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(Chapters 7, 8, and 9)

A. Graphing Exponential Functions

Functions containing variable exponents are known as exponential functions. These functions are in the form $y = ab^x$ where $a \neq 0$ and the base, b, is a positive number other than 1. Exponential functions can be used to model a variety of growth and decay problems in which the rate of change is exponential instead of constant.

1. Graph an Exponential Growth Model

Vocabulary

Exponential growth function A function in the form $y = ab^x$ where $a > 0$ and $b > 1$.

Growth factor The base, b, in an exponential growth function

Exponential growth model An equation of the form $y = a(1 + r)^t$ used to show the amount, y, of a real-life quantity that increases a fixed percent after t years (or other time period).

EXAMPLE **Write and graph an exponential growth model that describes the situation.**

A trading card had a value of \$10 in 2000 and increased by 15% per year since then.

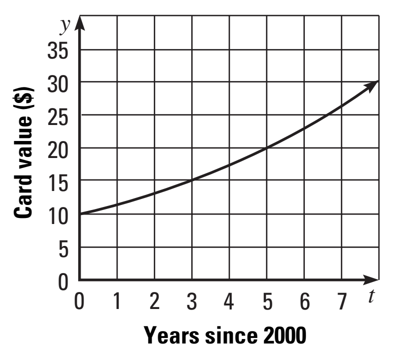
Solution:

The initial amount is $a = 10$ and the percent increase is $r = 0.15$. So the equation of the exponential growth model is:

In the exponential growth model, the growth factor, b, is shown by $1 + r$.

$y = a(1 + r)^t$	**Write the exponential growth model.**
$= 10(1 + 0.15)^t$	**Substitute 10 for *a* and 0.15 for *r*.**
$= 10(1.15)^t$	**Simplify.**

Plot a few points that lie on the graph of this function. Then draw a smooth curve through the points.



PRACTICE **Write and graph an exponential growth model that describes the situation.**

1. The number of registered users on a website was 500 in 1999. During the next 6 years, the number of registered users increased by about 40% each year.

2. A savings account at the bank earns 3% interest compounded annually. At the beginning of the year, you deposit \$200 into this account. You want to know what the value of this account will be in t years if no other deposits are made.

3. In 1998, there were 28 students enrolled in a gymnastics class. Since then, the enrollment rate increased about 12% each year. The gymnastics coach wants to

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know in what year the expected enrollment will reach about 100 students.

Vocabulary

2. Graph an Exponential Decay Model

Exponential decay function A function in the form $y = ab^x$ where $a > 0$ and $0 < b < 1$.

Exponential decay model An equation of the form $y = a(1 - r)^t$ used to show the amount, y, of a real-life quantity that decreases a fixed percent after t years (or other time period).

EXAMPLE

Decay facto The quantity $1 - r$ in an exponential decay model.

Write and graph an exponential decay model that describes the situation.

A new video game player costs $400. The value of the video game player decreases an average of 20% each year. You want to know what the value of the video game player will be t years from now.

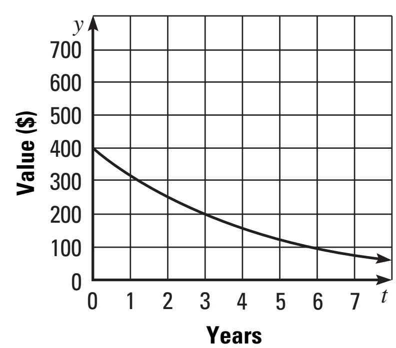
In the graph of the exponential decay model, the downward curve indicates the value of y approaches zero.

Solution:

The initial amount is $a = 400$ and the percent decrease is $r = 0.20$. So the equation of the exponential growth model is:

$y = a(1 - r)^t$	**Write the exponential decay model.**
$= 400(1 - 0.20)^t$	**Substitute 400 for *a* and 0.20 for *r*.**
$= 400(0.80)^t$	**Simplify.**

Plot a few points that lie on the graph of this function. Then draw a smooth curve through the points.



PRACTICE

Write and graph an exponential growth model that describes the situation.

4. The temperature of a 100°F object cools at a rate of 10% each hour.

5. A new medicine is being studied. The study shows the medicine loses 45% of it effectiveness each hour.

Vocabulary

The natural base e is a special irrational number with an approximate value of 2.718281828.

3. Graph Natural Base Functions

Natural base function A function in the form $y = ae^{rx}$ where $a > 0$ and if $r > 0$, the function is an exponential growth function and if $r < 0$, the function is an exponential decay function.

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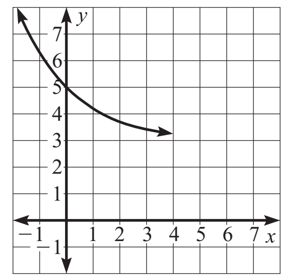
EXAMPLE **Graph the function $y = 2e^{-0.5x} + 3$. State the domain and range.**

Solution:

Because $a = 2$ is positive and $r = -0.5$ is negative, the function is an exponential decay function.

Plot the points $(-1, 6.3)$, $(0, 5)$, $(1, 4.2)$, and $(2, 3.7)$ and draw the curve.

The domain is all real numbers, and the range is $y > 3$.



PRACTICE **Graph the function and identify the domain and range of the function.**

6. $y = e^{2(x-1)}$ **7.** $y = 0.25e^{-x} - 1$ **8.** $y = 0.25e^{0.5x} + 2$

Quiz

Write and graph an exponential growth model that describes the situation.

1. The average tuition cost at a community college in 1990 was \$5000 per year. Each year since then, the tuition amount increased at a rate of 5%.

2. Five years ago, the cost of tickets for orchestra seats at a concert was \$30 each. Since then, the price of the tickets has increased an average of 12% each year.

3. The first issue of a health newsletter was published 6 years ago. At that time, the number of subscriptions to the newsletter was 600. Research shows that the number of subscriptions to the newsletter has increased at a rate of 16% each year since then.

Write and graph an exponential decay model that describes the situation.

4. The value of an investment has decreased at an average rate of 18% each year since 2001. You want to know the value of a \$10,000 investment made in 2001 t years from now.

5. In a controlled experiment, the speed at which a gear rotates is decreased at a rate of 10% each minute. At the start of the experiment, the gear rotates 200 revolutions per minute. You want to know the number of times the gear rotates t minutes from the start of the experiment.

Graph the function and identify the domain and range of the function.

6. $y = 3e^{-x+2}$ **7.** $y = 0.5e^{-0.5x}$ **8.** $y = e^{0.75x} - 4$

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B. Logarithmic and Exponential Equations

Inverse relationships between exponential and logarithmic functions can be used to graph logarithmic functions.

1. Graph Logarithmic Functions

Vocabulary

Logarithm of y with base b For positive numbers b and y where $b \neq 1$, $\log_b y = x$ if and only if $b^x = y$.

EXAMPLE **Find the inverse of the function.**

The natural log, ln, and *e* are inverses of each other. So, $\ln e^x = x$.

a. $y = 5^x$ **b.** $y = \ln(x - 2)$

Solution:

a. From the definition of logarithm, the inverse of $y = 5^x$ is $y = \log_5 x$.

b.

