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Study Guide

Angles and Degree Measure

Decimal degree measures can be expressed in **degrees(°)**, **minutes(′)**, and **seconds(″)**.

Example 1 **a. Change 12.520° to degrees, minutes, and seconds.**

$12.520° = 12° + (0.520 \cdot 60)'$	*Multiply the decimal portion of*
$= 12° + 31.2'$	*the degrees by 60 to find minutes.*
$= 12° + 31' + (0.2 \cdot 60)''$	*Multiply the decimal portion of*
$= 12° + 31' + 12''$	*the minutes by 60 to find seconds.*

12.520° can be written as 12° 31′ 12″.

b. Write 24° 15′ 33″ as a decimal rounded to the nearest thousandth.

$$24°\ 15'\ 33'' = 24° + 15'\left(\frac{1°}{60'}\right) + 33''\left(\frac{1°}{3600''}\right)$$
$$= 24.259°$$

24° 15′ 33″ can be written as 24.259°.

An angle may be generated by the rotation of one ray multiple times about the origin.

Example 2 **Give the angle measure represented by each rotation.**

a. 2.3 rotations clockwise

$2.3 \times -360 = -828$ *Clockwise rotations have negative measures.*

The angle measure of 2.3 clockwise rotations is −828°.

b. 4.2 rotations counterclockwise

$4.2 \times 360 = 1512$ *Counterclockwise rotations have positive measures.*

The angle measure of 4.2 counterclockwise rotations is 1512°.

If α is a nonquadrantal angle in standard position, its **reference angle** is defined as the acute angle formed by the terminal side of the given angle and the x-axis.

Reference Angle Rule	For any angle α, $0° < \alpha < 360°$, its reference angle α' is defined by a. α, when the terminal side is in Quadrant I, b. $180° - \alpha$, when the terminal side is in Quadrant II, c. $\alpha - 180°$, when the terminal side is in Quadrant III, and d. $360° - \alpha$, when the terminal side is in Quadrant IV.

Example 3 **Find the measure of the reference angle for 220°.**

Because 220° is between 180° and 270°, the terminal side of the angle is in Quadrant III.

$220° - 180° = 40°$

The reference angle is 40°.

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5-1

Practice

Angles and Degree Measure

Change each measure to degrees, minutes, and seconds.

1. 28.955° **2.** −57.327°

Write each measure as a decimal degree to the nearest thousandth.

3. 32° 28′ 10″ **4.** −73° 14′ 35″

Give the angle measure represented by each rotation.

5. 1.5 rotations clockwise **6.** 2.6 rotations counterclockwise

Identify all angles that are coterminal with each angle. Then find one positive angle and one negative angle that are coterminal with each angle.

7. 43° **8.** −30°

If each angle is in standard position, determine a coterminal angle that is between 0° and 360°, and state the quadrant in which the terminal side lies.

9. 472° **10.** −995°

Find the measure of the reference angle for each angle.

11. 227° **12.** 640°

13. ***Navigation*** For an upcoming trip, Jackie plans to sail from Santa Barbara Island, located at 33° 28′ 32″ N, 119° 2′ 7″ W, to Santa Catalina Island, located at 33.386° N, 118.430° W. Write the latitude and longitude for Santa Barbara Island as decimals to the nearest thousandth and the latitude and longitude for Santa Catalina Island as degrees, minutes, and seconds.

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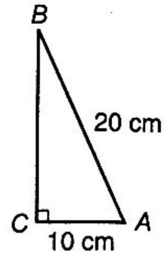
5-2

Study Guide

Trigonometric Ratios in Right Triangles

The ratios of the sides of right triangles can be used to define the **trigonometric** ratios known as the **sine, cosine,** and **tangent.**

Example 1 **Find the values of the sine, cosine, and tangent for $\angle A$.**



First find the length of $\overline{BC}$.

$(AC)^2 + (BC)^2 = (AB)^2$ *Pythagorean Theorem*

$10^2 + (BC)^2 = 20^2$ *Substitute 10 for AC and 20 for AB.*

$(BC)^2 = 300$

$BC = \sqrt{300}$ or $10\sqrt{3}$ *Take the square root of each side. Disregard the negative root.*

Then write each trigonometric ratio.

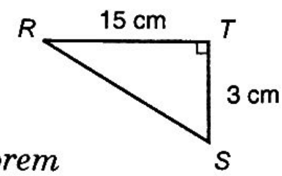
$\sin A = \frac{\textit{side opposite}}{\textit{hypotenuse}}$ $\cos A = \frac{\textit{side adjacent}}{\textit{hypotenuse}}$ $\tan A = \frac{\textit{side opposite}}{\textit{side adjacent}}$

$\sin A = \frac{10\sqrt{3}}{20}$ or $\frac{\sqrt{3}}{2}$ $\cos A = \frac{10}{20}$ or $\frac{1}{2}$ $\tan A = \frac{10\sqrt{3}}{10}$ or $\sqrt{3}$

Trigonometric ratios are often simplified but never written as mixed numbers.

Three other trigonometric ratios, called **cosecant, secant,** and **cotangent,** are reciprocals of sine, cosine, and tangent, respectively.

