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BENCHMARK 2
(Chapters 3 and 4)

A. Linear Systems (pp. 26–30)

1. Solve a System by Graphing

Vocabulary

Solution For a system of linear equations in two variables, an ordered pair (x, y) that satisfies each equation.

Consistent A system that has at least one solution.

Inconsistent A system that has no solution.

Independent A consistent system that has exactly one solution.

Dependent A consistent system that has infinitely many solutions.

EXAMPLE **Graph the linear system and estimate the solution. Then check the solution algebraically.**

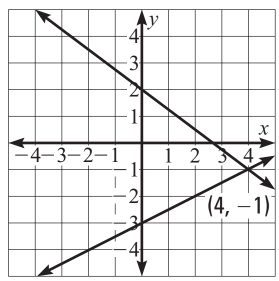
$3x + 4y = 8$

$x - 2y = 6$

Solution:

Begin by graphing both equations, as shown at the right. From the graph, the lines appear to intersect at $(4, -1)$.

Two distinct lines intersect in at most one point.



You can check this algebraically as follows.

Equation 1	Equation 2
$3x + 4y = 8$	$x - 2y = 6$
$3(4) + 4(-1) = 8$	$(4) - 2(-1) = 6$

The solution is $(4, -1)$.

PRACTICE **Graph the linear system and estimate the solution. Then check the solution algebraically.**

1. $3y = x$
$3y = -2x - 9$

2. $5x - 2y = 10$
$x - y = -1$

3. $3x - 5y = 15$
$y = 2x + 4$

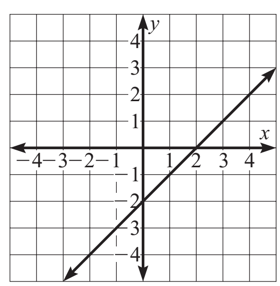
EXAMPLE **Solve the system. Then classify the system as *consistent and independent, consistent and dependent,* or *inconsistent*.**

$x - y = 2$

$-2x + 2y = -4$

Solution:

The graphs of the equations are the same line. Each point on the line is a solution, and the system has infinitely many solutions. Therefore, the system is consistent and dependent.



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BENCHMARK 2

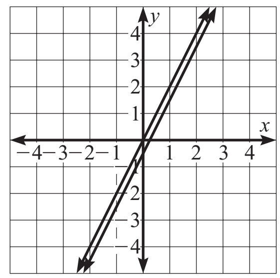
(Chapters 3 and 4)

EXAMPLE **Solve the system. Then classify the system as *consistent and independent, consistent and dependent,* or *inconsistent*.**

The graph of an inconsistent system of linear equations consists of parallel lines.

$2x - y = 0$

$4x - 2y = 1$



Solution:

The graphs of the equations are parallel. There are no solutions, and the system is inconsistent.

PRACTICE **Solve the system. Then classify the system as *consistent and independent, consistent and dependent,* or *inconsistent*.**

4. $4x + 6y = 18$
$6x + 9y = 18$

5. $x + y = 6$
$3x - 4y = 4$

6. $x - 2y = 4$
$-\frac{x}{2} + y = -2$

2. Use the Substitution Method

EXAMPLE **Solve the system using the substitution method.**

$x + 3y = 25$

$4x + 5y = 9$

Solution:

Step 1: **Solve** Equation 1 for x.

$x = 25 - 3y$ **Revised Equation 1**

Step 2: **Substitute** the expression for x into Equation 2 and solve for y.

$4(25 - 3y) + 5y = 9$

$100 - 7y = 9$

$y = 13$

Step 3: **Substitute**the value of y into revised Equation 1 and solve for x.

Use substitution to check your work.

$x = 25 - 3(13) = -14$

The solution is $(-14, 13)$. Check the solution by substituting into the original equations.

PRACTICE **Solve the system using the substitution method.**

7. $-5x + y = 1$
$9x - 2y = 4$

8. $3x - 2y = 17$
$-2x - 5y = 14$

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BENCHMARK 2

(Chapters 3 and 4)

3. Use the Elimination Method

EXAMPLE **Solve the system using the elimination method.**

$2x - 3y = 7$

$x + 2y = 3$

Solution:

Look for similarities between equations that you can exploit to eliminate a variable.

Step 1: Multiply Equation 2 by -2 so that the coefficient of x differ only in sign.

$-2x - 4y = -6$

Step 2: Add the revised equations and solve for y.

$$\begin{array}{r} 2x - 3y = 7 \\ -2x - 4y = -6 \\ \hline -7y = 1 \\ y = -\frac{1}{7} \end{array}$$

Step 3: Substitute the value of y into one of the original equations. Solve for x.

$$2x - 3\left(-\frac{1}{7}\right) = 7$$

$$2x + \frac{3}{7} = 7$$

$$2x = \frac{46}{7}$$

$$x = \frac{23}{7}$$

The solution is $\left(\frac{23}{7}, -\frac{1}{7}\right)$.

Check the solution by substituting into the original equations.

PRACTICE **Solve the system using the elimination method.**

9. $2x - y = 4$
$3x + 2y = 5$

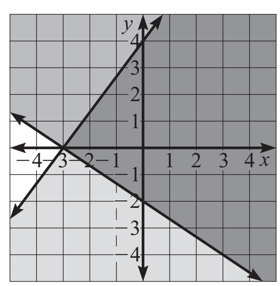
10. $3x - 2y = 4$
$2x + 4y = 6$

4. Graph a System of Two Inequalities

EXAMPLE **Graph the system of inequalities.**

$y \le \frac{4}{3}x + 4$

$y \ge -\frac{2}{3}x - 2$



Solution:

Check your solution by choosing a point on the coordinate plane and substituting into both inequalities. This helps to determine appropriate shading.

Step 1: Graph each inequality in the system. Use red for the first inequality and blue for the second inequality.

Step 2: Identify the region that is common to both graphs. It is the region that is shaded purple.