$y = \ln(x - 2)$	**Write original function.**
$x = \ln(y - 2)$	**Switch x and y.**
$e^x = y - 2$	**Write in exponential form.**
$y = e^x + 2$	**Solve for y.**

The inverse of $y = \ln(x - 2)$ is $y = e^x + 2$.

PRACTICE **Find the inverse of the function.**

1. $y = 2^x$ **2.** $y = e^{x+6}$ **3.** $y = \ln(2x) - 3$

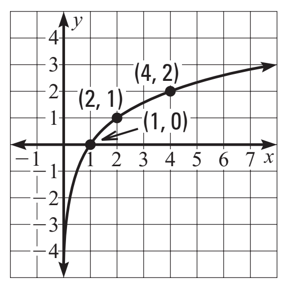
EXAMPLE **Graph the function.**

a. $y = \log_2 x$ **b.** $y = \log_{1/4} x$

Solution:

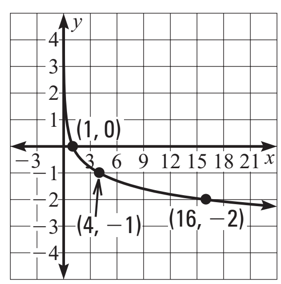
a. Plot several convenient points, such as (1, 0), (2, 1), and (4, 2). The y-axis is the vertical asymptote.

From *left* to *right*, draw a curve that starts just to the right of the y-axis and moves up through the plotted points.



b. Plot several convenient points, such as (1, 0), (4, −1), and (16, −2). The y-axis is the vertical asymptote.

From *left* to *right*, draw a curve that starts just to the right of the y-axis and moves down through the plotted points.



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PRACTICE **Graph the function.**

4. $y = \log_4 x$ **5.** $y = \log_{1/6} x$

2. Use Properties of Logarithms

EXAMPLE **Rewrite the logarithm in exponential form.**

Recall that the common logarithm is a logarithm with base 10. $\log_{10} x = \log x$

a. $\log_3 9 = 2$ **b.** $\log_{1/2} 8 = -3$

Solution:

a. By the definition of logarithm, the exponential form of $\log_3 9 = 2$ is $3^2 = 9$.

b. The exponential form of $\log_{1/2} 8 = -3$ is $\left(\frac{1}{2}\right)^{-3} = 8$.

PRACTICE **Rewrite the logarithm in exponential form.**

6. $\log_7 1 = 0$ **7.** $\log_x 625 = 4$ **8.** $\log_4 \left(\frac{1}{16}\right) = -2$

EXAMPLE **Evaluate the logarithm.**

a. $\log_3 27$ **b.** $\log_{49} 7$

Solution:

a. Ask yourself this question: 3 to what power gives 27? $3^3 = 27$, so $\log_3 27 = 3$.

b. Ask yourself this question: 49 to what power gives 7? $49^{1/2} = 7$, so $\log_{49} 7 = \frac{1}{2}$.

PRACTICE **Evaluate the logarithm.**

9. $\log_2 1$ **10.** $\log_{1/5} 25$ **11.** $\log_{64} 4$

EXAMPLE **Evaluate the logarithm using $\log_2 5 \approx 2.322$ and $\log_2 3 \approx 1.585$.**

Recall the properties of logarithms.

Product Property: $\log_b mn = \log_b m + \log_b n$

Quotient Property: $\log_b \frac{m}{n} = \log_b m - \log_b n$

Power Property: $\log_b m^n = n \log_b m$

a. $\log_2 \frac{3}{5}$ **b.** $\log_2 9$

Solution:

a. $\log_2 \frac{3}{5} = \log_2 3 - \log_2 5$ **Quotient property**

$= 1.585 - 2.322$ **Use the given values of each log.**

$= -0.737$ **Simplify.**

b. $\log_2 9 = \log_2 3^2$ **Write 9 as 3^2.**

$= 2 \log_2 3$ **Power property**

$= 2(1.585)$ **Use the given value of $\log_2 3$.**

$= 3.17$ **Simplify.**

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PRACTICE **Evaluate the logarithm using $\log_5 3 \approx 0.683$ and $\log_5 15 \approx 1.683$.**

12. $\log_5 45$ **13.** $\log_5 \frac{1}{5}$ **14.** $\log_5 27$

3. Use the Change-of-Base Formula

To change the base of a logarithmic expression for positive numbers a, b, and c with $b \neq 1$ and $c \neq 1$, use the formula:

$$\log_c a = \frac{\log_b a}{\log_b c}$$

EXAMPLE **Use the change-of-base formula to evaluate the logarithm.**

The common logarithm and the natural log can both be used to change the base of $\log_b x$.

a. $\log_6 3$ **b.** $\log_{15} 25$

Solution:

a. $\log_6 3 = \frac{\log 3}{\log 6} \approx \frac{0.477}{0.778} \approx 0.613$ **b.** $\log_{15} 25 = \frac{\log 25}{\log 15} \approx \frac{1.398}{1.176} \approx 1.189$

PRACTICE **Use the change-of-base formula to evaluate the logarithm.**

$\log_b x = \frac{\log x}{\log b}$ and $\log_b x = \frac{\ln x}{\ln b}$

15. $\log_3 20$ **16.** $\log_8 4$ **17.** $\log_{100} 25$

4. Solve Exponential and Logarithmic Equations

Vocabulary

Exponential equation An equation where the exponent is a variable expression.

Logarithmic equation An equation that involves a logarithm of a variable expression.

EXAMPLE **Solve the equation.**

a. $3^x = 33$ **b.** $\log_6 (5x + 6) = 2$

a. $3^x = 33$ **Write original equation.**

$\log_3 3^x = \log_3 33$ **Take $\log_3$ of each side.**

$x = \log_3 33$ **$\log_3 3^x = x$**

$x = \frac{\log 33}{\log 3}$ **Change-of-base formula**

$x \approx 3.183$ **Use a calculator to simplify.**

b. $\log_6 (5x + 6) = 2$ **Write original equation.**

$6^{\log_6 (5x+6)} = 6^2$ **Exponentiate each side using base 6.**

$5x + 6 = 36$ **$b^{\log_b x} = x$**

$5x = 30$ **Subtract 6 from each side.**

$x = 6$ **Divide each side by 5.**

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PRACTICE **Solve the equation.**

18. $5^x = 30$ **19.** $2^{x+4} = 45$ **20.** $4^{3x} - 5 = 27$

21. $9 \log_3 x = 4$ **22.** $\log_4 (3x - 8) = 0.5$

5. Write Exponential and Power Functions

EXAMPLE **Write an exponential function in the form $y = ab^x$ whose graph passes through the points (1, 10) and (3, 40).**

You can check that the exponential function is correct by substituting the given points (1, 10) and (3, 40) into the function.

STEP 1: Substitute the coordinates of the two given points into $y = ab^x$.

$10 = ab^1$ **Substitute 10 for *y* and 1 for *x*.**

$40 = ab^3$ **Substitute 40 for *y* and 3 for *x*.**

STEP 2: Solve for a in the first equation to obtain $a = \frac{10}{b}$. Then substitute this expression for a in the second equation.