Example 2 **Find the values of the six trigonometric ratios for $\angle R$.**



First determine the length of the hypotenuse.

$(RT)^2 + (ST)^2 = (RS)^2$ *Pythagorean Theorem*

$15^2 + 3^2 = (RS)^2$ *RT = 15, ST = 3*

$(RS)^2 = 234$

$RS = \sqrt{234}$ or $3\sqrt{26}$ *Disregard the negative root.*

$\sin R = \frac{\textit{side opposite}}{\textit{hypotenuse}}$ $\cos R = \frac{\textit{side adjacent}}{\textit{hypotenuse}}$ $\tan R = \frac{\textit{side opposite}}{\textit{side adjacent}}$

$\sin R = \frac{3}{3\sqrt{26}}$ or $\frac{\sqrt{26}}{26}$ $\cos R = \frac{15}{3\sqrt{26}}$ or $\frac{5\sqrt{26}}{26}$ $\tan R = \frac{3}{15}$ or $\frac{1}{5}$

$\csc R = \frac{\textit{hypotenuse}}{\textit{side opposite}}$ $\sec R = \frac{\textit{hypotenuse}}{\textit{side adjacent}}$ $\cot R = \frac{\textit{side adjacent}}{\textit{side opposite}}$

$\csc R = \frac{3\sqrt{26}}{3}$ or $\sqrt{26}$ $\sec R = \frac{3\sqrt{26}}{15}$ or $\frac{\sqrt{26}}{5}$ $\cot R = \frac{15}{3}$ or 5

5-2

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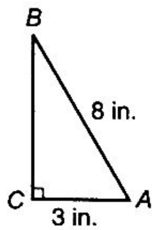
Practice

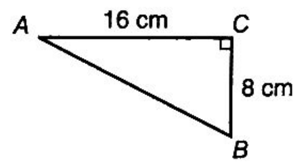
Trigonometric Ratios in Right Triangles

Find the values of the sine, cosine, and tangent for each ∠B.

1.



2.

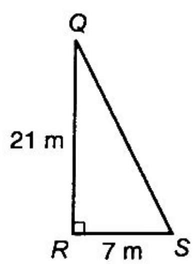


3. If $\tan\theta = 5$, find $\cot\theta$.

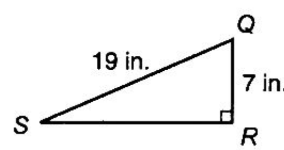
4. If $\sin\theta = \frac{3}{8}$, find $\csc\theta$.

Find the values of the six trigonometric ratios for each ∠S.

5.



6.



7. ***Physics*** Suppose you are traveling in a car when a beam of light passes from the air to the windshield. The measure of the angle of incidence is 55°, and the measure of the angle of refraction is 35° 15′. Use Snell's Law, $\frac{\sin\theta_i}{\sin\theta_r} = n$, to find the index of refraction n of the windshield to the nearest thousandth.

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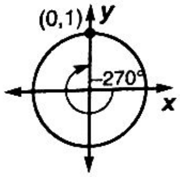
Study Guide

Trigonometric Functions on the Unit Circle

Example 1 **Use the unit circle to find cot (−270°).**



The terminal side of a −270° angle in standard position is the positive y-axis, which intersects the unit circle at (0, 1).

By definition, $\cot(-270°) = \frac{x}{y}$ or $\frac{0}{1}$.

Therefore, $\cot(-270°) = 0$.

Trigonometric Functions of an Angle in Standard Position			
	$\sin\theta = \frac{y}{r}$	$\cos\theta = \frac{x}{r}$	$\tan\theta = \frac{y}{x}$
	$\csc\theta = \frac{r}{y}$	$\sec\theta = \frac{r}{x}$	$\cot\theta = \frac{x}{y}$

Example 2 **Find the values of the six trigonometric functions for angle θ in standard position if a point with coordinates (−9, 12) lies on its terminal side.**

We know that $x = -9$ and $y = 12$. We need to find r.

$r = \sqrt{x^2 + y^2}$ *Pythagorean Theorem*

$r = \sqrt{(-9)^2 + 12^2}$ *Substitute −9 for x and 12 for y.*

$r = \sqrt{225}$ or 15 *Disregard the negative root.*

$\sin\theta = \frac{12}{15}$ or $\frac{4}{5}$ $\cos\theta = \frac{-9}{15}$ or $-\frac{3}{5}$ $\tan\theta = \frac{12}{-9}$ or $-\frac{4}{3}$

$\csc\theta = \frac{15}{12}$ or $\frac{5}{4}$ $\sec\theta = \frac{15}{-9}$ or $-\frac{5}{3}$ $\cot\theta = \frac{-9}{12}$ or $-\frac{3}{4}$

Example 3 **Suppose θ is an angle in standard position whose terminal side lies in Quadrant I. If $\cos\theta = \frac{3}{5}$, find the values of the remaining five trigonometric functions of θ.**

$r^2 = x^2 + y^2$ *Pythagorean Theorem*

$5^2 = 3^2 + y^2$ *Substitute 5 for r and 3 for x.*

$16 = y^2$

$\pm 4 = y$ *Take the square root of each side.*

Since the terminal side of θ lies in Quadrant I, y must be positive.