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BENCHMARK 2

(Chapters 3 and 4)

PRACTICE **Graph the system of inequalities.**

11. $y \le 4$
$y \ge x$

12. $5y - 10 \le -5x$
$y \le x - 5$

5. Solve a System of Three Linear Equations

EXAMPLE **Solve the system.**

The process used in solving a system of three equations is no different from what you used to solve a system of two equations.

$$4x + 2y + z = 8$$
$$-2x + 3y + 2z = 2$$
$$x + 6y + 4z = 4$$

Solution:

Step 1: Rewrite the system as a linear system in two variables.

$4x + 2y + z = 8$ **Add −2 times Equation 2**

$-4x - 6y - 4z = -4$ **to Equation 1.**

$-4y - 3z = 4$ **New Equation 1**

$-2x + 3y + 2z = 2$ **Add 2 times Equation 3**

$2x + 12y + 8z = 8$ **to Equation 2.**

$15y + 10z = 10$ **New E quation 2**

Step 2: Solve the new linear system for both of its variables.

$-40y - 30z = 40$ **Add 10 times new Equation 1**

$45y + 30z = 30$ **to 3 times new Equation 2.**

$5y = 70$

$y = 14$ **Solve for *y*.**

$z = -20$ **Substitute into new Equation 1 or 2 to find *z*.**

Step 3: Substitute $y = 14$ and $z = -20$ into an original equation and solve for x.

$$4x + 2y + z = 8$$
$$4x + 2(14) + (-20) = 8$$
$$x = 0$$

The solution is $x = 0$, $y = 14$, and $z = -20$ or the ordered triple $(0, 14, -20)$. Check this solution in each of the original equations.

PRACTICE **Solve the system.**

13. $-2x + 3y - 2z = 2$
$-4x + 4y + 4z = -4$
$2x - 3y + 4z = 3$

14. $x - 2y + z = 7$
$3x + y - z = 2$
$2x + 3y + 2z = 7$

BENCHMARK 6
A. Linear Systems

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BENCHMARK 2
(Chapters 3 and 4)

BENCHMARK 2
A. Linear Systems

Quiz

1. Graph the linear system and estimate the solution. Then check the solution algebraically.

$y = 5x - 2$
$y + 2x = 5$

2. Solve the system. Then classify the system as *consistent and independent, consistent and dependent,* or *inconsistent*.

$3x + 4y = 8$
$-3x - 4y = 10$

3. Solve the system using the substitution method.

$3x - 2y = 7$
$x + y = 4$

4. Solve the system using the elimination method.

$4x - 3y = 15$
$2x + y = 5$

5. Graph the system of inequalities.

$3x - y \le 6$
$x + y \le 6$

6. Solve the system.

$x + y + z = -2$
$2x - 3y + z = -11$
$-x + 2y - z = 8$

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BENCHMARK 2
(Chapters 3 and 4)

B. Matrices (pp. 31–35)

1. Perform Basic Matrix Operations

Vocabulary

Matrix A rectangular arrangement of numbers in rows and columns.

Elements The numbers in a matrix.

EXAMPLE **Perform the indicated operation, if possible.**

a. $\begin{bmatrix} -2 & 3 \\ 4 & -1 \end{bmatrix} + \begin{bmatrix} -1 & 4 \\ -2 & -5 \end{bmatrix}$ **b.** $\begin{bmatrix} 2 & 3 \\ 4 & -1 \\ 2 & -4 \end{bmatrix} - \begin{bmatrix} -1 & 2 \\ 5 & 4 \\ -5 & -3 \end{bmatrix}$

Solution:

a. $\begin{bmatrix} -2 & 3 \\ 4 & -1 \end{bmatrix} + \begin{bmatrix} -1 & 4 \\ -2 & -5 \end{bmatrix} = \begin{bmatrix} -2+(-1) & 3+4 \\ 4+(-2) & -1+(-5) \end{bmatrix} = \begin{bmatrix} -3 & 7 \\ 2 & -6 \end{bmatrix}$

b. $\begin{bmatrix} 2 & 3 \\ 4 & -1 \\ 2 & -4 \end{bmatrix} - \begin{bmatrix} -1 & 2 \\ 5 & 4 \\ -5 & -3 \end{bmatrix} = \begin{bmatrix} 2-(-1) & 3-2 \\ 4-5 & -1-4 \\ 2-(-5) & -4-(-3) \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ -1 & -5 \\ 7 & -1 \end{bmatrix}$

PRACTICE **Perform the indicated operation, if possible.**

Matrix addition is not always defined.

1. $\begin{bmatrix} 5 & -6 \\ -4 & 6 \end{bmatrix} + \begin{bmatrix} 2 & -4 \\ -4 & -7 \end{bmatrix}$ **2.** $\begin{bmatrix} 7 & -3 & -2 \\ -2 & -1 & -5 \end{bmatrix} - \begin{bmatrix} 1 & -3 & 3 \\ 2 & -4 & -2 \end{bmatrix}$

3. $\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} -1 & 2 & -1 \\ -3 & 4 & -2 \end{bmatrix}$

EXAMPLE **Perform the indicated operation, if possible.**

$-3\begin{bmatrix} -2 & -1 \\ 3 & 4 \end{bmatrix}$

Solution:

$-3\begin{bmatrix} -2 & -1 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} -3(-2) & -3(-1) \\ -3(3) & -3(4) \end{bmatrix} = \begin{bmatrix} 6 & 3 \\ -9 & -12 \end{bmatrix}$

PRACTICE **Perform the indicated operation, if possible.**

4. $5\begin{bmatrix} \frac{1}{10} & \frac{3}{10} \\ -\frac{2}{5} & -\frac{4}{5} \end{bmatrix}$ **5.** $-4\begin{bmatrix} -1 & -3 \\ 4 & 2 \end{bmatrix}$

2. Multiply Matrices

EXAMPLE

Matrix multiplication is not commutative.

a. Find AB if $A = \begin{bmatrix} 2 & 4 \\ -1 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$.

b. Find CD if $C = \begin{bmatrix} 1 & 3 \\ 2 & -2 \\ -3 & 2 \end{bmatrix}$ and $D = \begin{bmatrix} 3 & -1 \\ -2 & 3 \end{bmatrix}$.