$40 = \left(\frac{10}{b}\right)b^3$ **Substitute $\frac{10}{b}$ for *a* in the second equation.**

$40 = 10b^2$ **Simplify.**

$4 = b^2$ **Divide each side by 10.**

$2 = b$ **Take the positive square root because $b > 0$.**

STEP 3: Determine the value of a as $\frac{10}{2}$ or 5.

The exponential function is $y = 5 \cdot 2^x$.

EXAMPLE **Write a power function in the form $y = ax^b$ whose graph passes through the points (2, 6) and (4, 18).**

$6 = a \cdot 2^b$ and $18 = a \cdot 4^b$ **Substitute the values of each *x* and *y* into the function.**

$a = \frac{6}{2^b}$ **Solve for *a* using the first equation.**

$18 = \frac{6}{2^b} \cdot 4^b$ **Substitute the value of *a* in the second equation.**

$18 = 6 \cdot 2^b$ **Simplify.**

$3 = 2^b$ **Divide each side by 6.**

$\log_2 3 = \log_2 2^b$ **Take $\log_2$ of each side.**

$\frac{\log 3}{\log 2} = b$ **Change-of-base formula**

$b \approx 1.585$ **Use a calculator to simplify.**

$a = \frac{6}{2^{1.585}} = 2$ **Substitute the value of *b* to find *a*.**

The power function is $y = 2x^{1.585}$.

PRACTICE **23.** Write an exponential function whose graph passes through (1, 3) and (2, 1.5).

24. Write a power function whose graph passes through the points (2, −5) and (5, −10).

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Quiz

Find the inverse of the function.

1. $y = \log_7 x$ **2.** $y = e^{x+1}$ **3.** $y = \ln(x + 4)$

Graph the function.

4. $y = \log_5 x$ **5.** $y = \log_{1/2} x$

Evaluate the logarithm.

6. $\log_6 1$ **7.** $\log_{1/4} 64$ **8.** $\log_4 17$ **9.** $\log_7 2$

Evaluate the logarithm using $\log_4 3 \approx 0.792$ and $\log_4 8 = 1.5$.

10. $\log_4 24$ **11.** $\log_4 \frac{8}{3}$ **12.** $\log_4 \frac{9}{8}$

Solve the equation.

13. $7^x = 55$ **14.** $9^{0.5x} + 1 = 6$ **15.** $\log_5 4x = 2$

16. Write an exponential function whose graph passes through (1, 1) and (3, 16).

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C. Graphing Rational Functions

In direct variation, an increase in one variable causes an increase in another variable, and a decrease in one variable causes a decrease in another variable. With inverse variation, the reverse is true. An *increase* in one variable causes a *decrease* in another variable. Likewise, a *decrease* in one variable causes an *increase* in another variable.

1. Write an Inverse Variation Equation

Vocabulary

The equation $y = \frac{a}{x}$ is read "y varies inversely with x".

Inverse variation An equation with two variables, x and y, having the relationship $y = \frac{a}{x}$ where $a \neq 0$.

Constant of variation The constant value, a, in the equation $y = \frac{a}{x}$.

EXAMPLE

The variable y varies inversely with the variable x. When $x = 5$, $y = 3$. Write an equation relating x and y. Then find y when x is -30.

You can check that the inverse variation equation is correct. Substitute the original values for x and y and see if the equation is true.

Solution:

$y = \frac{a}{x}$	**Write the general equation for inverse variation.**
$3 = \frac{a}{5}$	**Substitute 5 for x and 3 for y.**
$15 = a$	**Solve for a.**

The inverse variation equation is $y = \frac{15}{x}$. When $x = -30$, $y = \frac{15}{-30} = -\frac{1}{2}$.

PRACTICE

Write an equation relating x and y. Then find y when $x = 4$.

1. y varies inversely with x. When $x = 2$, $y = 12$.
2. y varies inversely with x. When $x = 8$, $y = -2$.

2. Write a Joint Variation Equation

Vocabulary

Joint variation A form of *direct* variation in which one variable varies directly with the product of two or more variables, as in $z = axy$, where a is a nonzero constant.

EXAMPLE

The variable z varies jointly with the variables x and y. When $z = 24$, $x = -3$ and $y = 2$. Write an equation relating x, y, and z. Then find z when x is 4 and y is 3.

The equation $z = axy$ is read "z varies jointly with x and y"

Solution:

$z = axy$	**Write the general equation for joint variation.**
$24 = a(-3)(2)$	**Substitute 24 for z, -3 for x, and 2 for y.**
$24 = -6a$	**Simplify.**
$-4 = a$	**Solve for a.**

The joint variation equation is $z = -4xy$.
When $x = 4$ and $y = 3$, $z = -4(4)(3) = -48$.

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PRACTICE **Write a joint variation equation relating *x*, *y*, and *z*. Then find *z* when $x = 3$ and $y = 2$.**

3. When $x = 2$ and $y = 4$, $z = 64$.

4. When $x = -5$ and $y = 6$, $z = 10$.

3. Graph Simple Rational Functions

Vocabulary **Rational function** A function in the form $f(x) = \frac{p(x)}{q(x)}$ where $p(x)$ and $q(x)$ are polynomials and $q(x) \neq 0$.

EXAMPLE **Graph $y = \frac{3}{x + 4} + 2$. State the domain and range of the function.**

Solution:

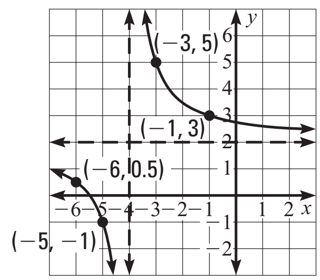
The domain of a rational function is all real numbers except for the vertical asymptote.

The range is all real numbers except for the horizontal asymptote.

STEP 1: **Draw** the asymptotes $x = -4$ and $y = 2$.

STEP 2: **Plot** points to the left of the vertical asymptote, such as $(-5, -1)$ and $(-6, 0.5)$, and to the right of the vertical asymptote, such as $(-3, 5)$ and $(-1, 3)$.

STEP 3: **Draw** the two branches of the hyperbola so that they approach the asymptotes and pass through the plotted points.



The domain is all real numbers except $x = -4$. The range is all real numbers except $y = 2$.

PRACTICE **Graph the function. State the domain and range.**

5. $y = \frac{5}{x} - 2$

6. $y = \frac{-2}{x - 3} + 5$

EXAMPLE **Graph $y = \frac{3x + 1}{x - 2}$. State the domain and range of the function.**

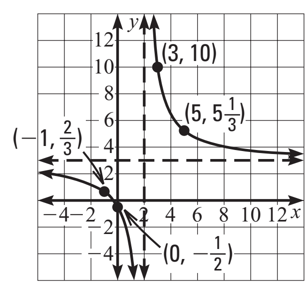
Solution:

Recall that in a rational function $y = \frac{(ax + b)}{(cx + d)}$, the vertical asymptote is the line $x = -\frac{d}{c}$. The horizontal asymptote is the line $y = \frac{a}{c}$.