$\sin\theta = \frac{4}{5}$ $\tan\theta = \frac{4}{3}$

$\csc\theta = \frac{5}{4}$ $\sec\theta = \frac{5}{3}$ $\cot\theta = \frac{3}{4}$

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5-3

Practice

Trigonometric Functions on the Unit Circle

Use the unit circle to find each value.

1. csc 90°

2. tan 270°

3. sin (−90°)

Use the unit circle to find the values of the six trigonometric functions for each angle.

4. 45°

5. 120°

Find the values of the six trigonometric functions for angle θ in standard position if a point with the given coordinates lies on its terminal side.

6. (−1, 5)

7. (7, 0)

8. (−3, −4)

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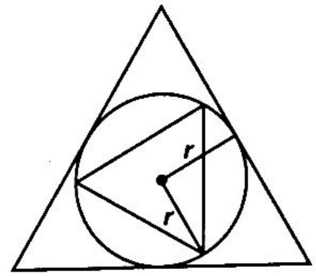
5-3

Enrichment

Areas of Polygons and Circles

A regular polygon has sides of equal length and angles of equal measure. A regular polygon can be inscribed in or circumscribed about a circle. For n-sided regular polygons, the following area formulas can be used.



Area of circle $A_C = \pi r^2$

Area of inscribed polygon $A_I = \frac{nr^2}{2} \times \sin \frac{360°}{n}$

Area of circumscribed polygon $A_C = nr^2 \times \tan \frac{180°}{n}$

Use a calculator to complete the chart below for a unit circle (a circle of radius 1).

	Number of Sides	Area of Inscribed Polygon	Area of Circle less Area of Polygon	Area of Circumscribed Polygon	Area of Polygon less Area of Circle
	3	1.2990381	1.8425545	5.1961524	2.0545598
1.	4				
2.	8				
3.	12				
4.	20				
5.	24				
6.	28				
7.	32				
8.	1000				

9. What number do the areas of the circumscribed and inscribed polygons seem to be approaching as the number of sides of the polygon increases?

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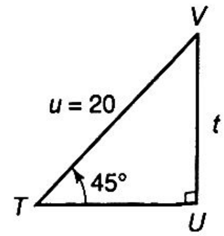
Study Guide

Applying Trigonometric Functions

Trigonometric functions can be used to solve problems involving right triangles.

Example 1 **If $T = 45°$ and $u = 20$, find t to the nearest tenth.**



From the figure, we know the measures of an angle and the hypotenuse. We want to know the measure of the side opposite the given angle. The sine function relates the side opposite the angle and the hypotenuse.

$\sin T = \frac{t}{u}$ — *$\sin = \frac{\text{side opposite}}{\text{hypotenuse}}$*

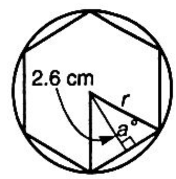
$\sin 45° = \frac{t}{20}$ — *Substitute 45° for T and 20 for u.*

$20 \sin 45° = t$ — *Multiply each side by 20.*

$14.14213562 \approx t$ — *Use a calculator.*

Therefore, t is about 14.1.

Example 2 ***Geometry*** **The apothem of a regular polygon is the measure of a line segment from the center of the polygon to the midpoint of one of its sides. The apothem of a regular hexagon is 2.6 centimeters. Find the radius of the circle circumscribed about the hexagon to the nearest tenth.**



First draw a diagram. Let a be the angle measure formed by a radius and its adjacent apothem. The measure of a is $360° \div 12$ or 30°. Now we know the measures of an angle and the side adjacent to the angle.

$\cos 30° = \frac{2.6}{r}$ — *$\cos = \frac{\text{side adjacent}}{\text{hypotenuse}}$*

$r \cos 30° = 2.6$ — *Multiply each side by r.*

$r = \frac{2.6}{\cos 30°}$ — *Divide each side by cos 30°.*

$r \approx 3.0022214$ — *Use a calculator.*

Therefore, the radius is about 3.0 centimeters.

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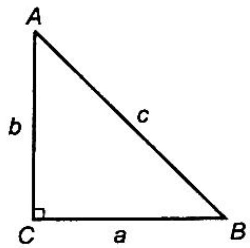
Practice

Applying Trigonometric Functions

Solve each problem. Round to the nearest tenth.



1. If $A = 55° \ 55'$ and $c = 16$, find a.

2. If $a = 9$ and $B = 49°$, find b.

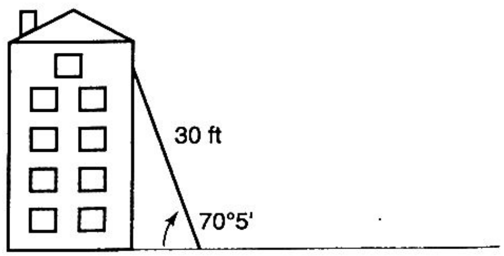
3. If $B = 56° \ 48'$ and $c = 63.1$, find b.

4. If $B = 64°$ and $b = 19.2$, find a.

5. If $b = 14$ and $A = 16°$, find c.

6. ***Construction*** A 30-foot ladder leaning against the side of a house makes a 70° 5′ angle with the ground.
   a. How far up the side of the house does the ladder reach?

   b. What is the horizontal distance between the bottom of the ladder and the house?