BENCHMARK 2
(Chapters 3 and 4)

Solution:

a. **Step 1:** **Multiply** the numbers in the first row of *A* by the numbers in the first column of *B*, add the products, and put the result in the first row, first column of *AB*.

Step 2: **Multiply** the numbers in the first row of *A* by the numbers in the second column of *B*, add the products, and put the results in the first row, second column of *AB*.

Step 3: **Multiply** the numbers in the second row of *A* by the numbers in the first column of *B*, add the products, and put the result in the second row, second column of *AB*.

Step 4: **Multiply** the numbers in the second row of *A* by the numbers in the second column of *B*, add the products, and put the results in the second row, second column of *AB*.

Step 5: **Simplify** the product matrix.

$$\begin{bmatrix} 2 & 4 \\ -1 & -2 \end{bmatrix} \cdot \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} (2(-1) + 4(-2)) & (2 \cdot 3 + 4 \cdot 4) \\ (-1(-1) + -2(-2)) & (-1 \cdot 3 + -2 \cdot 4) \end{bmatrix}$$

$$= \begin{bmatrix} -10 & 22 \\ 5 & -11 \end{bmatrix}$$

b. **Step 1:** **Multiply** the numbers in the first row of *C* by the numbers in the first column of *D*, add the products, and put the result in the first row, first column of *CD*.

Step 2: **Multiply** the numbers in the first row of *C* by the numbers in the second column of *D*, add the products, and put the results in the first row, second column of *CD*.

Step 3: **Multiply** the numbers in the second row of *C* by the numbers in the first column of *D*, add the products, and put the result in the second row, first column of *CD*.

Step 4: **Multiply** the numbers in the second row of *C* by the numbers in the second column of *D*, add the products, and put the results in the second row, second column of *CD*.

Step 5: **Multiply** the numbers in the third row of *C* by the numbers in the first column of *D*, add the products, and put the results in the third row, first column of *CD*.

Step 6: **Multiply** the numbers in the third row of *C* by the numbers in the second column of *D*, add the products, and put the results in the third row, second column of *CD*.

Step 7: **Simplify** the product matrix.

$$\begin{bmatrix} 1 & 3 \\ 2 & -2 \\ -3 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 & -1 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} (1 \cdot 3 + 3(-2)) & (1(-1) + 3 \cdot 3) \\ (2 \cdot 3 + 2(-2)) & (2(-1) + -2 \cdot 3) \\ (-3 \cdot 3 + 2(-2)) & (-3(-1) + 2 \cdot 3) \end{bmatrix} = \begin{bmatrix} -3 & 8 \\ 10 & -8 \\ -13 & 9 \end{bmatrix}$$

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BENCHMARK 2

(Chapters 3 and 4)

PRACTICE

6. Find AB if $A = \begin{bmatrix} -5 & 3 \\ -2 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & 2 \\ -1 & -3 \end{bmatrix}$.

7. Find CD if $C = \begin{bmatrix} -2 & 3 & 4 \\ -1 & 3 & -4 \end{bmatrix}$ and $D = \begin{bmatrix} -2 & 3 \\ -2 & 4 \\ 3 & -4 \end{bmatrix}$.

3. Evaluate Determinants

Vocabulary

Determinant The determinant of a 2 × 2 matrix is written $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$ and is defined as the difference $ad - bc$.

EXAMPLE **Evaluate the determinants.**

Only square matrices have determinants.

a. $\begin{vmatrix} -2 & 3 \\ -4 & 3 \end{vmatrix}$ **b.** $\begin{vmatrix} 1 & 2 & 3 \\ 3 & -1 & 1 \\ 4 & 1 & 2 \end{vmatrix}$

Solution:

a. $\begin{vmatrix} -2 & 3 \\ -4 & 3 \end{vmatrix} = (-2(3) - 3(-4)) = 6$

b. $\begin{vmatrix} 1 & 2 & 3 \\ 3 & -1 & 1 \\ 4 & 1 & 2 \end{vmatrix}\begin{matrix} 1 & 2 \\ 3 & -1 \\ 4 & 1 \end{matrix} = (-2 + 8 + 9) - (-12 + 1 + 12) = 14$

PRACTICE **Evaluate the determinants.**

8. $\begin{vmatrix} 7 & 16 \\ 3 & 8 \end{vmatrix}$ **9.** $\begin{vmatrix} 10 & 50 \\ -5 & 25 \end{vmatrix}$ **10.** $\begin{vmatrix} 6 & 7 & 4 \\ -2 & -4 & 3 \\ 1 & 1 & 1 \end{vmatrix}$

4. Use Cramer's Rule

EXAMPLE **Use Cramer's rule to solve this system:**

$2x - 3y = 9$

$x + 5y = -2$

Solution:

Cramer's Rule provides an alternative approach to solving systems of linear equations.

Step 1: Evaluate the determinant of the coefficient matrix.

$$\begin{vmatrix} 2 & -3 \\ 1 & 5 \end{vmatrix} = 2(5) - (-3)(1) = 13$$

Step 2: Apply Cramer's rule because the determinant is not zero.

$$x = \frac{\begin{vmatrix} 9 & -3 \\ -2 & 5 \end{vmatrix}}{13} = \frac{39}{13} = 3 \quad y = \frac{\begin{vmatrix} 2 & 9 \\ 1 & -2 \end{vmatrix}}{13} = \frac{-13}{13} = -1$$

The solution is (3, −1).

Check this solution in the original equations.