STEP 1: **Draw** the asymptotes $x = 2$ and $y = 3$.

STEP 2: **Plot** points to the left of the vertical asymptote, such as $(0, -0.5)$ and $(-1, 0.67)$, and to the right of the vertical asymptote, such as $(5, 5.3)$ and $(3, 10)$.

STEP 3: **Draw** the two branches of the hyperbola so that they approach the asymptotes and pass through the plotted points.



The domain is all real numbers except $x = 2$. The range is all real numbers except $y = 3$.

PRACTICE **Graph the function. State the domain and range.**

7. $y = \frac{5x}{x + 4}$

8. $y = \frac{2x + 1}{2x - 1}$

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4. Graph General Rational Functions

EXAMPLE **Graph the function.**

a. $y = \dfrac{2x}{x^2 + 4}$ **b.** $y = \dfrac{x^2}{x^2 - 1}$ **c.** $y = \dfrac{x^2 + 3x + 2}{x - 2}$

Solution:

Remember that the horizontal asymptote depends on the degree of the polynomials of the function.

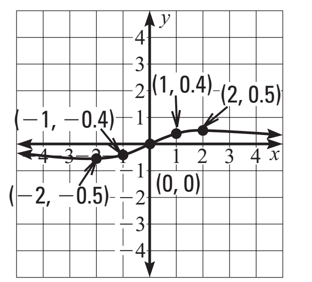
For $f(x) = \dfrac{p(x)}{q(x)}$, with the degree of $p(x) = m$ and the degree of $q(x) = n$, $f(x)$ has these horizontal asymptotes:

for $m < n$, $y = 0$ for

$m = n$, $y = \dfrac{a_m}{b_n}$

for $m > n$, no horizontal asymptote.

a.

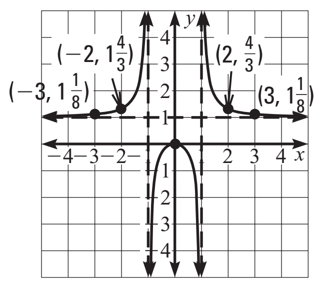


The numerator has a zero at 0, so the x-intercept is 0. The denominator has no real zeros, so there is no vertical asymptote.

The horizontal asymptote is $y = 0$ (the x-axis) since the degree of the numerator, 1, is less than the degree of the denominator, 2.

The graph passes through points $(-2, -0.5)$, $(-1, -0.4)$, $(0, 0)$, $(1, 0.4)$, and $(2, 0.5)$.

b.

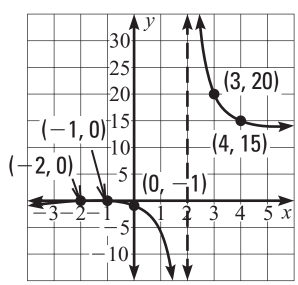


The numerator has a zero at 0, so the x-intercept is 0. The denominator has zeros at 1 and -1, so the vertical asymptotes are at $x = 1$ and $x = -1$.

The horizontal asymptote is $y = 1$ since both the degree of the numerator and the denominator is 2.

The graph passes through points $(-3, 1.125)$, $(-2, 1.333)$, $(0, 0)$, $(2, 1.333)$, and $(3, 1.125)$.

c.



The numerator has zeros at -1 and -2. The denominator has a zero at 2, so the vertical asymptote is at $x = 2$.

There is no horizontal asymptote since the degree of the numerator, 2, is greater than the degree of the denominator, 1.

The graph passes through points $(-2, 0)$, $(-1, 0)$, $(0, -1)$, $(3, 20)$, and $(4, 15)$.

PRACTICE **Graph the function.**

9. $y = \dfrac{x - 3}{x^2 - 1}$ **10.** $y = \dfrac{x^2 + 4x}{x + 2}$

Quiz

***y* varies inversely with *x*. Write an equation relating *x* and *y*. Then find *y* when *x* = −2.**

1. $y = 6$ when $x = 3$ **2.** $y = -1$ when $x = 4$ **3.** $y = 5$ when $x = 6$

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***z* varies jointly with *x* and *y*. Write a joint varies equation relating *x*, *y*, and *z*. Find *z* when $x = 5$ and $y = -1$.**

4. $z = 12$ when $x = 2$ and $y = 8$

5. $z = -32$ when $x = -4$ and $y = 2$

Graph the function.

6. $y = \frac{8x + 1}{2x + 6}$

7. $y = \frac{x}{2x^2 - 8}$

8. $y = \frac{x^2}{x + 3}$

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D. Rational Expressions and Equations

A *simplified rational expression* has no common factors, other than ±1, in its numerator and denominator. To simplify a rational expression, factor the numerator and denominator. Then divide out all common factors.

1. Simplify Rational Expressions

EXAMPLE

Simplify: $\dfrac{x^2 + 6x + 9}{x^2 - x - 12}$

Notice that the common factor $x + 3$ appears twice in the numerator and once in the denominator. That factor is divided out from the numerator and denominator only once.

Solution:

$$\frac{x^2 + 6x + 9}{x^2 - x - 12} = \frac{(x + 3)(x + 3)}{(x + 3)(x - 4)}$$ **Factor numerator and denominator.**

$$= \frac{(x + 3)}{(x - 4)}$$ **Divide out common factor (x + 3).**

PRACTICE

Simplify the expression.

Be sure to fully factor the numerator and denominator before dividing out common factors.

1. $\dfrac{2x^2}{4x^2 - 8x}$ **2.** $\dfrac{x^2 - 16}{x^2 + 9x + 20}$ **3.** $\dfrac{3x^2 - 3x}{x^2 - 1}$

2. Multiply and Divide Rational Expressions

EXAMPLE

Multiply: $\dfrac{x^2 + x}{x^2 + 4x + 3} \cdot \dfrac{x^2 + 2x - 3}{x^2 - 1}$

To multiply rational expressions, multiply numerators, then multiply denominators. Write the new fraction in simplified terms.

Solution:

$$\frac{x^2 + x}{x^2 + 4x + 3} \cdot \frac{x^2 + 2x - 3}{x^2 - 1} = \frac{x(x + 1)}{(x + 1)(x + 3)} \cdot \frac{(x + 3)(x - 1)}{(x + 1)(x - 1)}$$ **Factor.**

$$= \frac{x(x + 1)(x + 3)(x - 1)}{(x + 1)(x + 3)(x + 1)(x - 1)}$$ **Multiply.**

$$= \frac{x}{(x + 1)}$$ **Divide out common factors.**

PRACTICE

Multiply the expression.

4. $\dfrac{5x^2y^4}{6x^3y} \cdot \dfrac{3x^2y^2}{10xy^5}$ **5.** $\dfrac{x^2 - 6x}{6x^2 + 12x} \cdot \dfrac{3x^2 - 12}{x - 6}$

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BENCHMARK 4

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EXAMPLE **Divide:** $\frac{6x}{2x-6} \div \frac{4x-12}{x^2-6x+9}$

To divide rational expressions, multiply the first expression by the reciprocal of the second expression.