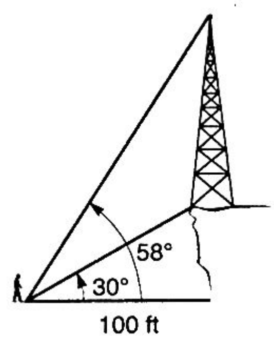


7. ***Geometry*** A circle is circumscribed about a regular hexagon with an apothem of 4.8 centimeters.
   a. Find the radius of the circumscribed circle.

   b. What is the length of a side of the hexagon?

   c. What is the perimeter of the hexagon?

8. ***Observation*** A person standing 100 feet from the bottom of a cliff notices a tower on top of the cliff. The angle of elevation to the top of the cliff is 30°. The angle of elevation to the top of the tower is 58°. How tall is the tower?



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5-5

Study Guide

Solving Right Triangles

When we know a trigonometric value of an angle but not the value of the angle, we need to use the inverse of the trigonometric function.

Trigonometric Function	Inverse Trigonometric Relation
$y = \sin x$	$x = \sin^{-1} y$ or $x = \arcsin y$
$y = \cos x$	$x = \cos^{-1} y$ or $x = \arccos y$
$y = \tan x$	$x = \tan^{-1} y$ or $x = \arctan y$

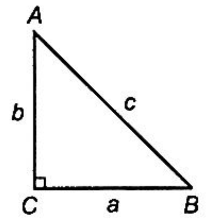
Example 1 **Solve $\tan x = \sqrt{3}$.**

If $\tan x = \sqrt{3}$, then x is an angle whose tangent is $\sqrt{3}$.

$x = \arctan \sqrt{3}$

From a table of values, you can determine that x equals 60°, 240°, or any angle coterminal with these angles.

Example 2 **If $c = 22$ and $b = 12$, find B.**



In this problem, we know the side opposite the angle and the hypotenuse. The sine function relates the side opposite the angle and the hypotenuse.

$\sin B = \frac{b}{c}$ — $sin = \frac{\text{side opposite}}{\text{hypotenuse}}$

$\sin B = \frac{12}{22}$ — *Substitute 12 for b and 22 for c.*

$B = \sin^{-1}\left(\frac{12}{22}\right)$ — *Definition of inverse*

$B \approx 33.05573115$ or about 33.1°.

Example 3 **Solve the triangle where $b = 20$ and $c = 35$, given the triangle above.**

$$a^2 + b^2 = c^2$$
$$a^2 + 20^2 = 35^2$$
$$a = \sqrt{825}$$
$$a \approx 28.72281323$$

$$\cos A = \frac{b}{c}$$
$$\cos A = \frac{20}{35}$$
$$A = \cos^{-1}\left(\frac{20}{35}\right)$$
$$A \approx 55.15009542$$

$$55.15009542 + B \approx 90$$
$$B \approx 34.84990458$$

Therefore, $a \approx 28.7$, $A \approx 55.2°$, and $B \approx 34.8°$.

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5-5

Practice

Solving Right Triangles

Solve each equation if $0° \le x \le 360°$.

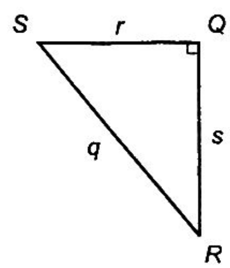
1. $\cos x = \frac{\sqrt{2}}{2}$ **2.** $\tan x = 1$ **3.** $\sin x = \frac{1}{2}$

Evaluate each expression. Assume that all angles are in Quadrant I.

4. $\tan\left(\tan^{-1}\frac{\sqrt{3}}{3}\right)$ **5.** $\tan\left(\cos^{-1}\frac{2}{3}\right)$ **6.** $\cos\left(\arcsin\frac{5}{13}\right)$

Solve each problem. Round to the nearest tenth.

7. If $q = 10$ and $s = 3$, find S.

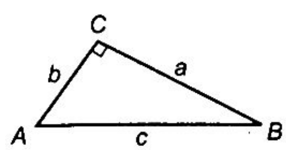


8. If $r = 12$ and $s = 4$, find R.

9. If $q = 20$ and $r = 15$, find S.

Solve each triangle described, given the triangle at the right. Round to the nearest tenth, if necessary.

10. $a = 9, B = 49°$



11. $A = 16°, c = 14$

12. $a = 2, b = 7$

13. ***Recreation*** The swimming pool at Perris Hill Plunge is 50 feet long and 25 feet wide. The bottom of the pool is slanted so that the water depth is 3 feet at the shallow end and 15 feet at the deep end. What is the angle of elevation at the bottom of the pool?

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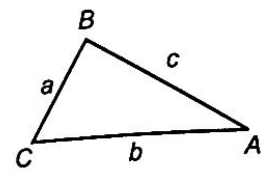
5-6

Study Guide

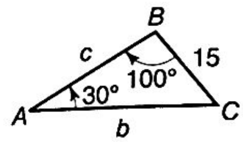
The Law of Sines



Given the measures of two angles and one side of a triangle, we can use the **Law of Sines** to find one unique solution for the triangle.

