -3|x - 1| + 2$ **6.** $f(x) = 2|x| - 5$

**Using the same graph of the function $y = f(x)$ from the above example, sketch the graph of the given functions.**

**7.** $y = f(x) - 3$ **8.** $y = -3f(x)$ **9.** $y = 2f(x - 1)$