BENCHMARK 2
B. Matrices

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BENCHMARK 2
(Chapters 3 and 4)

BENCHMARK 2 B. Matrices

PRACTICE **Use Cramer's rule to solve each system.**

11. $5x + 4y = -3$
$3x - 5y = -24$

12. $2x - 3y = 17$
$3x + y = 9$

5. Find the Inverse of a Matrix

EXAMPLE **Find the inverse of $A = \begin{bmatrix} -1 & 4 \\ 3 & 2 \end{bmatrix}$.**

Only square matrices have inverses, but not all square matrices have inverses.

Solution:

$$A^{-1} = -\frac{1}{14}\begin{bmatrix} 2 & -4 \\ -3 & -1 \end{bmatrix} = \begin{bmatrix} -\frac{1}{7} & \frac{2}{7} \\ \frac{3}{14} & \frac{1}{14} \end{bmatrix}$$

PRACTICE **Find the inverse of the matrix.**

13. $\begin{bmatrix} 3 & 8 \\ -1 & 5 \end{bmatrix}$

14. $\begin{bmatrix} -1 & -2 \\ 0 & -1 \end{bmatrix}$

15. $\begin{bmatrix} -2 & 2 \\ 1 & -1 \end{bmatrix}$

6. Solve a Matrix Equation

EXAMPLE **Solve the matrix equation *AX* = *B* for the 2 × 2 matrix.**

$$\begin{bmatrix} 5 & -2 \\ 1 & 1 \end{bmatrix} X = \begin{bmatrix} 5 & 3 \\ 8 & 2 \end{bmatrix}$$

Multiplying a matrix by its inverse is analogous to multiplying a real number by its reciprocal.

Solution:

Begin by finding the inverse of A.

$$\frac{1}{7}\begin{bmatrix} 1 & 2 \\ -1 & 5 \end{bmatrix} = \begin{bmatrix} \frac{1}{7} & \frac{2}{7} \\ -\frac{1}{7} & \frac{5}{7} \end{bmatrix}$$

To solve the equation for X, multiply both sides of the equation by A^{-1} on the left.

$$\begin{bmatrix} \frac{1}{7} & \frac{2}{7} \\ -\frac{1}{7} & \frac{5}{7} \end{bmatrix}\begin{bmatrix} 5 & -2 \\ 1 & 1 \end{bmatrix} X = \begin{bmatrix} \frac{1}{7} & \frac{2}{7} \\ -\frac{1}{7} & \frac{5}{7} \end{bmatrix}\begin{bmatrix} 5 & 3 \\ 8 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} X = \begin{bmatrix} 3 & 1 \\ 5 & 1 \end{bmatrix}$$

$$X = \begin{bmatrix} 3 & 1 \\ 5 & 1 \end{bmatrix}$$

PRACTICE **Solve the matrix equation *AX* = *B* for the 2 × 2 matrix *X*.**

16. $\begin{bmatrix} 2 & -1 \\ 13 & 1 \end{bmatrix} X = \begin{bmatrix} 2 & -2 \\ 4 & -1 \end{bmatrix}$

17. $\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} X = \begin{bmatrix} -2 & -4 \\ -1 & 3 \end{bmatrix}$

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BENCHMARK 2

(Chapters 3 and 4)

7. Solve a Linear System Using Inverse Matrices

EXAMPLE **Use an inverse matrix to solve the linear system.**

A system of linear equations can be translated into a matrix problem which can be solved using inverses.

$3x + 5y = 7$

$6x - y = -8$

Solution:

Step 1: Write the linear system as a matrix equation $AX = B$.

$$\begin{bmatrix} 3 & 5 \\ 6 & -1 \end{bmatrix} X = \begin{bmatrix} 7 \\ -8 \end{bmatrix}$$

Step 2: Find the inverse matrix A.

$$-\frac{1}{33}\begin{bmatrix} -1 & -5 \\ -6 & 3 \end{bmatrix}$$

Step 3: Multiply the matrix of constant by on the left.

$$X = A^{-1}B = -\frac{1}{33}\begin{bmatrix} -1 & -5 \\ -6 & 3 \end{bmatrix}\begin{bmatrix} 7 \\ -8 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

The solution of the system is $(-1, 2)$. Check this in the original equations.

PRACTICE **Use an inverse matrix to solve the linear system.**

18. $-3x + y = 8$
$x + y = 4$

19. $y - 6x = 1$
$-15x + 2y = -4$

Quiz

Perform the indicated operation, if possible.

1. $\begin{bmatrix} 4 & 5 \\ 2 & -1 \end{bmatrix} + \begin{bmatrix} -2 & -3 \\ 4 & 5 \end{bmatrix}$

2. $\begin{bmatrix} -3 \\ 2 \\ 1 \end{bmatrix} - \begin{bmatrix} -10 \\ 11 \\ 3 \end{bmatrix}$

3. $-4\begin{bmatrix} 2 & -2 \\ -1 & 3 \\ -4 & -2 \end{bmatrix}$

4. Find AB if $A = \begin{bmatrix} -2 & -1 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -1 \\ 2 & -2 \end{bmatrix}$.

5. Find the determinant and the inverse for matrix $A = \begin{bmatrix} 2 & -3 \\ 4 & -1 \end{bmatrix}$.

6. Use an inverse matrix to solve the linear system.

$2x + 5y = 1$

$-x - 3y = 2$

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BENCHMARK 2
(Chapters 3 and 4)

BENCHMARK 2 C. Graphing Quadratics

C. Graphing Quadratic Functions (pp. 36–41)

1. Find the Minimum or Maximum Value

Vocabulary

Quadratic function A function that can be written in the standard form $y = ax^2 + bx + c$ where $a \neq 0$.

Parabola The graph of a quadratic function.

Vertex The lowest or highest point on a parabola.

Axis of symmetry A line that divides the parabola into mirror images and passes through the vertex.

EXAMPLE

A quadratic function has either a maximum or a minimum. It cannot have both.

Tell whether the given function has a minimum value or a maximum value. Then find the minimum or maximum value.

a. $y = x^2 + 8x - 20$ **b.** $y = -2x^2 - 12x + 3$

Solution:

a. Because $a > 0$, the function has a minimum value. To find it, calculate the coordinates of the vertex.