Law of Sines	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Example 1 **Solve $\triangle ABC$ if $A = 30°$, $B = 100°$, and $a = 15$.**



First find the measure of $\angle C$.
$C = 180° - (30° + 100°)$ or $50°$

Use the Law of Sines to find b and c.

$$\frac{a}{\sin A} = \frac{b}{\sin B} \qquad \frac{c}{\sin C} = \frac{a}{\sin A}$$

$$\frac{15}{\sin 30°} = \frac{b}{\sin 100°} \qquad \frac{c}{\sin 50°} = \frac{15}{\sin 30°}$$

$$\frac{15 \sin 100°}{\sin 30°} = b \qquad c = \frac{15 \sin 50°}{\sin 30°}$$

$$29.54423259 \approx b \qquad c \approx 22.98133329$$

Therefore, $C = 50°$, $b \approx 29.5$, and $c \approx 23.0$.

The area of any triangle can be expressed in terms of two sides of a triangle and the measure of the included angle.

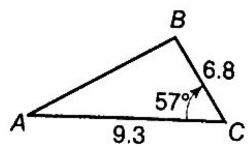
Area (K) of a Triangle	$K = \frac{1}{2}bc \sin A$	$K = \frac{1}{2}ac \sin B$	$K = \frac{1}{2}ab \sin C$

Example 2 **Find the area of $\triangle ABC$ if $a = 6.8$, $b = 9.3$, and $C = 57°$.**

$K = \frac{1}{2}ab \sin C$

$K = \frac{1}{2}(6.8)(9.3) \sin 57°$

$K \approx 26.51876336$



The area of $\triangle ABC$ is about 26.5 square units.

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5-6

Practice

The Law of Sines

Solve each triangle. Round to the nearest tenth.

1. $A = 38°, B = 63°, c = 15$

2. $A = 33°, B = 29°, b = 41$

3. $A = 150°, C = 20°, a = 200$

4. $A = 30°, B = 45°, a = 10$

Find the area of each triangle. Round to the nearest tenth.

5. $c = 4, A = 37°, B = 69°$

6. $C = 85°, a = 2, B = 19°$

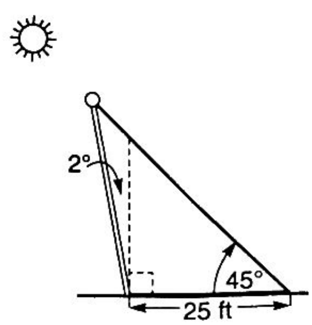
7. $A = 50°, b = 12, c = 14$

8. $b = 14, C = 110°, B = 25°$

9. $b = 15, c = 20, A = 115°$

10. $a = 68, c = 110, B = 42.5°$

11. ***Street Lighting*** A lamppost tilts toward the sun at a 2° angle from the vertical and casts a 25-foot shadow. The angle from the tip of the shadow to the top of the lamppost is 45°. Find the length of the lamppost.



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5-7

Study Guide

The Ambiguous Case for the Law of Sines

If we know the measures of two sides and a nonincluded angle of a triangle, three situations are possible: no triangle exists, exactly one triangle exists, or two triangles exist. A triangle with two solutions is called the **ambiguous case**.

Case 1: $A < 90°$ for a, b, and A	
$a < b \sin A$	no solution
$a = b \sin A$	one solution
$a \geq b$	one solution
$b \sin A < a < b$	two solutions
Case 2: $A \geq 90°$	
$a \leq b$	no solution
$a > b$	one solution

Example **Find all solutions for the triangle if $a = 20$, $b = 30$, and $A = 40°$. If no solutions exist, write *none*.**

Since $40° < 90°$, consider Case 1.
$b \sin A = 30 \sin 40°$
$b \sin A \approx 19.28362829$
Since $19.3 < 20 < 30$, there are two solutions for the triangle.
Use the Law of Sines to find B.

$$\frac{20}{\sin 40°} = \frac{30}{\sin B} \qquad \frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\sin B = \frac{30 \sin 40°}{20}$$

$$B = \sin^{-1}\left(\frac{30 \sin 40°}{20}\right)$$

$$B \approx 74.61856831$$

So, $B \approx 74.6°$. Since we know there are two solutions, there must be another possible measurement for B. In the second case, B must be less than 180° and have the same sine value. Since we know that if $\alpha < 90$, $\sin \alpha = \sin (180 - \alpha)$, $180° - 74.6°$ or $105.4°$ is another possible measure for B. Now solve the triangle for each possible measure of B.

Solution I

$C \approx 180° - (40° + 74.6°)$ or $65.4°$

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{20}{\sin 40°} \approx \frac{c}{\sin 65.4°}$$

$$c \approx \frac{20 \sin 65.4°}{\sin 40°}$$

$$c \approx 28.29040558$$

One solution is $B \approx 74.6°$, $C \approx 65.4°$, and $c \approx 28.3$.

Solution II

$C \approx 180° - (40° + 105.4°)$ or $34.6°$

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{20}{\sin 40°} \approx \frac{c}{\sin 34.6°}$$

$$c \approx \frac{20 \sin 34.6°}{\sin 40°}$$

$$c \approx 17.66816088$$

Another solution is $B \approx 105.4°$, $C \approx 34.6°$, and $c \approx 17.7$.