Solution:

$$\frac{6x}{2x-6} \div \frac{4x-12}{x^2-6x+9} = \frac{6x}{2x-6} \cdot \frac{x^2-6x+9}{4x-12}$$ **Multiply by reciprocal.**

$$= \frac{6x}{2(x-3)} \cdot \frac{(x-3)(x-3)}{4(x-3)}$$ **Factor.**

$$= \frac{6x(x-3)(x-3)}{2(x-3)(4)(x-3)}$$ **Multiply.**

$$= \frac{3x}{4}$$ **Divide out common factors.**

PRACTICE **Divide the expression.**

6. $\frac{x+5}{x^2+5x} \div \frac{x-2}{5x^2-10x}$

7. $\frac{x^2+5x+4}{x-1} \div \frac{x^2-16}{x^2-5x+4}$

3. Add and Subtract Rational Expressions

EXAMPLE **Perform the indicated operation and simplify, if possible.**

Check to see if a rational expression can be simplified before adding.

$\frac{x}{10x^2}$ simplifies to $\frac{1}{10x}$

This will make adding the rational expressions easier.

a. $\frac{x}{10x^2} + \frac{6}{5x^2-10x}$

b. $\frac{x-1}{2x+6} - \frac{2}{x^2-9}$

Solution:

a. $$\frac{x}{10x^2} + \frac{6}{5x^2-10x} = \frac{x}{10x^2} + \frac{6}{5x(x-2)}$$ **Factor denominators.**

$$= \frac{x}{10x^2} \cdot \frac{x-2}{x-2} + \frac{6}{5x(x-2)} \cdot \frac{2x}{2x}$$ **LCD is $10x^2(x-2)$.**

$$= \frac{x^2-2x}{10x^2(x-2)} + \frac{12x}{10x^2(x-2)}$$ **Multiply.**

$$= \frac{x^2+10x}{10x^2(x-2)}$$ **Add.**

$$= \frac{x(x+10)}{10x^2(x-2)}$$ **Factor.**

$$= \frac{x+10}{10x(x-2)}$$ **Divide out common factor.**

b. $$\frac{x-1}{2x+6} - \frac{2}{x^2-9} = \frac{x-1}{2(x+3)} - \frac{2}{(x+3)(x-3)}$$ **Factor denominators.**

$$= \frac{x-1}{2(x+3)} \cdot \frac{(x-3)}{(x-3)} - \frac{2}{(x+3)(x-3)} \cdot \frac{2}{2}$$ **LCD is $2(x+3)(x-3)$.**

$$= \frac{x^2-4x+3}{2(x+3)(x-3)} - \frac{4}{2(x+3)(x-3)}$$ **Multiply.**

$$= \frac{x^2-4x-1}{2(x+3)(x-3)}$$ **Subtract.**

This expression cannot be simplified, so $\frac{x-1}{2x+6} - \frac{2}{x^2-9} = \frac{x^2-4x-1}{2(x+3)(x-3)}$.

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PRACTICE **Perform the indicated operation and simplify, if possible.**

8. $\dfrac{2}{x^2 + 3x + 2} + \dfrac{x}{2x + 2}$

9. $\dfrac{x - 5}{x^2 - 8x + 15} - \dfrac{x + 5}{x^2 + 2x - 15}$

4. Simplify Complex Fractions

Vocabulary **Complex fraction** a fraction containing a fraction in its numerator, denominator, or both.

EXAMPLE **Simplify:** $\dfrac{\frac{4}{x} + 1}{\frac{8}{x} - \frac{x}{2}}$

One way to simplify complex fractions is to multiply the numerator and denominator by the LCD of every fraction within the complex fraction.

Solution:

The LCD of every fraction in the complex fraction is $2x$.

$\dfrac{\frac{4}{x} + 1}{\frac{8}{x} - \frac{x}{2}} = \dfrac{\frac{4}{x} + 1}{\frac{8}{x} - \frac{x}{2}} \cdot \dfrac{2x}{2x}$ **Multiply numerator and denominator by the LCD.**

$= \dfrac{8 + 2x}{16 - x^2}$ **Simplify.**

$= \dfrac{2(4 + x)}{(4 - x)(4 + x)}$ **Factor the numerator and denominator.**

$= \dfrac{2}{4 - x}$ **Simplify.**

PRACTICE **Simplify the complex fraction.**

10. $\dfrac{6}{\frac{3}{x} - \frac{4}{x}}$

11. $\dfrac{\frac{1}{x} + 2}{\frac{-4}{x} - 8}$

12. $\dfrac{\frac{4}{x - 1}}{\frac{3}{x - 1} - \frac{1}{x}}$

5. Solve a Rational Equation by Cross-Multiplying

EXAMPLE **Solve:** $\dfrac{6}{x + 4} = \dfrac{2}{2x - 7}$

Solution:

$\dfrac{6}{x + 4} = \dfrac{2}{2x - 7}$ **Write original equation.**

$6(2x - 7) = 2(x + 4)$ **Cross multiply.**

$12x - 42 = 2x + 8$ **Distributive property**

$10x - 42 = 8$ **Subtract 2x from each side.**

$10x = 50$ **Add 42 to each side.**

$x = 5$ **Divide each side by 10.**

The solution is 5.

Check the solution by substituting it into the original equation.

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PRACTICE

13. $\frac{8}{3x} = \frac{4}{2x + 1}$

14. $\frac{2}{x - 6} = \frac{-5}{x + 1}$

6. Solve a Rational Equation with Two Solutions

EXAMPLE

Solve: $\frac{3}{x + 2} = \frac{3x^2}{x^2 - 4} - \frac{2x}{x - 2}$

Solution:

Write each denominator in factored form to find the LCD $(x + 2)(x - 2)$. Multiply each factor by the LCD.

$$\frac{3}{x + 2} \cdot (x + 2)(x - 2) = \frac{3x^2}{x^2 - 4} \cdot (x + 2)(x - 2) - \frac{2x}{x - 2} \cdot (x + 2)(x - 2)$$

Be sure to check for extraneous solutions by substituting the results into the original equation.

$$3(x - 2) = 3x^2 - 2x(x + 2)$$
$$3x - 6 = 3x^2 - 2x^2 - 4x$$
$$3x - 6 = x^2 - 4x$$
$$0 = x^2 - 7x + 6$$
$$0 = (x - 1)(x - 6) \text{ so } x - 1 = 0 \text{ or } x - 6 = 0$$
$$x = 1 \text{ or } \quad x = 6$$

Checking the possible solutions shows that the solutions are 1 and 6.

PRACTICE

15. Solve the equation $\frac{2}{x} - \frac{5}{x - 1} = \frac{x}{x - 1}$. Check for extraneous solutions.

Quiz

Simplify the expression.

1. $\frac{x^2 + 2x - 15}{x + 5}$

2. $\frac{x^2 + 4x - 12}{x^2 - x - 2}$

3. $\frac{\frac{8}{x} - \frac{3}{x}}{5x}$

4. $\frac{\frac{6}{x + 2}}{\frac{4}{x + 2} - \frac{2}{x - 2}}$

Perform the indicated operation and simplify, if possible.