$x = -\frac{b}{2a} = -\frac{8}{2} = -4$

Now substitute this value of x into the function.

$y = (-4)^2 + 8(-4) - 20 = -36$

The minimum value is $(-4, -36)$.

b. Because $a < 0$, the function has a maximum value. To find it, calculate the coordinates of the vertex.

$x = -\frac{b}{2a} = -\frac{-12}{-4} = -3$

Then substitute this value of x into the function.

$y = (-2)(-3)^2 - 12(-3) + 3 = 21$

The maximum value is $(-3, 21)$.

PRACTICE

Tell whether each function has a minimum value or a maximum value. Then find the minimum or maximum value.

1. $y = x^2 - 4$ **2.** $y = -x^2 - 4x + 6$ **3.** $y = -2x^2 + 3x - 6$

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BENCHMARK 6

(Chapters 3 and 4)

2. Graph a Quadratic in Standard Form

EXAMPLE **Graph $y = 2x^2 + 4x - 5$**

Solution:

Just as two points determine a line, three non-collinear points determine a parabola. So always plot and label three points when graphing parabolas.

Step 1: Identify the coefficients of the function. The coefficients are $a = 2$, $b = 4$, $c = -5$. Because $a > 0$, the parabola opens up.

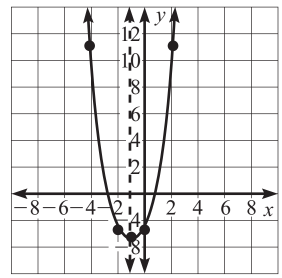
Step 2: Find the vertex. Calculate the x-coordinate.

$x = -\frac{b}{2a} = -\frac{4}{4} = -1$

Then find the y-coordinate of the vertex.

$y = 2(-1)^2 + 4(-1) - 5 = -7$

So the vertex is $(-1, -7)$. Plot this point.



Step 3: Draw the axis of symmetry $x = -1$.

Step 4: Identify the y-intercept c, which is -5. Plot the point $(0, -5)$. Then reflect this point in the axis of symmetry to plot another point $(-2, -5)$.

Step 5: Evaluate the function for another value of x, such as $x = -4$.

$y = 2(-4)^2 + 4(-4) - 5 = 11$

Plot the point $(-4, 11)$ and its reflection $(2, 11)$ in the axis of symmetry.

Step 6: Draw a parabola through the plotted points.

PRACTICE **Graph the function. Label the vertex and axis of symmetry.**

4. $y = x^2 + 4x - 2$ **5.** $y = -x^2 - 6x + 2$ **6.** $y = -4x^2 - 10$

3. Graph a Quadratic Function in Vertex Form

Vocabulary **Vertex form** A form of a quadratic function written $y = a(x - h)^2 + k$.

EXAMPLE **Graph $y = \frac{1}{2}(x + 2)^2 - 3$**

Solution:

Vertex form is the easiest form of the equation of a parabola to work with when graphing.

Step 1: Identify the constants $a = \frac{1}{2}$, $h = -2$, and $k = -3$. Because $a > 0$, the parabola opens up.

Step 2: Plot the vertex $(h, k) = (-2, -3)$ and draw the axis of symmetry $x = -2$.

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BENCHMARK 2
(Chapters 3 and 4)

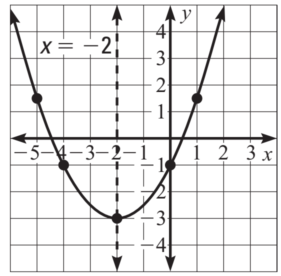
Step 3: Evaluate the function for two values of x.

$x = 0$: $y = \frac{1}{2}(0 - (-2))^2 - 3 = -1$

$x = 1$: $y = \frac{1}{2}(1 - (-2))^2 - 3 = 1.5$

Plot the points $(0, -1)$ and $(1, 1.5)$ and their reflections in the axis of symmetry.

Step 4: Draw a parabola through the plotted points.



PRACTICE **Graph each. Label the vertex and axis of symmetry.**

7. $y = 2(x + 1)^2 - 4$ **8.** $y = -3(x - 2)^2 + 1$ **9.** $y = -(x + 3)^2 - 2$

4. Graph a Quadratic in Intercept Form

Vocabulary **Intercept Form** A form of a quadratic function written $y = a(x - p)(x - q)$.

EXAMPLE **Graph $y = 2(x - 3)(x + 1)$**

Solution:

Now is a good time to review factoring trinomials, since these are quadratic functions.

Step 1: Identify the x-intercepts. Because $p = 3$ and $q = -1$, the x-intercepts occur at the points $(3, 0)$ and $(-1, 0)$.

Step 2: Find the coordinates of the vertex.

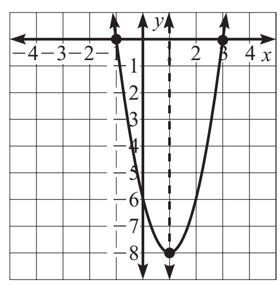
$$x = \frac{p + q}{2} = \frac{3 + (-1)}{2} = 1$$

$$y = 2(1 - 3)(1 + 1) = -8$$

So, the vertex is $(1, -8)$.

Step 3: Draw the axis of symmetry $x = 1$.

Step 4: Draw a parabola through the vertex and the points where the x-intercepts occur.



PRACTICE **Graph each. Label the vertex, axis of symmetry, and *x*-intercepts.**

10. $y = -(x + 2)(x + 1)$ **11.** $y = 3(x - 4)(x - 2)$ **12.** $y = -3(x + 1)(x + 3)$

BENCHMARK 2 C. Graphing Quadratics

Name ______________________ Date ____________

BENCHMARK 6

(Chapters 3 and 4)

5. Rewrite Quadratic Functions

EXAMPLE **Write in standard form.**

$y = -3(x + 2)(x - 1)$

Solution:

$y = -3(x^2 - x + 2x - 2)$

$y = -3(x^2 + x - 2)$

$y = -3x^2 - 3x + 6$

EXAMPLE **Write $f(x) = \frac{1}{2}(x - 2)^2 - 3$ in standard form.**

Each of the different forms for quadratics has advantages depending on the situation. Therefore, you should be able to convert from one form to another easily.