5-7

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Practice

The Ambiguous Case for the Law of Sines

Determine the number of possible solutions for each triangle.

1. $A = 42°, a = 22, b = 12$

2. $a = 15, b = 25, A = 85°$

3. $A = 58°, a = 4.5, b = 5$

4. $A = 110°, a = 4, c = 4$

Find all solutions for each triangle. If no solutions exist, write none. Round to the nearest tenth.

5. $b = 50, a = 33, A = 132°$

6. $a = 125, A = 25°, b = 150$

7. $a = 32, c = 20, A = 112°$

8. $a = 12, b = 15, A = 55°$

9. $A = 42°, a = 22, b = 12$

10. $b = 15, c = 13, C = 50°$

11. ***Property Maintenance*** The McDougalls plan to fence a triangular parcel of their land. One side of the property is 75 feet in length. It forms a 38° angle with another side of the property, which has not yet been measured. The remaining side of the property is 95 feet in length. Approximate to the nearest tenth the length of fence needed to enclose this parcel of the McDougalls' lot.

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5-8

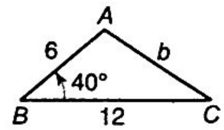
Study Guide

The Law of Cosines

When we know the measures of two sides of a triangle and the included angle, we can use the **Law of Cosines** to find the measure of the third side. Often times we will use both the Law of Cosines and the Law of Sines to solve a triangle.

Law of Cosines	$a^2 = b^2 + c^2 - 2bc \cos A$ $b^2 = a^2 + c^2 - 2ac \cos B$ $c^2 = a^2 + b^2 - 2ab \cos C$

Example 1 **Solve $\triangle ABC$ if $B = 40°$, $a = 12$, and $c = 6$.**



$b^2 = a^2 + c^2 - 2ac \cos B$ *Law of Cosines*
$b^2 = 12^2 + 6^2 - 2(12)(6) \cos 40°$
$b^2 \approx 69.68960019$
$b \approx 8.348029719$
So, $b \approx 8.3$.

$\frac{b}{\sin B} = \frac{c}{\sin C}$ *Law of Sines*

$\frac{8.3}{\sin 40°} = \frac{6}{\sin C}$

$\sin C = \frac{6 \sin 40°}{8.3}$

$C = \sin^{-1}\left(\frac{6 \sin 40°}{8.3}\right)$

$C \approx 27.68859159$

So, $C \approx 27.7°$.
$A \approx 180° - (40° + 27.7°) \approx 112.3°$

The solution of this triangle is $b \approx 8.3$, $A \approx 112.3°$, and $C \approx 27.7°$.

Example 2 **Find the area of $\triangle ABC$ if $a = 5$, $b = 8$, and $c = 10$.**

First, find the semiperimeter of $\triangle ABC$.

$s = \frac{1}{2}(a + b + c)$

$s = \frac{1}{2}(5 + 8 + 10)$

$s = 11.5$

Now, apply Hero's Formula

$k = \sqrt{s(s - a)(s - b)(s - c)}$
$k = \sqrt{11.5(11.5 - 5)(11.5 - 8)(11.5 - 10)}$
$k = \sqrt{392.4375}$
$k \approx 19.81003534$

The area of the triangle is about 19.8 square units.

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5-8

Practice

The Law of Cosines

Solve each triangle. Round to the nearest tenth.

1. $a = 20, b = 12, c = 28$

2. $a = 10, c = 8, B = 100°$

3. $c = 49, b = 40, A = 53°$

4. $a = 5, b = 7, c = 10$

Find the area of each triangle. Round to the nearest tenth.

5. $a = 5, b = 12, c = 13$

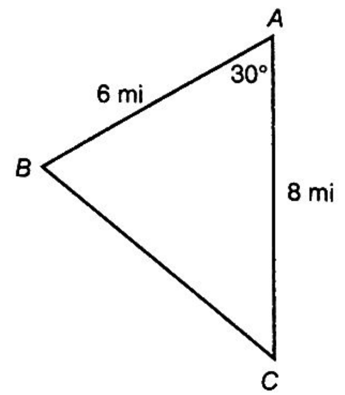
6. $a = 11, b = 13, c = 16$

7. $a = 14, b = 9, c = 8$

8. $a = 8, b = 7, c = 3$

9. The sides of a triangle measure 13.4 centimeters, 18.7 centimeters, and 26.5 centimeters. Find the measure of the angle with the least measure.

10. ***Orienteering*** During an orienteering hike, two hikers start at point A and head in a direction 30° west of south to point B. They hike 6 miles from point A to point B. From point B, they hike to point C and then from point C back to point A, which is 8 miles directly north of point C. How many miles did they hike from point B to point C?



NAME ______________________ DATE ______________ PERIOD ________

Chapter 5

Chapter 5 Test, Form 1A

Write the letter for the correct answer in the blank at the right of each problem.