5. $\frac{4x^2}{x - 4} \div \frac{2x}{3x^2 - 12x}$

6. $\frac{x^2 + 3x - 18}{2x^2 - 18} \cdot \frac{x^2 + x - 6}{x^2 + 4x - 12}$

7. $\frac{2}{3x^2 + 3x} + \frac{3}{x^2 - 5x - 6}$

8. $\frac{x}{x^2 + 2x + 1} - \frac{1}{1 - x^2}$

Solve the equation.

9. $\frac{1}{x + 6} = \frac{-3}{x - 6}$

10. $\frac{x}{x + 3} = \frac{2}{3x + 5}$

11. $\frac{1}{2} + \frac{3}{x} = \frac{8}{2x}$

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BENCHMARK 4

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E. Parabolas and Circles

You learned how to apply the Pythagorean Theorem, $a^2 + b^2 = c^2$, to find missing side lengths of right triangles. Skills used applying that formula, such as squaring numbers and taking square roots, can also be applied to the topics shown below.

1. Use the Distance and Midpoint Formulas

Vocabulary

Distance formula The formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ used to find the distance between two points (x_1, y_1) and (x_2, y_2).

Midpoint formula The formula $M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ used to find the point equidistant from two endpoints $A(x_1, y_1)$ and $B(x_2, y_2)$.

EXAMPLE **A line segment joins points $A(1, 7)$ and $B(-4, 2)$.**

a. What is the length of $\overline{AB}$? **b.** What is the midpoint of $\overline{AB}$?

Solution:

a. Let $(x_1, y_1) = (1, 7)$ and let $(x_2, y_2) = (-4, 2)$.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-4 - 1)^2 + (2 - 7)^2}$$

$$= \sqrt{(-5)^2 + (-5)^2} = \sqrt{25 + 25} = \sqrt{50} = 5\sqrt{2}$$

$\overline{AB}$ is $5\sqrt{2}$ units.

Plot the endpoints and midpoint on a coordinate grid to check if the midpoint seems reasonable.

b. $M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{1 + (-4)}{2}, \frac{7 + 2}{2}\right) = \left(\frac{-3}{2}, \frac{9}{2}\right)$

The midpoint of $\overline{AB}$ is $\left(\frac{-3}{2}, \frac{9}{2}\right)$.

PRACTICE **The vertices of a triangle are $Q(8, 0)$, $R(3, 4)$, and $S(5, -6)$. Find the length and midpoint of the side of the triangle.**

1. $\overline{QR}$ **2.** $\overline{SR}$ **3.** $\overline{SQ}$

2. Graph an Equation of a Parabola

Vocabulary

Parabola The set of all points equidistant from a fixed line and a fixed point not on the line.

The focus and directrix each lie $|p|$ units from the vertex of a parabola.

Focus The fixed point $(0, p)$ for parabolas of the form $x^2 = 4py$ or the fixed point $(p, 0)$ for parabolas of the form $y^2 = 4px$.

Directrix The fixed line $y = -p$ for parabolas of the form $x^2 = 4py$ or the fixed line $x = -p$ for parabolas of the form $y^2 = 4px$.

Axis of symmetry The vertical axis, $x = 0$, for parabolas of the form $x^2 = 4py$ or the horizontal axis, $y = 0$, for parabolas of the form $y^2 = 4px$.

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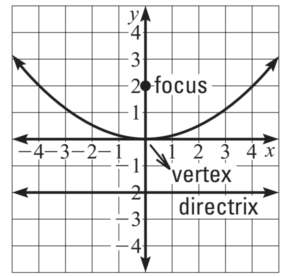
EXAMPLE **Graph $x^2 = 8y$. Identify the focus, directrix, and axis of symmetry.**

Solution:

The equation is already written in standard form, $x^2 = 4py$, where $4p = 8$, so $p = 2$. The focus is $(0, p)$, or $(0, 2)$. The directrix is $y = -p$, or $y = -2$. The axis of symmetry is the vertical axis, $x = 0$.



Make a table of values and plot the points.

x	−2	−1	0	1	2
y	0.5	0.125	0	0.125	0.5

PRACTICE **Graph the equation. Identify the focus, directrix, and axis of symmetry.**

4. $\frac{1}{3}x^2 = y$ **5.** $y^2 = -4x$ **6.** $x = \frac{1}{12}y^2$

3. Graph an Equation of a Circle

Vocabulary

Circle The set of all points in a plane equidistant from a fixed point.

Center The fixed point in a circle that all points are equidistant from.

Radius The distance between the center and any point on the circle.

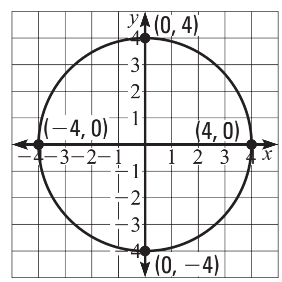
EXAMPLE **Graph $x^2 = 16 - y^2$. Identify the radius of the circle.**

Solution:

The standard form of a circle with its center at the origin is $x^2 + y^2 = r^2$.

Write the equation in standard form as $x^2 + y^2 = 16$. The center of the circle is at the origin. Since $r^2 = 16$, the radius $r = \sqrt{16} = 4$.

Plot several convenient points that are 4 units from the origin, such as (0, 4), (4, 0), (0, −4), and (−4, 0). Draw the circle that passes through these points.



PRACTICE **Graph the equation. Identify the radius of the circle.**

7. $x^2 + y^2 = 4$ **8.** $y^2 = -x^2 + 9$ **9.** $y^2 - 32 = -x^2$

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BENCHMARK 4
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4. Write a Circular Model

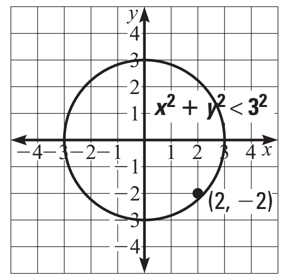
EXAMPLE **A restaurant will deliver meals within a 3 mile radius of its location. You live 2 miles east and 2 miles south of the restaurant. Are you within the restaurant's delivery area?**

Solution:

Drawing a diagram of the delivery area is a good way to visually check your answer.

Write an inequality for the delivery area of the restaurant. This region is all points that satisfy the inequality below.

$x^2 + y^2 < 3^2$



The location 2 miles east and 2 miles south is shown by the point (2, −2). Substitute this point into the equality above.

$x^2 + y^2 < 3^2$	**Inequality from above**
$(2)^2 + (-2)^2 \overset{?}{<} 3^2$	**Substitute for *x* and *y*.**
$8 < 9$	**The inequality is true.**

So, you are within the delivery range.

PRACTICE **Determine if the locations are within the delivery range.**

10. 2.5 miles west and 1.5 miles north

11. 2.75 miles north and 1.25 miles east

5. Graph a Translated Conic Section

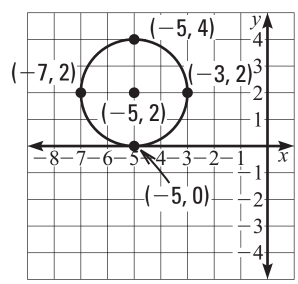
EXAMPLE **Graph $(x + 5)^2 + (y - 2)^2 = 4$.**

Solution:

The equation $(x - h)^2 + (y - k)^2 = r^2$ describes a circle translated *h* units along the *x*-axis and *k* units along the *y*-axis.