Solution:

$f(x) = \frac{1}{2}(x - 2)(x - 2) - 3$

$f(x) = \frac{1}{2}(x^2 - 4x + 4) - 3$

$f(x) = \frac{1}{2}x^2 - 2x + 2 - 3$

$f(x) = \frac{1}{2}x^2 - 2x - 1$

PRACTICE **Write the quadratic function in standard form.**

13. $y = 3(x - 2)(x + 2)$

14. $y = -2(x - 3)(x - 4)$

15. $y = -3(x + 1)^2 - 1$

16. $f(x) = 3(x - 2)^2 - 2$

6. Solve a Quadratic Inequality by Graphing

EXAMPLE **Solve $3x^2 - 6x + 2 \geq 0$ by graphing.**

Solution:

A solution to an inequality is a value that makes the inequality true.

The solution consists of the x-values for which the graph of $3x^2 - 6x + 2 \geq 0$ lies on or above the x-axis. Find the graph's x-intercepts by using the quadratic formula to solve for x.

$0 = 3x^2 - 6x + 2$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(3)(2)}}{2(3)}$$

$$x = \frac{6 \pm \sqrt{12}}{6}$$

$x \approx 0.4226$ and $x \approx 1.5774$

Sketch a parabola that opens up and has 0.4226 and 1.5774 as x-intercepts. The graph lies on or above the x-axis to the left of (and including) 0.4226 and to the right of (and including) 1.5774.

The solution of the inequality is approximately $x \leq 0.4226$ and $x \geq 1.5774$

PRACTICE **Solve each inequality using a graph.**

17. $x^2 - 4x + 2 \geq 0$

18. $2x^2 - 3x - 4 \leq 0$

BENCHMARK 2
C. Graphing Quadratics

Name ______________________________ Date ______________

BENCHMARK 2

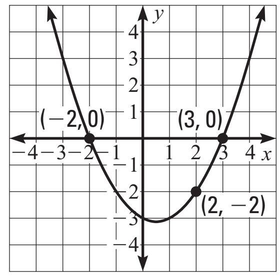
(Chapters 3 and 4)

7. Write a Quadratic Model

EXAMPLE **Write a quadratic function for the parabola shown.**

Use the appropriate form of the equation of a parabola depending on the information you are given.

Solution:



Use intercept form because the x-intercepts are given.

$y = a(x - p)(x - q)$

$y = a(x + 2)(x - 3)$

Use the other given point, (2, −2), to find a.

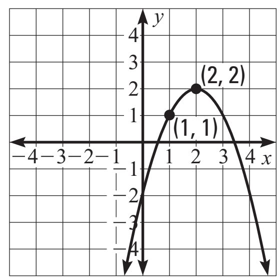
$-2 = a(2 + 2)(2 - 3)$

$a = \frac{1}{2}$

A quadratic function for the parabola is $y = \frac{1}{2}(x + 2)(x - 3)$.

EXAMPLE **Write a quadratic function for the parabola shown.**

Solution:



Use vertex form because the vertex is given.

$y = a(x - h)^2 + k$

$y = a(x - 2)^2 + 2$

Use the other given point (1, 1) to find a.

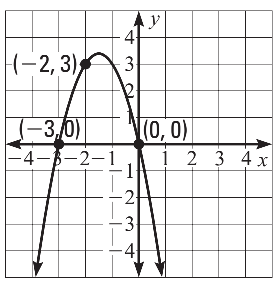
$1 = a(1 - 2)^2 + 2$

$a = -1$

A quadratic function for the parabola is $y = -(x - 2)^2 + 2$.

PRACTICE **Write a quadratic function for the parabola shown.**

19.



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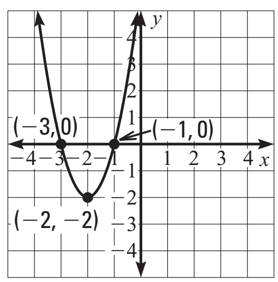

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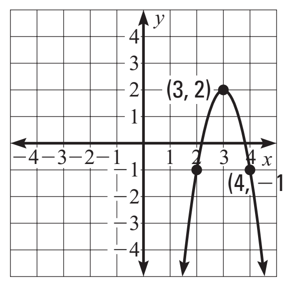


Name ______________________________ Date ______________

BENCHMARK 6

(Chapters 3 and 4)

Quiz

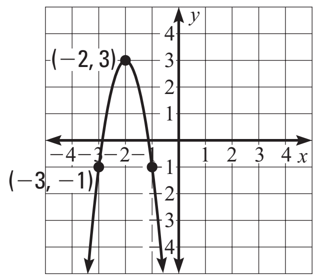
1. Tell whether the function $y = -3x^2 - 9x + 1$ has a minimum value or a maximum value. Then find the minimum or maximum value.

2. Graph $y = -2(x + 3)^2 - 3$. Label the vertex and axis of symmetry.

3. Graph $y = 2(x - 3)(x + 2)$. Label the vertex, axis of symmetry, and x-intercepts.

4. Write $y = -3(x + 1)^2 - 1$ in standard form.

5. Write a quadratic function for the parabola shown.



BENCHMARK 2
C. Graphing Quadratics

Name ______________________ Date ______________

BENCHMARK 2
(Chapters 3 and 4)

D. Solving Quadratic Equations (pp. 42–44)

1. Factor Quadratic Trinomials

Vocabulary

Monomial An expression that is either a number, a variable, or the product of a number and one or more variables.

Binomial The sum of two monomials.

Trinomial Sum of three monomials.