1. Change 128.433° to degrees, minutes, and seconds. **1.** ________

A. 128° 25′ 58″ **B.** 128° 25′ 59″ **C.** 128° 25′ 92″ **D.** 128° 26′ 00″

2. Write 43° 18′ 35″ as a decimal to the nearest thousandth of a degree. **2.** ________

A. 43.306° **B.** 43.308° **C.** 43.309° **D.** 43.310°

3. Give the angle measure represented by 3.25 rotations clockwise. **3.** ________

A. −1170° **B.** −90° **C.** 90° **D.** 1170°

4. Identify all coterminal angles between −360° and 360° for the angle −420°. **4.** ________

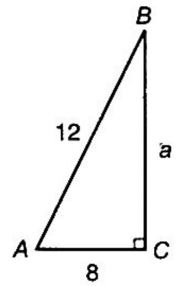
A. −60° and 300° **B.** −30° and 330°
C. 30° and −330° **D.** 60° and −300°

5. Find the measure of the reference angle for 1046°. **5.** ________

A. −56° **B.** 56° **C.** 34° **D.** −34°

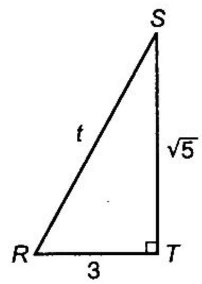
6. Find the value of the tangent for $\angle A$. **6.** ________

A. $\frac{2\sqrt{5}}{2}$ **B.** $\frac{\sqrt{5}}{2}$
C. $\frac{2}{3}$ **D.** $\frac{\sqrt{5}}{3}$



7. Find the value of the secant for $\angle R$. **7.** ________

A. $\frac{\sqrt{70}}{5}$ **B.** $\frac{3\sqrt{14}}{14}$
C. $\frac{\sqrt{5}}{3}$ **D.** $\frac{\sqrt{14}}{3}$



8. Which of the following is equal to csc θ ? **8.** ________

A. $\frac{1}{\sin\theta}$ **B.** $\frac{1}{\cos\theta}$ **C.** $\frac{1}{\tan\theta}$ **D.** $\frac{1}{\sec\theta}$

9. If cot θ = 0.85, find tan θ . **9.** ________

A. 0.588 **B.** 0.85 **C.** 1.176 **D.** 1.7

10. Find cos (−270°). **10.** ________

A. undefined **B.** −1 **C.** 1 **D.** 0

11. Find the exact value of sec 300°. **11.** ________

A. −2 **B.** $-\frac{2\sqrt{3}}{3}$ **C.** 2 **D.** $\frac{2\sqrt{3}}{3}$

12. Find the value of csc θ for angle θ in standard position if the point at (5, −2) lies on its terminal side. **12.** ________

A. $-\frac{\sqrt{29}}{2}$ **B.** $-\frac{2\sqrt{29}}{29}$ **C.** $\frac{\sqrt{29}}{5}$ **D.** $\frac{5\sqrt{29}}{29}$

13. Suppose θ is an angle in standard position whose terminal side lies in Quadrant II. If $\sin\theta = \frac{12}{13}$, find the value of sec θ . **13.** ________

A. $-\frac{5}{13}$ **B.** $-\frac{13}{5}$ **C.** $-\frac{12}{5}$ **D.** $\frac{13}{12}$

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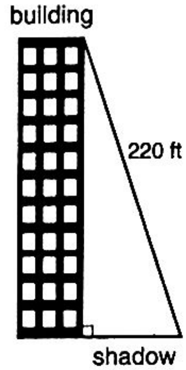
Chapter 5 Test, Form 1A (continued)

For Exercises 14 and 15, refer to the figure. The angle of elevation from the end of the shadow to the top of the building is 63° and the distance is 220 feet.



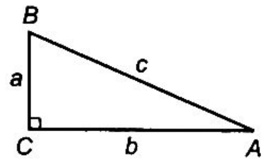
14. Find the height of the building to the nearest foot. **14.** ______
A. 100 ft **B.** 196 ft
C. 432 ft **D.** 112 ft

15. Find the length of the shadow to the nearest foot. **15.** ______
A. 100 ft **B.** 196 ft
C. 432 ft **D.** 112 ft

16. If $0° \le x \le 360°$, solve the equation $\sec x = -2$. **16.** ______
A. 150° and 210° **B.** 210° and 330°
C. 120° and 240° **D.** 240° and 300°

17. Assuming an angle in Quadrant I, evaluate $\csc\left(\cot^{-1}\frac{4}{3}\right)$. **17.** ______
A. $\frac{3}{5}$ **B.** $\frac{5}{3}$ **C.** $\frac{4}{5}$ **D.** $\frac{5}{4}$

18. Given the triangle at the right, find B to the nearest tenth of a degree if $b = 10$ and $c = 14$. **18.** ______
A. 44.4° **B.** 35.5°
C. 54.5° **D.** 45.6°



For Exercises 19 and 20, round answers to the nearest tenth.

19. In $\triangle ABC$, $A = 27° 35'$, $B = 78° 23'$, and $c = 19$. Find a. **19.** ______
A. 8.6 **B.** 9.2 **C.** 12.8 **D.** 19.4

20. If $A = 42.2°$, $B = 13.6°$, and $a = 41.3$, find the area of $\triangle ABC$. **20.** ______
A. 138.8 units^2 **B.** 493.8 units^2 **C.** 327.4 units^2 **D.** 246.9 units^2

21. Determine the number of possible solutions if $A = 62°$, $a = 4$, and $b = 6$. **21.** ______
A. none **B.** one **C.** two **D.** three

22. Determine the greatest possible value for B if $A = 30°$, $a = 5$, and $b = 8$. **22.** ______
A. 23.1° **B.** 53.1° **C.** 126.9° **D.** 96.9°

For Exercises 23-25, round answers to the nearest tenth.