The vertex of such a circle is (h, k).

STEP 1: **Compare** the given equation to the standard form of an equation of a circle. The graph is a circle with its center at $(h, k) = (-5, 2)$ and radius $r = \sqrt{4} = 2$.



STEP 2: **Plot** the center. Then plot several other points that are each 2 units from the center of the circle:

$(-5 + 2, 2) = (-3, 2)$ $(-5 - 2, 2) = (-7, 2)$

$(-5, 2 + 2) = (-5, 4)$ $(-5, 2 - 2) = (-5, 0)$

STEP 3: **Draw** a circle through the points.

PRACTICE **Graph the equations. Identify the radius.**

12. $(x - 5)^2 + (y + 4)^2 = 9$

13. $(x + 2)^2 + (y + 3)^2 = 16$

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BENCHMARK 4

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Quiz

Find the length and midpoint of the segment connecting the points.

1. (2, 3) and (4, 4) **2.** (−5, 4) and (3, −2) **3.** (−1, −7) and (5, −6)

Graph the equation. Identify the focus, directrix, and axis of symmetry.

4. $y^2 = x$ **5.** $\frac{1}{2}x^2 + y = 0$ **6.** $y = -3x^2$

Graph the equation. Identify the radius of the circle.

7. $x^2 + y^2 = 100$ **8.** $x^2 = 64 - y^2$

9. $-x^2 = y^2 - 25$ **10.** $(x + 2)^2 + (y + 7)^2 = 1$

11. $(x - 3)^2 + (y + 3)^2 = 36$ **12.** $(x + 1)^2 + y^2 = 49$

A auto service station will tow a car if it breaks down with an 8 mile radius of the station. Determine if the cars are located within the towing region of the service station.

13. 6 miles east and 5 miles south of the station.

14. 7 miles west and 3 miles north of the station.

15. 4.5 miles east and 6.5 miles north of the station.

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BENCHMARK 4
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F. Ellipses and Hyperbolas

You learned how to graph equations of conic sections for parabolas and circles. Here you will graph equations for other conic sections, namely ellipses and hyperbolas.

1. Graph an Equation of an Ellipse

Vocabulary

The equation of an ellipse with its center at the origin and a horizontal major axis is: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. The equation of an ellipse with a vertical major axis is: $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$.

Ellipse The set of all fixed points in a plane P where the sum of the distances between P and two fixed points is constant.

Foci The two fixed points in an ellipse.

Vertices Points on an ellipse located on the line connecting the foci.

Major axis The axis that joins the vertices.

Co-vertices Points on an ellipse located on the line perpendicular to the major axis through the center of the ellipse.

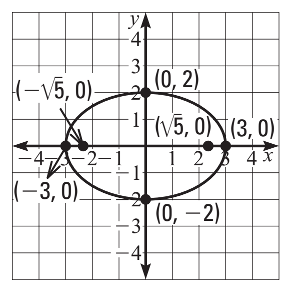
Minor axis The axis that joins the co-vertices.

EXAMPLE **Graph the equation $\frac{x^2}{9} + \frac{y^2}{4} = 1$. Identify the vertices, co-vertices, and foci.**

Solution:

The lengths of the major and minor axes are $2a$ and $2b$, respectively, where $a > b > 0$.

STEP 1: Identify the vertices, co-vertices, and foci.
$a^2 = 9$ and $b^2 = 4$, so $a = 3$ and $b = 2$
Since the denominator of the x^2-term is greater than that of the y^2-term, the major axis is horizontal.



The vertices of the ellipse are at $(\pm a, 0) = (\pm 3, 0)$.
The co-vertices are at $(0, \pm b) = (0, \pm 2)$.

The foci, c, can be found using $c^2 = a^2 - b^2$.
$c^2 = a^2 - b^2 = 3^2 - 2^2 = 9 - 4 = 5$, so $c = \sqrt{5}$.
The foci are at $(\pm\sqrt{5}, 0)$ or about $(\pm 2.2, 0)$.

STEP 2: Draw the ellipse centered at the origin that passes through the vertices and co-vertices.

PRACTICE **Graph the equation. Identify the vertices, co-vertices, and foci of the ellipse.**

1. $\frac{x^2}{36} + \frac{y^2}{64} = 1$

2. $16x^2 + 4y^2 = 64$

EXAMPLE

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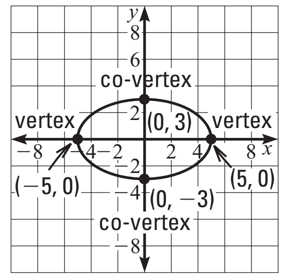
2. Write an Equation of an Ellipse

EXAMPLE **Write an equation of the ellipse with a vertex at (5, 0), a co-vertex at (0, −3), and center at (0, 0).**



Solution:

Sketching the ellipse with the given vertex, co-vertex, and center is a good way to check your final equation.

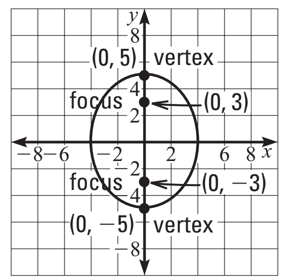
Since ellipses are symmetrical, the other vertex is at (−5, 0) and the other co-vertex is at (0, 3). The vertex is on the x-axis, so the major axis is horizontal with $a = 5$. The co-vertex is on the y-axis, so the minor axis is vertical with $b = 3$.

An equation, therefore, is $\frac{x^2}{5^2} + \frac{y^2}{3^2} = 1$ or $\frac{x^2}{25} + \frac{y^2}{9} = 1$.

PRACTICE **Write an equation of the ellipse with the given characteristics and center at (0, 0).**

3. Vertex: (0, 6); co-vertex (4, 0) **4.** Vertex (−9, 0); co-vertex (0, 1)

EXAMPLE **Write an equation of the ellipse with a vertex at (0, −5) and a focus at (0, 3).**



Solution:

Be careful not to confuse the formula for finding the foci, $c^2 = a^2 - b^2$, with the formula for the Pythagorean Theorem, $c^2 = a^2 + b^2$.

Sketch the ellipse. The given vertex and focus lie on y-axis. So, the major axis is vertical, with $a = 5$ and $c = 3$. Use the equation $c^2 = a^2 - b^2$ to find b.

$3^2 = 5^2 - b^2$

$b^2 = 5^2 - 3^2 = 25 - 9 = 16$

$b = \sqrt{16} = 4$

An equation, therefore, is $\frac{x^2}{4^2} + \frac{y^2}{5^2} = 1$ or $\frac{x^2}{16} + \frac{y^2}{25} = 1$.

PRACTICE **Write an equation of the ellipse with the given characteristics and center at (0, 0).**

5. Vertex: (0, 7); foci (0, 5) **6.** Vertex (−6, 0); foci (2, 0)

3. Graph an Equation of a Hyperbola

Vocabulary

A hyperbola has two asymptotes that contain the diagonals of a rectangle centered at the hyperbola's center.