EXAMPLE **Factor the expression.**

Many trinomials cannot be factored. But you need to try all possibilities before you can conclude that that is the case.

a. $x^2 - 2x - 24$ **b.** $x^2 - 10x + 21$

Solution:

a. You want $x^2 - 2x - 24 = (x + m)(x + n)$ where $mn = -24$ and $m + n = -2$

$m = -6$ and $n = 4$. So $x^2 - 2x - 24 = (x - 6)(x + 4)$.

b. You want $x^2 - 10x + 21 = (x + m)(x + n)$ where $mn = 21$ and $m + n = -10$

$m = -7$ and $n = -3$. So $x^2 - 10x + 21 = (x - 7)(x - 3)$.

PRACTICE **Factor the quadratic trinomial.**

1. $y = x^2 - 14x + 49$ **2.** $y = x^2 - 3x - 28$ **3.** $y = x^2 + x - 6$

2. Use Special Factoring Patterns

EXAMPLE **Factor the expression using special patterns.**

WARNING: $a^2 - b^2 \neq (a - b)^2$

a. $y = x^2 - 64$ **b.** $y = x^2 + 14x + 49$ **c.** $y = x^2 - 10x + 25$

Solution:

a. $y = x^2 - 64 = (x + 8)(x - 8)$ **Difference of two squares**

b. $y = x^2 + 14x + 49 = (x + 7)^2$ **Perfect square trinomial**

c. $y = x^2 - 10x + 25 = (x - 5)^2$ **Perfect square trinomial**

PRACTICE **Factor the expression using special patterns.**

4. $y = x^2 + 4x + 4$ **5.** $y = x^2 - 81$ **6.** $y = x^2 - 6x + 9$

3. Solve Quadratic Equations by Factoring

EXAMPLE

a. Factor $6x^2 - x - 2$. **b.** Factor $10x^2 + 13x + 4$.

When factoring $ax^2 + bx + c$ there can be many different combinations to consider if neither a nor c is prime.

Solution:

a. The correct factorization is $(2x + 1)(3x - 2)$.

b. The correct factorization is $(2x + 1)(5x + 4)$.

Name ______________________ Date ______________

BENCHMARK 2
(Chapters 3 and 4)

PRACTICE **Factor each expression.**

7. $2x^2 + 10x + 12$ **8.** $2x^2 - 7x - 15$ **9.** $6x^2 - 11x + 3$

4. Use Properties of Square Roots

Vocabulary **Radical** The expression $\sqrt{s}$.

EXAMPLE **Simplify the expression.**

When simplifying radicals, find the greatest perfect square that is a factor of the value under the radical.

a. $\sqrt{40}$ **b.** $\sqrt{2}\sqrt{10}$ **c.** $\sqrt{\frac{-9}{16}}$ **d.** $\sqrt{\frac{18}{25}}$

Solution:

a. $\sqrt{40} = \sqrt{4}\sqrt{10} = 2\sqrt{10}$

b. $\sqrt{2}\sqrt{10} = \sqrt{20} = \sqrt{4}\sqrt{5} = 2\sqrt{5}$

c. $\sqrt{\frac{-9}{16}} = \frac{i\sqrt{9}}{\sqrt{16}} = \frac{3i}{4}$

d. $\sqrt{\frac{18}{25}} = \frac{\sqrt{18}}{\sqrt{25}} = \frac{\sqrt{18}}{5} = \frac{\sqrt{9}\sqrt{2}}{5} = \frac{3\sqrt{2}}{5}$

PRACTICE **Simplify the expression.**

10. $\sqrt{54}$ **11.** $\sqrt{3}\sqrt{12}$

12. $\sqrt{\frac{25}{49}}$ **13.** $\sqrt{\frac{14}{9}}$

EXAMPLE **Simplify the expression.**

a. $\sqrt{\frac{7}{3}}$ **b.** $\frac{2}{3 + \sqrt{5}}$

Solution:

a. $\sqrt{\frac{7}{3}} = \frac{\sqrt{7}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{21}}{3}$

b. $\frac{2}{3 + \sqrt{5}} = \frac{2}{3 + \sqrt{5}} \cdot \frac{3 - \sqrt{5}}{3 - \sqrt{5}} = \frac{6 - 2\sqrt{5}}{9 + 3\sqrt{5} - 3\sqrt{5} - 5} = \frac{6 - 2\sqrt{5}}{4} = \frac{3 - \sqrt{5}}{2}$

PRACTICE **Simplify the expression.**

14. $\sqrt{\frac{11}{5}}$ **15.** $\sqrt{\frac{12}{5}}$

16. $\frac{3}{2 + \sqrt{3}}$ **17.** $\frac{5}{4 - \sqrt{6}}$

5. Add, Subtract, and Multiply Complex Numbers

Vocabulary **Complex number** A number $a + bi$ where a and b are real numbers.

Imaginary number If $b \neq 0$, then $a + bi$ is an imaginary number.

EXAMPLE **Write the expression as a complex number in standard form.**

Adding and subtracting complex numbers is similar to adding and subtracting binomials. But remember, *i* is not a variable.

a. $(3 - i) + (4 + 2i)$ **b.** $(6 + 2i) - (3 + 2i)$ **c.** $8 - (3 + 4i) - 6i$

Solution:

a. $(3 - i) + (4 + 2i) = (3 + 4) + (-1 + 2)i = 7 + i$

b. $(6 + 2i) - (3 + 2i) = (6 - 3) + (2 - 2)i = 3$

c. $8 - (3 + 4i) - 6i = (8 - 3) + (-4 - 6)i = 5 - 10i$

BENCHMARK 2
D. Solving Quadratics

Name ______________________ Date ______________

BENCHMARK 2
(Chapters 3 and 4)

PRACTICE **18.** $(4 + 5i) - (2 - 6i)$ **19.** $(3 + 4i) + (6 - 4i)$ **20.** $3 - (4 + 3i) - 6i$

6. Solve Quadratic Equations by Finding Square Roots

EXAMPLE **a.** Solve $4x^2 + 7 = 55$. **b.** Solve $2x^2 + 12 = 4$.