23. In $\triangle ABC$, $A = 47°$, $b = 12$, and $c = 8$. Find a. **23.** ______
A. 6.3 **B.** 8.7 **C.** 8.8 **D.** 18.4

24. In $\triangle ABC$, $a = 7.8$, $b = 4.2$, and $c = 3.9$. Find B. **24.** ______
A. 15.1° **B.** 148.7° **C.** 78.9° **D.** 16.2°

25. If $a = 22$, b = 14, and $c = 30$, find the area of $\triangle ABC$. **25.** ______
A. 33 units^2 **B.** 121.0 units^2 **C.** 130.2 units^2 **D.** 143.8 units^2

Bonus The terminal side of an angle θ in standard position coincides with the line $4x + y = 0$ in Quadrant II. Find sec θ to the nearest thousandth. **Bonus:** ______
A. −0.243 **B.** −4.123 **C.** 0.243 **D.** 4.123

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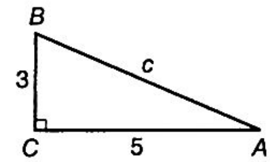
Chapter 5, Quiz A (Lessons 5-1 and 5-2)

1. Change 47.283° to degrees, minutes, and seconds. 1. ________
2. Write 122° 43′ 12″ as a decimal to the nearest thousandth of a degree. 2. ________
3. Give the angle measure represented by 2.25 rotations counterclockwise. 3. ________
4. Identify all coterminal angles between −360° and 360° for the angle 480°. 4. ________
5. Find the measure of the reference angle for 323°. 5. ________
6. Find the value of the sine for ∠A. 6. ________
7. Find the value of the tangent for ∠A. 7. ________
8. Find the value of the secant for ∠A. 8. ________



Exercises 6–8

9. If $\cot \theta = -\frac{2}{3}$, find $\tan \theta$. 9. ________
10. If $\sin \theta = 0.5$, find $\csc \theta$. 10. ________

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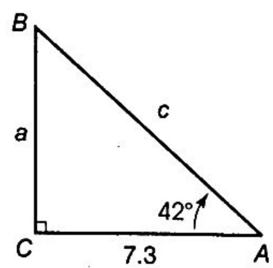
Chapter 5, Quiz B (Lessons 5-3 and 5-4)

Use the unit circle to find each value.

1. sin (−90°) 1. ________
2. csc 180° 2. ________
3. Find the exact value of cos 210°. 3. ________
4. Find the exact value of tan 135°. 4. ________
5. Find the value of $\sec \theta$ for angle θ in standard position if the point at (4, −5) lies on its terminal side. 5. ________
6. Suppose θ is an angle in standard position whose terminal side lies in Quadrant III. If $\tan \theta = \frac{5}{12}$, find the value of $\sin \theta$. 6. ________

Refer to the figure. Find each value to the nearest tenth.

7. Find a. 7. ________
8. Find c. 8. ________



A 100-foot cable is stretched from a stake in the ground to the top of a pole. The angle of elevation is 57°.

9. Find the height of the pole to the nearest tenth. 9. ________
10. Find the distance from the base of the pole to the stake to the nearest tenth. 10. ________

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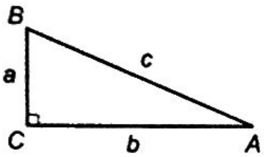
Chapter 5

Chapter 5, Quiz C (Lessons 5-5 and 5-6)

1. If $0° \leq x \leq 360°$, solve: $\csc x = -2$. **1.** ______________

2. Assuming an angle in Quadrant I, evaluate $\tan\left(\sec^{-1}\frac{13}{5}\right)$. **2.** ______________

3. Given right triangle ABC, find B to the nearest tenth of a degree if $b = 7$ and $c = 12$. **3.** ______________



Find each value. Round to the nearest tenth.

4. In $\triangle ABC$, $A = 58° 21'$, $C = 97° 07'$, and $b = 23.8$. Find a. **4.** ______________

5. If $B = 29.5°$, $C = 64.5°$, and $a = 18.8$, find the area of $\triangle ABC$. **5.** ______________

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Chapter 5

Chapter 5, Quiz D (Lessons 5-7 and 5-8)

1. Determine the number of possible solutions for $\triangle ABC$ if $A = 28°$, $a = 4$, and $b = 11$. **1.** ______________

Find each value. Round to the nearest tenth.

2. For $\triangle ABC$, determine the least possible value for B if $A = 49°$, $a = 12$, and $b = 15$. **2.** ______________

3. In $\triangle ABC$, $B = 32°$, $a = 11$, and $c = 2.4$. Find b. **3.** ______________

4. In $\triangle ABC$, $a = 3.1$, $b = 5.4$, and $c = 4.7$. Find C. **4.** ______________

5. If $a = 28.2$, $b = 36.5$, and $c = 40.1$, find the area of $\triangle ABC$. **5.** ______________