Hyperbola The set of all fixed points in a plane P where the *difference* of the distances between P and two fixed points is constant.

Foci The two fixed points of a hyperbola.

Vertices The points on the hyperbola where the line through the foci intersects the hyperbola.

Transverse axis The axis that joins the vertices.

Center The midpoint of the transverse axis.

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EXAMPLE **Graph $16y^2 - 4x^2 = 64$. Identify the vertices, foci, and asymptotes of the hyperbola.**

Solution:

STEP 1: Rewrite the equation in standard form.

$16y^2 - 4x^2 = 64$ **Write original equation.**

$\frac{16y^2}{64} - \frac{4x^2}{64} = 1$ **Divide each side by 64.**

$\frac{y^2}{4} - \frac{x^2}{16} = 1$ **Simplify.**

The hyperbola with form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ has a horizontal transverse axis and asymptotes at $y = \pm\left(\frac{b}{a}\right)x$.

The hyperbola with form $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ has a vertical transverse axis and asymptotes at $y = \pm\left(\frac{a}{b}\right)x$.

STEP 2: Identify the vertices, foci, and asymptotes. $a^2 = 4$ and $b^2 = 16$, so $a = 2$ and $b = 4$ Since the y^2-term is positive, the transverse axis is vertical and the vertices are at $(0, \pm 2)$.

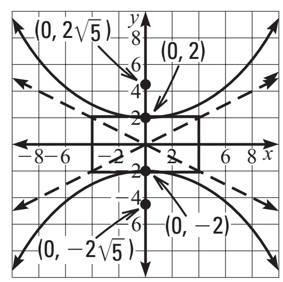
The foci, c, can be found using $c^2 = a^2 + b^2$.

$c^2 = a^2 + b^2 = 2^2 + 4^2 = 4 + 16 = 20$, so $c = \sqrt{20} = 2\sqrt{5}$.

The foci are at $(0, \pm 2\sqrt{5})$, or about $(0, \pm 4.5)$.

The asymptotes are $y = \pm\frac{a}{b}x$, or $y = \pm\frac{2}{4}x = \pm\frac{1}{2}x$

STEP 3: Draw the hyperbola. First draw a rectangle centered at the origin that is $2a = 2(2)$ or 4 units high and $2b = 2(4)$ or 8 units wide. The asymptotes pass through the opposite corners of the rectangle. Draw the hyperbola passing through the vertices and approaching the asymptotes.



PRACTICE **Graph the equation. Identify the vertices, foci, and asymptotes of the hyperbola.**

7. $\frac{x^2}{4} - \frac{y^2}{49} = 1$

8. $4y^2 - 9x^2 = 36$

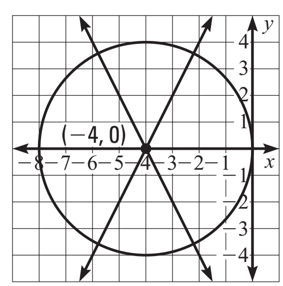
4. Identify Symmetries in Conic Sections

EXAMPLE **Identify the line(s) of symmetry for the conic sections.**

a. $(x + 4)^2 + y^2 = 16$ **b.** $y^2 = 4x$ **c.** $\frac{x^2}{9} + \frac{y^2}{4} = 1$

Solution:

a. Any line through the center, $(-4, 0)$, of the circle is a line of symmetry



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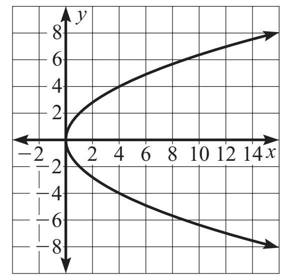
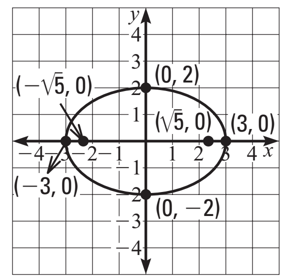
b. Parabolas of the form $y^2 = 4px$ have a horizontal axis of symmetry, or the line $y = 0$.



c. Any horizontal or vertical line through the center of the ellipse is a line of symmetry. The center is at (0, 0), so the symmetry lines are at $x = 0$ and $y = 0$.



PRACTICE **Identify the line(s) of symmetry for the conic sections.**

9. $(x - 3)^2 + (y + 6)^2 = 9$

10. $\frac{x^2}{16} + \frac{y^2}{25} = 1$

11. $\frac{x^2}{49} - \frac{y^2}{36} = 1$

5. Classify Conic Sections

Vocabulary

General second-degree equation The equation $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ used to describe any conic.

Discriminant The expression $B^2 - 4AC$ used to identify the type of conic.

EXAMPLE **Classify the conic.**

A circle has a discriminant > 0, $B = 0$, and $A = C$.

An ellipse has a discriminant < 0 and either $B \neq 0$ or $A \neq C$.

A parabola has a discriminant = 0.

A hyperbola has a discriminant > 0.

a. $x^2 - y^2 + 2x - 6y - 43 = 0$

b. $y^2 - 5x - 8y + 16 = 0$

Solution:

a. $A = 1, B = 0$, and $C = -1$, so the value of the discriminant is:
$B^2 - 4AC = 0^2 - 4(1)(-1) = 0 + 4 = 4$
Because $B^2 - 4AC > 0$, the conic is a hyperbola.

b. $A = 0, B = 0$, and $C = 1$, so the value of the discriminant is:
$B^2 - 4AC = 0^2 - 4(0)(1) = 0$
Because $B^2 - 4AC = 0$, the conic is a parabola.

PRACTICE **Classify the conic.**

12. $4x^2 + y^2 + 20x - 2y + 25 = 0$

13. $x^2 + y^2 + 4x - 6y - 23 = 0$

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Quiz

Graph the equation. Identify the vertices, co-vertices, and foci of the ellipse.

1. $\frac{x^2}{25} + \frac{y^2}{49} = 1$ **2.** $\frac{x^2}{82} + \frac{y^2}{9} = 1$ **3.** $x^2 + 4y^2 = 4$

Write an equation of the ellipse with the given characteristics and center at (0, 0).

4. Vertex: $(-6, 0)$; co-vertex $(0, -2)$

5. Vertex: $(0, -8)$; co-vertex $(5, 0)$

6. Vertex: $(5, 0)$; foci $(-4, 0)$

7. Vertex $(0, -9)$; foci $(0, 6)$

Graph the equation. Identify the vertices, foci, and asymptotes of the hyperbola.

8. $\frac{y^2}{9} - \frac{x^2}{25} = 1$ **9.** $16x^2 - 9y^2 = 144.$

Identify the line(s) of symmetry for the conic sections.

10. $(x + 5)^2 + (y + 1)^2 = 4$ **11.** $x^2 = -2y$

12. $64x^2 + 4y^2 = 256$

Classify the conic.

13. $x^2 + y^2 + 4x - 10y + 21 = 0$ **14.** $x^2 + 4y^2 + 24y + 20 = 0$

15. $2x^2 - 3y^2 - 4x - 18y - 49 = 0$ **16.** $x^2 + 14x - 2y + 51 = 0$