Solution:

a. $4x^2 + 7 = 55$

$4x^2 = 48$

$x^2 = 12$

$x = \pm\sqrt{12} = \pm\sqrt{4}\sqrt{3} = \pm 2\sqrt{3}$

The solutions are $2\sqrt{3}$ and $-2\sqrt{3}$.

b. $2x^2 + 12 = 4$

$2x^2 = -8$

$x^2 = -4$

$x = \pm\sqrt{-4} = \pm i\sqrt{4} = \pm 2i$

PRACTICE **21.** $4x^2 + 10 = 34$ **22.** $5x^2 - 8 = 22$ **23.** $-3x^2 - 4 = 14$

Quiz

Factor each.

1. $x^2 - 7x + 10$

2. $x^2 - 16x + 64$

3. $6x^2 - 7x - 3$

4. $x^2 - 121$

Simplify each expression.

5. $\sqrt{75}$

6. $\sqrt{\frac{4}{7}}$

7. $\frac{7}{3 - \sqrt{2}}$

8. $(3 - 4i) - (7 + 2i)$

Solve for *x*.

9. $3x^2 - 12 = -32$

Name ______________________ Date ____________

BENCHMARK 2
(Chapters 3 and 4)

E. Quadratic Formula (pp. 45–47)

1. Make a Perfect Square Trinomial

EXAMPLE

This process is known as "completing the square" because you are building a perfect square trinomial from an existing trinomial.

Find the value of c that makes $x^2 + 14x + c$ a perfect square trinomial. Then write the expression as the square of a binomial.

Solution:

Step 1: Find half the coefficient of x. $\frac{14}{2} = 7$

Step 2: Square the result of Step 1: $7^2 = 49$.

Step 3: Replace c with the result from Step 2: $x^2 + 14x + 49$.

Written as the square of a binomial, it looks like this: $(x + 7)^2$.

PRACTICE

Find the value of c that makes each a perfect square trinomial. Then write the expression as the square of a binomial.

1. $x^2 - 6x + c$ **2.** $x^2 + 8x + c$ **3.** $x^2 + 10x + c$

2. Solve a Quadratic Equation by Completing the Square

EXAMPLE

Although completing the square requires some work, it makes solving quadratic equations simple.

a. Solve $x^2 + 4x - 8 = 0$ by completing the square.

$$x^2 + 4x = 8$$
$$x^2 + 4x + 4 = 8 + 4$$
$$(x + 2)^2 = 12$$
$$x + 2 = \pm\sqrt{12}$$
$$x = \pm\sqrt{12} - 2$$
$$x = -2 \pm 2\sqrt{3}$$

The solutions are $-2 + 2\sqrt{3}$ and $-2 - 2\sqrt{3}$.

b. Solve $3x^2 + 6x - 9 = 0$ by completing the square.

$$3x^2 + 6x = 9$$
$$x^2 + 2x = 3$$
$$x^2 + 2x + 1 = 3 + 1$$
$$(x + 1)^2 = 4$$
$$x + 1 = \pm\sqrt{4}$$
$$x = \pm 2 - 1$$

The solutions are -3 and 1.

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BENCHMARK 2
(Chapters 3 and 4)

BENCHMARK 2
E. Quadratic Formula

PRACTICE **Solve each by completing the square.**

4. $x^2 - 8x - 2 = 0$ **5.** $x^2 - 10x + 5 = 0$

6. $4x^2 + 24x - 16 = 0$ **7.** $6x^2 + 12x - 18 = 0$

3. Use the Discriminant

EXAMPLE **Find the discriminant of the quadratic equation and give the number and type of solutions of the equation. Find all solutions.**

a. $x^2 + 3x + 10$ **b.** $x^2 - 10x + 20$ **c.** $x^2 - 4x + 4$

If the discriminant is greater than zero, there are two real solutions. If the discriminant is less than zero, there are two imaginary solutions. If the discriminant is equal to zero there is one real solution.

Solution:

a. Step 1: Find the discriminant.

$b^2 - 4ac = 3^2 - 4(1)(10) = -31$

Because the discriminant is less than zero, conclude that there are exactly two imaginary solutions.

Step 2: Find the solutions by using the quadratic formula.

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-3 \pm \sqrt{-31}}{2} = \frac{-3 \pm i\sqrt{31}}{2}$$

b. Step 1: Find the discriminant.

$b^2 - 4ac = (-10)^2 - 4(1)(20) = 20$

Because the discriminant is greater than zero, conclude that there are exactly two real solutions.

Step 2: Find the solutions by using the quadratic formula.

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{10 \pm \sqrt{20}}{2} = \frac{10 \pm 2\sqrt{5}}{2} = \frac{5 \pm \sqrt{5}}{1}$$

c. Step 1: Find the discriminant.

$b^2 - 4ac = (-4)^2 - 4(1)(4) = 0$

Because the discriminant is zero, conclude that there is exactly one real solution.

Step 2: Find the solutions by using the quadratic formula.

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{4 \pm \sqrt{0}}{2} = 2$$

PRACTICE **Find the discriminant of the quadratic equation and give the number and type of solutions of the equation. Find all solutions.**

8. $x^2 - 14x + 49 = 0$ **9.** $x^2 + 4x - 10 = 0$ **10.** $4x^2 + 3x + 4 = 0$

Name ______________________ Date ____________

BENCHMARK 2

(Chapters 3 and 4)

Quiz

Find the value of *c* that makes each a perfect square trinomial. Then write the expression as the square of a binomial.

1. $x^2 - 12x + c$

2. $x^2 - 22x + c$

Solve each quadratic by completing the square.

3. $x^2 + 8x - 10 = 0$

4. $2x^2 - 12x + 4 = 0$

Find the discriminant of the quadratic equation and give the number and type of solutions of the equation. Find all solutions.

5. $x^2 - 4x + 6 = 0$

6. $x^2 - 16x + 64 = 0$